GEOPHYSICAL PROSPECTING

Papers and Discussions Presented at Meetings Held at New York, February, 1928, and at Boston, August, 1928 v. 81

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PREFACE

The sudden realization of the possibilities for practical service from geophysical methods of prospecting for ores or other valuable bodies has recently aroused very general interest on the part of mining and oil men, geologists and physicists. Although few of the methods now actively practiced are entirely new, relatively little use was made of them until within the last few years. Their development, stimulated by success in a number of notable cases, has been most rapid. Their range of application has been increased, new methods have been devised to utilize physical properties of rocks and ores previously neglected, and better results have been gained by old methods with improved instruments.

The basic principles underlying the work are for the most part relatively simple. When a particular physical field is influenced by a known body of specific properties, the distortion to be expected can generally be estimated with reasonable accuracy from theoretical considerations or experimental evidence. The practical value of the work, however, depends on the reverse operation; *viz.*, the estimation of the properties of a concealed body from the measured distortion of a known field. This, obviously, is a far more difficult problem, and an exact solution, in any except the simplest cases, is generally to be found only when intersecting lines of evidence are available from other independent observations and from geological relations.

Geophysical methods of prospecting bring to economic geologists not only new opportunities but also new responsibilities. The interdependence of the fields is too apparent to need elaboration. Measurable disturbances of a physical field are very rarely caused by the valuable material itself. The evidence generally relates merely to geologic features, such as changes in character of rock, structure of formations or mineral associations, which may be of significance in terms of ore or oil, and the physicist is dependent, therefore, on geological reasoning for final interpretation of his results. The new and convenient methods of applied geophysics, however, have provided the geologists with means of enlarging the range and accuracy of many observations, and they should not fail to take full advantage of them, not only in commercial work, but in purely scientific investigations.

The demand of geophysicists for exact data concerning rocks and rock bodies has caused geologists to take stock of their supply of information. Valuable material is on hand and readily available, but more definite knowledge is needed of many critical properties of rocks and ores, such as elasticity, electrical conductivity, and even density, not merely in

laboratory specimens but in formational units and under various geologic conditions. This field for research is not new but the needs of applied geophysics bring it immediate importance and should prove to be a healthy stimulant.

The aim of the Committee on Geophysical Methods of Prospecting, in presenting this first collection of papers, has been to make available to the mining public sound information concerning the principles underlying the various methods now in active use, to provide examples of current practice and results, and to afford opportunity for specialists to publish recent achievements and to receive the benefit of criticism and discussion by their colleagues.

In selecting papers for the volume, the diversity of interest among our members was recognized; consequently, if a certain unevenness in quality is noted, it should be kept in mind that the various articles are not all intended for the same audience. A few frankly address the specialist, and use appropriate shorthand in their expressions, whereas others aim to present phases of the subject to mining men who desire merely an understanding of general principles of the methods to afford a basis of judgment of their economic value in particular problems.

The response for contributions of papers and discussion from members and others who are active in geophysical prospecting has been most generous, and the thanks of the committee are due to them for the valuable time they have given, often in the midst of active professional work. The frank interchange of ideas and scientific information on the part of many has been gratifying, and it is hoped that it will continue in the future, in spite of the present competitive conditions. Complete confidence of the mining profession in the value of geophysical prospecting is likely to be gained only if all technical and scientific aspects of the work are openly discussed.

The response made to the effort of the Committee on Geophysical Methods of Prospecting to bring together existing data as to principles, methods and results, and to stimulate discussion and informed criticism, has been most gratifying. The Committee was established in 1927 and offered its first program in February, 1928. So long was the list of papers and so active the discussion, that an extra session needed to be provided beyond the two first planned. More than 200 members and guests took part. With this evidence of keen interest, it was decided to make geophysics the major subject at the Boston meeting, held August 29, 30, 31 and again with excellent support. At Boston an exhibition of instruments and methods as well as technical sessions was organized and widely attended. The papers and discussions of these two meetings form the basis of this report. A third program has already been organized and will be offered at New York on February 20, 1929, in the course of the next annual meeting of the Institute.

Donald H. McLaughlin
Chairman, Committee on Geophysical
Methods of Prospecting

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Geophysical Exploration for Ores

By Max Mason,† Chicago, Ill.

(New York Section Meeting, October, 1927*)

In 1923 a Western mining company was experimenting with the device of an inventor designed to locate buried ores by radio. Because the progress was slow and the results were confusing, the company began to doubt the usefulness of the method and invited me to review the whole question of the application of physics to ore detection. I was fortunate in securing the cooperation of several physicists who had worked with me during the war on the problem of submarine detection. This group has been increased and has been studying the problem of physical exploration for ores since that time.

It was evident that the problem is a big one, with many angles of approach. Its general features may be set down, in rough outline, as follows: The soils, rocks and ores hidden beneath the surface differ one from another in many respects. These differences may be made to furnish a clew for the physicists working from the surface. To bring these differences into action the physicist creates some kind of an effect which penetrates into the ground and is distorted and reflected when it meets boundaries between different sorts of underground structure. In other words, he sends a message down, and the rocks and ore send back signals in reply. In picturing this process, we should not regard the ground as dense and impenetrable. The kind of messages used pass through the earth about as readily as a sound wave travels the air.

If, then, the fundamental procedure is to shout down questions in the hope that an orebody will hear and answer back to us, it is clear that a large part of the expert's study must relate to the kind of questions best suited to the temperament and intelligence of orebodies. It will be easier for the ore to reply to some of our questions than to others, and it is for us to find the right questions. In certain cases, we are spared the necessity of using a messenger, because nature has already provided one. For example, we already have a terrestrial magnetic field which automatically and continually conveys messages from underneath. We also have available the earth's gravitational field which furnishes us

* This paper was presented at the joint meeting of the New York Sections of the American Institute of Mining and Metallurgical Engineers and the Mining and Metallurgical Society of America at the Machinery Club, New York, Oct. 27, 1927 and was also presented at the Annual Meeting of the A. I. M. E. in February, 1928.

† President, University of Chicago.

information any time we care to record it. In other cases (which are in point of fact the most important ones) the ore is too polite to talk unless spoken to, and we therefore have to stimulate it with an artificial field. Whether natural or artificial, the utilization of the appropriate kind of exciting field is of the first importance in the process of physical ore detection.

It is equally apparent that we must be able to translate the message which the earth sends back to us in order that it may tell a story. This process of translation or diagnosis will often be complicated, for every part of the underground structure sends back its own answer, and we must

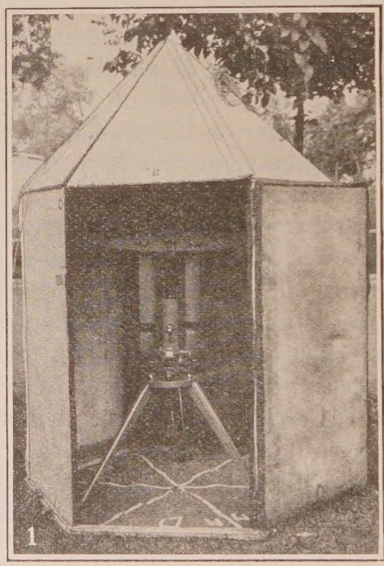

FIG. 1.—EÖTVÖS BALANCE IN SPECIAL HEAT-INSULATING HUT.

separate one from the next and bring order out of a confusing babble of replies. This interpretation of the orebody's message is perhaps the most difficult part of the problem. One must know in what language the ground will speak, how to distinguish the Chinese of the surface soils from the Greek of the ore. It was obvious from the beginning of our work that the interpretation of messages was going to play a major rôle in ore detection, and our experience during the subsequent years has only emphasized this fact.

From the beginning of our work we adopted a policy of emphasis on the fundamentals of the project. We desired to explore all the possible paths which might reasonably be hoped to lead to results of practical significance. This involved a review of the prior work on geophysics, a study of theory and apparatus, and tests and comparisons in the field

and over known orebodies. We attempted to apply the fundamentals of physics in an unbiased investigation of all possible physical means of searching for ores. In the four years which have been devoted to this

FIG. 2.—TWO TYPES OF PORTABLE MAGNETOMETERS.

work important progress on both the theoretical and practical aspects of the problem has been obtained, and as a result the whole situation has become greatly clarified in our minds. I hope I may convey to you

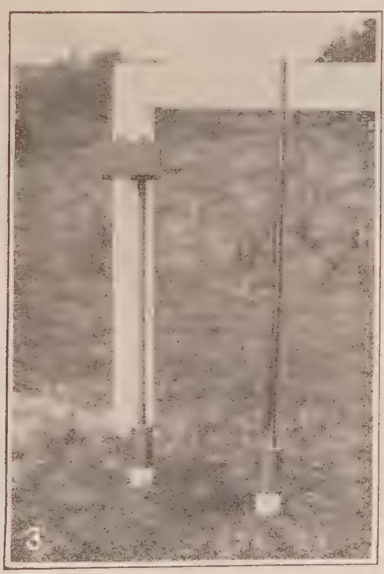

FIG. 3.—EQUIPMENT FOR SELF-POTENTIAL SURVEYS.

our conception of the work of the physicist in exploration, of the possibilities and limitations of his tools, of the manner in which practical success is being achieved, and of the present status of this development.

The acoustic method—which is, broadly speaking, the study of the echoes reflected by orebodies from incident sound waves—early proved rather disappointing. Perhaps the future will bring success with methods which work on a somewhat similar principle (the seismic methods), but the difficulties in such applications are of a serious nature. The seismic and acoustic methods depend upon the distortion, reflection, or change in velocity of an artificially produced small earthquake wave, or of a sound wave. In the neighborhood of most orebodies the rock conditions are complicated by fracture zones, by faults or folds, and, in general, by many irregularities. Such conditions will usually produce greater distortions in a seismic wave than the ore itself. In districts such as the oil

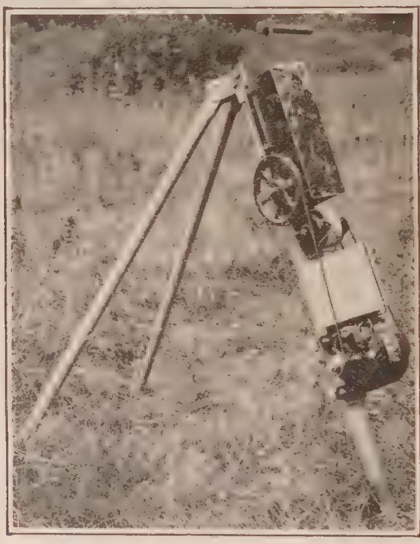

Fig. 4.—Hand-driven electric alternator for surface potential method.

fields of Texas, where the structure is simple, seismic methods have been successful in the discovery of oil domes, but extensions of these methods to ores have not yet been of practical significance.

Another physical method is the gravitational one, which makes use of the earth's natural gravitational field. Each cubic foot of rock, ore, or soil exerts a gravitational pull which is proportional to its weight, but which rapidly decreases as the distance from the observer becomes greater. Distances being equal, the heavy ores exert a stronger attraction. The instrument which is now most widely used in gravitational measurements is the torsion balance of Baron Eötvös. The instrument is called a "balance" for the following reason: It is so constructed that it gives no response when the subsoil beneath it is uniform in density or where successive horizontal layers are uniform. If there is an excess

or deficiency of density on one side or the other, the instrument becomes unbalanced and gives a reading. The Eötvös balance is of such extremely high sensitivity that it is necessary to set it up over ground which has previously been smoothed for a radius of about 6 ft. Moreover, a topographic survey of the neighborhood must be made and the effect of near-by surface irregularities computed and corrections made therefor. The balance is relatively expensive and slow in application, for one can obtain observations at only four stations per 24-hr. day. Its usefulness is, therefore, greatly restricted and is confined chiefly to the confirmation

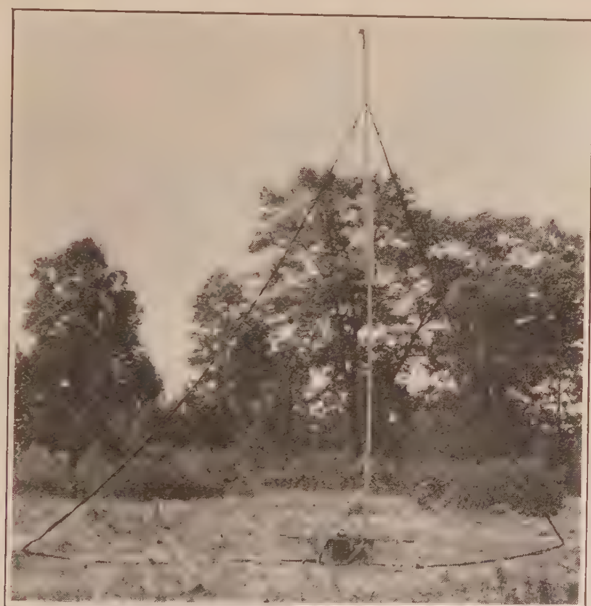

Fig. 5.—Sending equipment used in inductive method. Note the triangular loop.

and extension of the results of other methods, or for special purposes where relatively simple large-scale structures are of interest. It will thus be clear to you that the success of the Eötvös balance in such districts as the oil fields of Texas cannot be duplicated in most mining regions. We have made tests with the balance in two mining districts and believe that it has useful, though quite restricted, applications in ore work.

In common with the gravitational method, the magnetic method is concerned with the reception and interpretation of messages which nature automatically supplies. It is the oldest and most familiar application of physics to mining exploration, but in recent years it has become of greater importance than ever before. The pocket dip needle of the

geologist has found increasing application in the rapid and cheap survey of areas showing pronounced magnetic characteristics. Where the magnetic variations in the earth's field are small and where increased accuracy and sensitivity are desired, field instruments are now available which far surpass, in reliability, speed, and accuracy, those of a dozen years ago. The discovery and interpretation of concealed intrusives, faults, and structures in general, by magnetic methods, as well as the delineation of magnetic orebodies, have all undergone pronounced improvement as a result of the present increased interest in geophysics.

Fig. 6.—Receiving equipment used in the inductive method.

In many cases, magnetic methods offer a more powerful tool than any other in the exploration of hidden geological structures.

Besides the magnetic methods, the electrical methods alone are suited to reconnaissance exploration for metallic minerals, and I assume that you are chiefly interested in these methods. The electrical method which I shall first describe is similar to the magnetic and the gravitational methods, in that no artificial source is required to provoke a response from the ore. This is the self-potential method about which Mr. Kelly spoke to your society last May. Mr. Kelly explained, you will recall, that orebodies, particularly sulfides, undergo oxidation and that the upper zones are usually in a state of activity different from that of the

lower ones. The difference in chemical activity leads to a current flow tending to neutralize the difference of potential. This current spreads through the ground for considerable distances, but it is all supposed to pass through the orebody, just as all the current from an ordinary dry cell passes through the cell, regardless of the complications of the external circuit. In fact, the dry cell is the best simple analogy to an orebody supplying current in the manner mentioned by Mr. Kelly. By tracing the current it should be an easy matter to find the ore. The apparatus used in tracing the current flow is called the self-potential apparatus, because it measures the potentials naturally associated with the current flow.

The best early work with the self-potential method was done by Carl Barus in 1882 at the Comstock Lode. He applied the method almost in its present form. About 1913 Prof. C. Schlumberger, of Paris, revived interest in self-potential work and began a series of careful and extensive experiments in its theory and application, which contributed much to the knowledge of this method.

Other Electrical Methods

All other electrical methods require an artificial exciting means in order to elicit a response from ores. Such methods may be divided into two main types, the surface potential and the inductive.

The exciting means in the surface potential method is the flow of electric current through the soil. A dynamo may be used as the source; current from it is led into the earth at one point, and out at another, by means of grounded metal stakes called electrodes. The underground current flow follows the lines of least resistance, and tends to concentrate along any conductors, such as orebodies, which may be in the neighborhood. The occurrence of underground concentrations is indicated by the nature of the current distribution at the surface.

Professor Schlumberger made creditable contributions to the study of artificial current distribution at the surface as influenced by ores. Hans Lundberg and his associates were also active pioneers in the development of this method, and introduced the use of straight extended electrodes, grounded at intervals along their length, instead of point electrodes. The current was thereby introduced into the ground evenly along the length of the electrode. The two electrodes roughly bounded the region of exploration, and were often very long, say 3,000 ft., and separated nearly as far, say 2500 ft. The current paths at the surface between the long parallel electrodes are, in the absence of disturbing factors, straight and parallel; hence their distortions are readily apparent. Mr. Lundberg's extended electrode system has received wide application and has created much interest in electrical prospecting.

The Inductive Method

The other, or inductive type of electrical method, includes most of those special methods now in use. In this method an alternating magnetic field is created, which is similar to the natural magnetic field of the earth, except for the fact that it reverses periodically at a high frequency. This frequency is usually of the order of 1000 cycles per second, or in the acoustic range, but it may be higher—as high as a radio frequency. The magnetic field penetrates the rocks and soils of the earth nearly as readily as it does the air above. However, when it encounters a conductor, let us say an orebody, a new magnetic field of the same frequency and general characteristics is emitted by the conductor. This new field radiates from the conductor in all directions in the same general manner as did the original one from the original source. It is called the secondary, or induced magnetic, field, and we therefore refer to this general method as the inductive method of electrical prospecting. The secondary field which is created through the presence of the ore constitutes its answer to our question, and is of course the one we wish to measure and study. At any point, however, both the primary and secondary fields are necessarily coexistent, and all we can do is to measure the total and then estimate what amount is due to the ore.

H. R. Conklin and others deserve credit, both for the early recognition of the possibilities of this method and for contributions toward its practical development.

Advantages of the Inductive Method

The inductive method has the following basic advantages over the other electrical methods:

1. It requires no grounded electrodes. Good electrical contact with the ground is difficult in rocky and barren regions, on ice-covered lakes and in deep snow, and in dry sand and gravel. The application of the inductive method is independent of such surface conditions.

2. A related advantage is important. With the surface-potential method, current flow must, of course, reach the ore, and be concentrated in it, in order to produce the desired distortion at the surface. If highly insulating layers exist below relatively conducting surface soils, nearly all the flow will be shut off from the deeper regions and be confined to the conducting surface soils. The ore will get no chance to indicate its presence. A similar situation may occur in the use of the self-potential method, for obviously the normal field of current flow as generated by the oxidization of the ore will be much modified by an intervening insulator. However, in the inductive method, the magnetic field penetrates an insulating layer nearly as readily as air and, therefore, the intervention of an insulating region is unimportant.

3. Both the surface potential and inductive methods depend primarily on the conductivity of the ore. This dependence is, however, of a fundamentally different type in the two cases. In the case of the surface-potential method it may be said that no absolute scale for the measurement of conductivity exists. To illustrate, let us assume a definite topography with the two grounded electrodes in position. Now, imagine the conductivity of every portion of the subsoil multiplied by the same factor, say one thousand. The current paths will remain unaltered by such treatment.[1] In other words, the current distribution is determined by the *ratio* of the conductivities of adjacent regions, not by the absolute values. Moreover, the influence of this ratio upon the distribution undergoes a saturation effect, in that higher values of the ratio produce only slightly greater concentrations. When the conductivity ratio exceeds 10 or 20, about 90 per cent. of the possible influence is realized, and higher values are of no appreciable avail in strengthening the indications caused by the conductor.

In practice, it is unfortunately true that ratios of conductivity of 10 or more are obtained between neighboring soils. The chief cause of such difference is moisture content; which in turn depends upon type of soil and upon topography. These differences, then, are about as effective in their influence upon the surface potential method as a difference of 10,000-fold between ore and soil. In the inductive method this difficulty is absent, for, in surface covers of high electrical resistance, the terrain response depends nearly directly upon the *absolute* conductivities and not upon the ratio of adjoining portions. Therefore, regardless of the value of the ratio, the entire response will be small whenever the conductivities are slight. Fortunately, the conductivity of cover is usually small, and thus the influence of topography is much less pronounced. The inductive method is thus better able to distinguish between the enormous conductivity differences between ores and cover and the far lesser variations occurring in barren land.

Of the six methods which I have outlined, we have been studying and using all except the seismic. Although our major interest has been in the development of the inductive method, we have found practical value in all the others. In most of our surveys we use the self-potential, magnetic, and inductive methods in combination with one another. The gravitational and surface-potential methods are reserved for special conditions, and as special checks.

I think you will now be interested in seeing pictures of applications of the methods I have described.

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[1] This is not rigorously true, but holds for the conductivities and frequencies customarily used in practical ore-detection work.

Fig. 1 shows the Eötvös gravitational balance in its special heat-insulating hut. This equipment, as I have said, is limited in speed to about four stations per 24-hr. day.

Fig. 2 shows two types of portable magnetometers for the measurement of the terrestrial magnetic field. One is the Askania balance, the other the Gepege. The Askania balances are very satisfactory. They are reliable and rapid in use, and we often average 80 observations per day with them.

Fig. 3 shows equipment for self-potential surveys. The two electrodes are porous cups, filled with copper sulfate solution. The potential difference existing between the two electrodes which are placed on the ground is measured by a potentiometer, which is mounted above the left-hand electrode. You will observe how simple and portable is this equipment.

Fig. 4 shows a hand-driven electric alternator for supplying current to the ground for the surface-potential method.

Fig. 5 relates to the inductive method, which is our primary one in reconnaissance work. It shows the power source (at center), which is a light gas engine set, and the triangular sending loop. The entire equipment is readily portable by two men.

Fig. 6 shows the receiving equipment used in the inductive method. It consists of the rotatable coil mounted and leveled on the tripod, the amplifier in the box and the head phones. The direction of the magnetic field is measured with this equipment. For the measurement of intensity a more elaborate form of apparatus is required.

I do not wish to confuse the situation by illustrating equipment which is either in the experimental stage or suited only for specialized purposes. The equipment for the methods which I have mentioned is not cumbersome and complicated. It is of strongly constructed type, and capable of rapid use under most field conditions.

I shall now touch upon our customary type of survey, employing the magnetic, self-potential and inductive methods. These particular three methods are chosen, when conditions are suitable, because they are rapid and relatively economical, and because they are entirely independent of one another in the physical properties upon which they depend.

In the first place, it will do no harm to recall that the geologist must select and limit the area desirable for electrical exploration. The work thus begins with geology. Furthermore, it needs geology during its course in the field. Lastly, after the data are all taken, the important work of interpretation of results and of forming a logical and self-consistent picture requires all the aid geology can give. The conclusions must fit the known geological, as well as physical, facts.

In some cases the mining companies supply all the necessary geological assistance. Often, however, it will be expedient for geologists associated

with physical exploration groups to assume this responsibility. Such a course has the advantage of putting the whole exploration problem into the hands of one group, experienced in all phases of the work, and accustomed to studying together the various kinds of problems which arise. I think the advantages of such cooperation need no recommendation. Prof. Warren Meade, of the University of Wisconsin, is directing the geological phases of our work.

The area chosen for survey is laid out in long straight parallel traverses separated in conformity with the size of possible orebodies, such that at least one traverse will, in all probability, pass above any body large enough for commercial importance. Often a spacing of 500 ft. is suitable; sometimes it is much less. The observations are made at 100-ft. intervals along the traverse, and at each station readings with the three independent methods are usually taken—the inductive method, the self-potential method, and the terrestrial magnetic method. Our usual physical exploration thus involves brush-cutting, the transit survey of the traverses, and the chaining of the observing stations, together with the operation of the inductive method by two observers; of a magnetic balance by one observer, and of the self-potential apparatus by one observer and a helper. In all, excluding axemen, such a survey will require a personnel of five engineers and two assistants.

We have attempted to improve the speed and efficiency of exploration in the use of these three methods. In some hundred miles of traverses in the Sudbury nickel basin, near Sudbury, Ont., we have averaged 72 acres of reconnaissance work per day, with each of the three methods. The work was done in a period of about four months, and this average of 72 acres per day is the gross daily average for some 7000 acres, including time lost because of weather conditions and other unfavorable circumstances. At times 20,000 ft. of traverse have been done per field day with the inductive method, and this through heavily covered and rough country. This corresponds to about 250 acres per day. The methods may therefore be applied with speed.

The Physicist's Work

I have outlined the basic problem of physical exploration and described to you methods by which practical work may be carried out. A critical valuation and understanding of the real nature of these methods can be obtained only through the scientific studies to which they owe their birth and development. I should now like to view these scientific studies through the physicist's eyes, and to show you the means by which practical methods are evolved and by which progress in the basic problems of physical exploration is being attained.

In outline, we have seen that the physicist's problems are of two kinds —those of excitation and those of reception and interpretation. We are

not here concerned about the details of apparatus for producing or receiving a field. The real limitations of ore detection lie in indications of suitable strength as compared with other disturbances similar in kind. We therefore ask: Granting perfect apparatus, what can physics do, and what can't it do, in detecting ores as they really occur?

There are three basic ways by which physics gathers information about the detectability of ores: (1) The mathematical method; (2) the laboratory and experimental method; (3) the knowledge gained from widespread practical field experience. Each is of prime importance.

Mathematics Applied to Detecting Ores

The first of these methods, the mathematical, tells us exactly what occurs when the source field which we create meets a postulated orebody, or other change in structure. This would appear to be all we want to know, but unfortunately there are difficulties. We find the mathematician cannot supply solutions except for a few simple cases. When we ask for the answer he says he cannot give it but can supply something nearly as good. He then changes our actual problem around so that he will be able to solve it. A lump of massive ore becomes to him a smooth sphere, the irregular cover becomes homogeneous, the hills and valleys are smoothed to a level surface. Or if the ore seems to occur as a sheet, it takes the form of an extended plane. Other shapes of bodies are approximated by various kinds of ellipsoids. The geometry must always be simplified.

Although the solutions available pertain only to such idealized cases, much useful information results from them. For they at least show us the best that may be hoped for under simple conditions, and how strong and of what character the message sent to us by certain simple orebody shapes will be. Moreover, we may approximate the answers for more complicated problems by a judicious combination and manipulation of the results for the simple cases.

In dealing with these idealized problems in ore detection, the physicist becomes impressed with the fact that his solutions pertain to more general problems than the particular one with which he started. There is, in fact, an interesting similarity between the gravitational problems, the terrestrial magnetic ones, the direct current ones, and electromagnetic or inductive ones. I have no doubt the same similarity can also be discovered in certain seismic applications. Thus the physicist is led to value a broad point of view in approaching the subject. It may be said, indeed, that the basic study in geophysics is the propagation and distortion of physical fields, whether they be gravitational, magnetic, or electromagnetic. To make my meaning clearer, let us consider the detection of a sphere by the various methods. We arrive at the following results: A

magnetic sphere in the earth's magnetic field, a conducting sphere in the field of flow of a direct electric current, or a conducting sphere under the inductive influence of a magnetic field, all give the same kind of mathematical solutions. The gravitational effect of a heavy sphere on the Eötvös balance is, moreover, closely related to the effects in the three cases.

Unfortunately, the inductive method of exploration, which we consider the most valuable one, is also the most difficult of mathematical analysis. The physicist at present has to make greater simplifications than he wishes, in order to get problems he can solve. However, he uses these answers, when obtained, chiefly as a guide book to show the general characteristics of the case, and employs additional aids, as we shall see, which make the situation better than it may appear.

I have said that the mathematician can predict the response of certain simple types of ore deposits to the fields we choose to impress upon them; and that these solutions may often be extended to tell us approximately what will happen under more complex conditions. I must now, unfortunately, mention a difference between such problems and the real ones of a survey. For we have been putting the question thus: Granted a buried sphere, or other deposit, can its response be predicted? Answer, "Yes." But we should have put the question other end to: Granted the field party turns in an observed response, can we determine exactly what caused the observed data? Answer, "No," strictly speaking, for we cannot solve the problem backward and so find the cause of the results. It is fundamental in electrodynamics that, to find out what is inside a region, one has to get measurements all around the boundary surfaces. Of course, the ground surface alone is accessible in electrical surveys. We cannot measure the fields below and at the sides of the ore.

How the Physicist Proceeds

In practice, the physicist is forced to proceed as follows: He obtains, digests and catalogs as many of the mathematical solutions of idealized cases as possible, being careful to check these solutions by appropriate experiments. Then he tries to fit the observed field data to the typical results in his file records, and draws approximate conclusions as to the shape and position of the ore. If he is led to believe the indications are caused by several different and separate conductors, he has to consider them separately by a process of subtraction.

In simple conditions his predictions will be closely correct, but it is always necessary to make some assumptions or guesses in the process of interpretation. Of course, geological knowledge, experience and common sense are at the bottom of such guesses. For example, the field data may indicate a deposit roughly spherical in shape. Then, if the physicist assumes that the topographical effects are either known or negligible,

that the cover is uniform and disturbing factors are absent, he may accurately locate the center of the sphere from theory. It would also be useful to know how large a sphere had been discovered. Unfortunately, this question cannot be accurately answered. Before the physicist can do so he has to decide what the conductivity is, for radius and conductivity jointly determine the magnitude of the response.[2] One might hope that this difficulty would be clarified by the use of different frequencies for the impressed field, but unfortunately this will not help.

In brief the situation is this: The form of the response field at the surface will tell us that the source of the disturbance is a sphere, and also tell where its center is. Determination of the radius and conductivity is not unique, and estimates are therefore without logical basis. Although rigorous analysis is without avail, we may guess a reasonable value for the conductivity, and thereby obtain an estimate of the radius.

My purpose in spending so much time on this simple problem of the sphere is to show you that, even when conditions are exceedingly simple, there are difficulties in telling *all* about the case. You can imagine how much more complicated interpretations must become when several bodies are present or when shapes are irregular and structure is complicated. Some operators of electrical exploration methods offer to do things for us which are entirely outside the bounds of known science. They are the spiritualists of geophysics, and obtain messages which no others can decipher. The honest and informed physicist must admit the limitations of his science.

Under shallow cover, we may expect reasonably close delineation of the position of ores, but it will always be difficult to distinguish between the conductivity of the substance and the total amount present. Naturally these enter together, and it is hard to separate them. As the depth of cover increases, the determination of ore boundaries becomes more and more uncertain.

Use of Models

The second, or experimental, means of studying the basic detectability of ores is one which has proved indispensable in many other applications of physics. I refer especially to small-scale experimentations with models. We all know that the use of models in wind tunnels and in towing basins, for example, has contributed untold information to the science of the airplane, and to naval architecture. I believe that the use of models will play an important rôle in ore exploration. But before we can use models intelligently we must know the equations and laws which apply. The model is not simply a duplicate of the larger structure made on a reduced scale, as is sometimes thought to be the condition in

[2] It is theoretically possible to determine the conductivity and radius separately, but, practically speaking, sufficient accuracy and certainty are not yet attained.

ore-detection work. Account must be taken of the new values of the electromagnetic quantities when the scale is changed. In the towing of a ship model, or the test of an airplane wing in a wind tunnel, the resistance is not directly obtained through multiplying by the model scale. Similarly in ore models, where one factor, such as size, is changed, the other factors must be altered in correspondence. Models are models, but you must know how to work them.

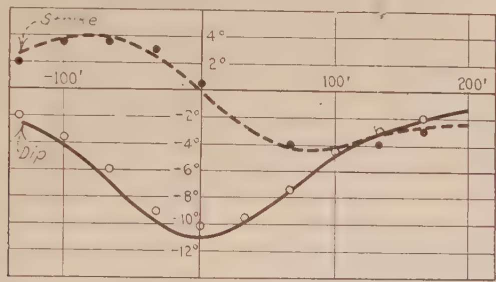

Fig. 7.—Curves showing the theoretical distortion of an electromagnetic field caused by a conducting sphere.

Models, when correctly used, serve to tie together in a valuable manner the exact predictions of mathematical theory, the extensions of theory based on judgment, and the actual experience in the field. They contribute new information which might easily pass unnoticed through studies of theory and practice alone.

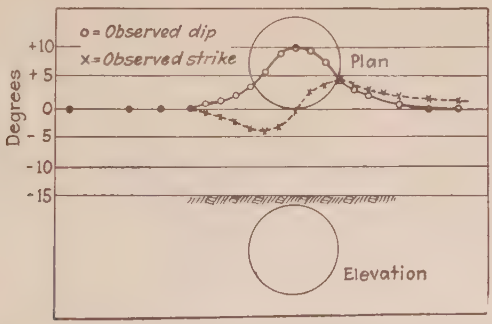

Fig. 8.—A model experiment, showing the distortion of an electromagnetic field caused by a conducting sphere.

I think I can make their value clearer by means of illustrations. We will again take the example of a conducting spherical orebody in the presence of an inductive field such as we use in practice. The mathematics for this case predicts the sort of curves shown in Fig. 7. Here the sphere is assumed to be 100 ft. in radius, with center under 120 ft. of cover, composed of solid pyrite. We suppose that our line of traverse passes, not over its center, but directly over its extreme outer edge, as

shown in the plan. The energizing source is assumed to be far away—2000 ft. to the left, and of 1000 cycles frequency. Then theory predicts the following distortion in the incident magnetic field along the traverse. The field is normally horizontal, but is caused by the sphere to dip, as shown by the solid curve, until it obtains a maximum of 11° opposite the center of the sphere. The dotted curve shows the change in the strike of the incident field. This is less, amounting to about 4°, but it reverses over the sphere, so the total change is 8°, or nearly as great as on the dip. Besides the changes in direction, we will have a change in intensity of the field. In the particular traverse chosen we will get relatively small changes in intensity, but by another choice of position the intensity changes become much more apparent. In the traverse shown, a maximum variation of only about 3 per cent. is obtained.

One other factor needs to be considered, and then we have told the entire story of the response of the given sphere under the given excitation. This is the phase change which has been often mentioned in connection with electrical prospecting. However, in the case of our present problem, the phase shift is small—of about 4°—and is substantially constant in value at all points within a mile of the sphere. In this particular case the phase shift cannot be regarded as the primary indication.

Now let us see whether our theoretical solution is correct. We make up a model sphere of about 3 ft. diameter and we make small-scale measurements of the strike and dip of the source field in its neighborhood. The results of these measurements are shown in Fig. 8. It is clear that the curves resemble the theoretical ones. To show how good the agreement is, the experimental points are replotted to a large scale, for comparison with the predictions. In Fig. 9 the solid disks are experimental settings on the strike, and the circles those for the dip. The agreement is within 30 minutes at most values and is within the accuracy of the work. We see then that we are working the theory successfully and the models correctly. We have not yet, however, connected this subject with a real orebody, and I think you will be interested in seeing this next step. As an example, we shall use the Falconbridge or Longyear nickel body, in the Sudbury, Ontario, basin. It has been well drilled, and is known to be a nearly straight long sheet, of a dip of about 90°, and probably of great extension in depth.

At the Edison Kettle hole, its shape is shown in plan and section at the right of Fig. 10, as drawn by Hugh Roberts from the drill data. A model of this orebody was made on a scale of about 300 to 1, and of a section shown at the left. This is roughly a mean between the two sections indicated by the drill. The cover is 60 ft., the thickness of the upper lip 40 ft. and at the widest part 120 ft. With this model, the distortion of the incident inductive field was measured, and the curves shown were obtained. The solid curve represents variations in dip; the dotted

one those in strike. Actual distortions obtained in the field are shown by the circles. It is seen that the model results and the actual data agree within a few degrees in most places. Moreover, by the model curves, a position for the quartzite-ore contact is indicated which agrees within 5 ft. of the result obtained by Mr. Roberts. When we consider that

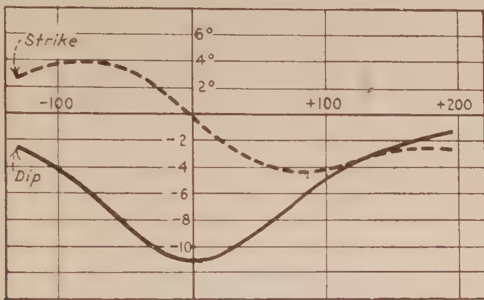

FIG. 9.—COMPARISON OF THEORETICAL AND EXPERIMENTAL DISTORTION OF AN ELECTROMAGNETIC FIELD CAUSED BY A CONDUCTING SPHERE.

the real orebody probably departs somewhat from the section assumed, the agreement is surely gratifying.

Fig. 11 will illustrate another valuable rôle of the model. This illustration is much like that in Fig. 10, and represents a small continuous orebody of similar type 500 ft. long, 100 ft. deep, and under 40 ft. of cover. Now let us suppose the ore were disseminated through a thin, perfectly

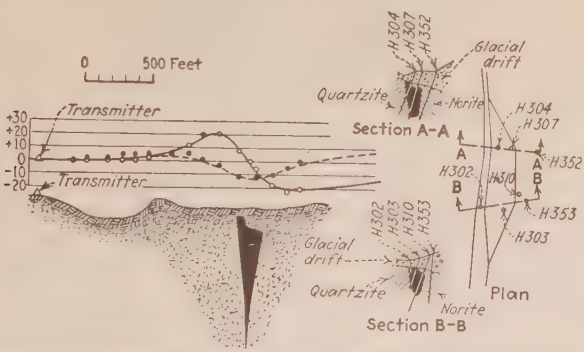

FIG. 10.—COMPARISON OF RESULTS OBTAINED FROM A MODEL WITH FIELD DATA GAINED IN APPLYING THE INDUCTIVE METHOD OF DETECTION TO THE LONGYEAR OREBODY.

insulating intrusive dike, and that the particles failed to make contact one with another. We know that under such conditions the inductive reaction would be enormously diminished, but the results of the tests are nevertheless striking. Let us, for example, preserve the same amount of ore, but cut it up into small squares, each one insulated from the next. Let the squares be of generous size, for a disseminated ore—say 9 ft. on a side. The resulting response is shown in the lower curve, taken from a

model experiment. The curves become straight lines, and the response is essentially nil.

I need not dwell upon the utility of the third key to the ore-detection problem namely, the information to be gained from practical field experience. As in everything else, the knowledge gained from first-hand contact with the actual work is bound to be of prime importance. Comparisons of results with each of the methods I have mentioned when they are applied over the same part of an orebody have seldom been given. I would, therefore, like to illustrate by actual field data the typical responses which occur.

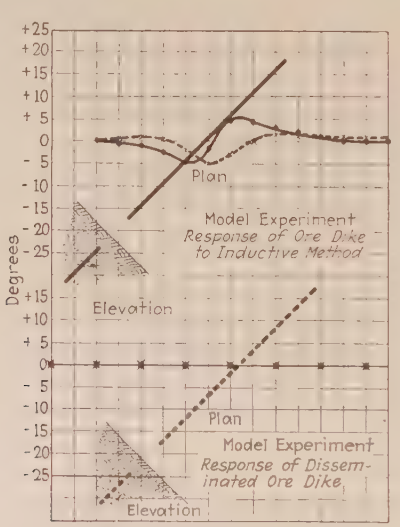

FIG. 11.—CURVES INDICATING THE RESPONSE OF AN ORE DIKE TO THE INDUCTIVE METHOD. A MODEL EXPERIMENT.

The orebody to be used in this illustration is the Falconbridge body. At drill hole No. 24 it has a thickness of about 30 ft., with 115 ft. of overburden, as shown in Fig. 12. In Fig. 13, the transverse component of the gravitational gradient obtained with an Eötvös balance is shown. The smooth curve is a weighted one drawn through the observed points, which are shown as circles. These points vary considerably about the curve, owing chiefly to the random effects of the large boulders of the glacial drift. In the absence of ore, the norite quartzite contact should give a symmetrical curve. The lack of symmetry in this curve may be attributed to ore, although results are not as definite as could be wished, because of the incidental variations caused by the presence of boulders. The curve in Fig. 14 illustrates the detection of this ore by the magnetic dip needle. Here the angle of dip in degrees is plotted against distance in feet along the traverse. It is evident that the well-defined peak furnishes an accurate determination of the position of the vein.

Fig. 15 shows two traverses over the ore at drill hole No. 24 taken by means of the self-potential method. The profiles are here flat and fail to indicate the ore. However, in order to show what this method can do over another part of the ore, another profile is shown in Fig. 16. This was taken about 3000 ft. west in the Edison Kettle hole where the cover is 60 ft. thick and the ore 120 ft. thick at its widest part. It is a repetition of S. F. Kelly's traverse No. 5.[3] His curve is shown dotted. It is seen that our curve agrees well, though taken four years later. The negative peak occurs above the ore.

The next illustration figures refer to the surface potential method. Fig. 17 shows four equipotential ovals near drill hole No. 24. The black

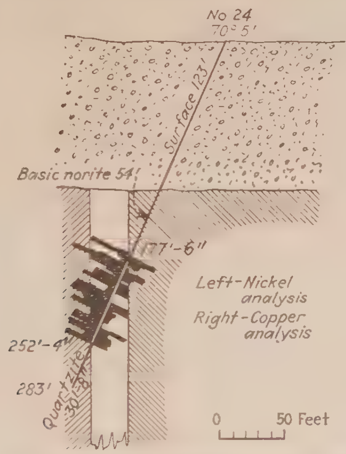

FIG. 12.—CROSS-SECTION OF DRILL HOLE NO. 24, ACCORDING TO THE LONGYEAR MAP.

bars and the arrows refer to a quantitative manner of interpreting the results, which will be described in a subsequent paper. In brief, the theory requires that the ore vein should lie between the four bars given. It is clear that the actual ore is to be found closely central between them.

Fig. 18 shows Mr. Lundberg's well-known extended electrode system as applied at drill hole No. 24. The spreading of the equipotential lines across this vein is evident, and locates its strike and position. The dotted curves are repetitions of the solid ones taken after a heavy rainfall. They agree sufficiently closely with the previous equipotential lines.

The inductive method at drill hole No. 24 was tested at several different frequencies. At 60,000 cycles the curve of Fig. 19 was obtained. These curves represent changes in the strike and dip of the magnetic vector obtained in a straight traverse across the ore. They localize the ore vein within some 50 feet.

[3] S. F. Kelly: Experiments in Electrical Prospecting. *Engng. & Min. Jnl* (1922) **114**, 623, 673, 976.

The inductive method was also tested at 800 cycles, using a horizontal loop 200 ft. in diameter and situated 1000 ft. north of the ore. Systematic changes of direction in both the strike and dip of the magnetic vector were obtained in crossing the vein.

Fig. 20 shows another traverse using a small vertical loop source and is of the same nature as the one previously shown in connection with the models over the Edison Kettle hole. The crosses and circles show the distortions in the strike and dip respectively of the incident field as the ore is crossed in a straight line traverse. These results are strictly in accord with expectations and have been confirmed by work with a model of the orebody. In fact, the results obtained with the model are shown by the dotted and solid curves. As in the previous model work, a satisfactory agreement is obtained. This form of the inductive method has proved especially valuable.

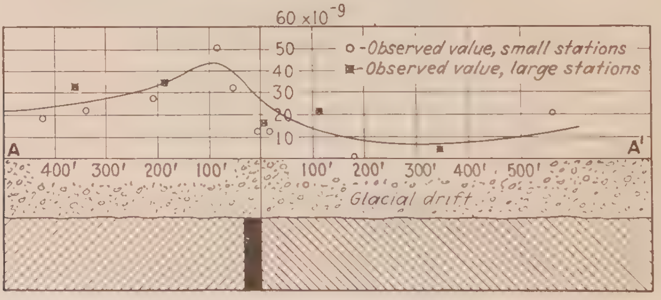

Fig. 13.—The transverse component of the gravitational gradient obtained with the Eötvös balance on the Falconbridge orebody.

I hope this will suffice to show the close and important relation which exists between the three main modes of approach to solutions of the basic exploration problem: the mathematical one; the experimental one, especially through models; and the practical field work. These three means, taken together, are competent to tell us much about the detectability of ores. They are the tools by which we investigate that difficult basic question, "What are the possibilities and limitations of physical methods of ore detection," or "What ores are to be regarded as detectable and why."

Detectability by Electrical Method—Four Factors

The detectability of orebodies by electrical methods is dependent upon only four factors: (1) electrical conductivity of the ores; (2) continuity of the orebody; (3) size of the orebody; and (4) the distance of the orebody below the surface.

1. Ores which have conductivity sufficiently high for successful prospecting by electrical methods have often been listed. They are the native metals and the sulfides (except sphalerite), the tellurides, arsen-

ides and antimonides. The non-conductors are the carbonates, silicates and oxides, except magnetite and some of the oxides of manganese. It will perhaps be well to add that the absence of conductors other than ore is desirable. For example, graphite is a good conductor, and when graphite slates are present results resembling the indications of ore may be obtained. The problem is naturally simplified if it is known that ores are the only conductors present.

2. The continuity of the mineralized parts of the orebody determines the effective conductivity of the whole mass in as important a way as the conductivity of the pure mineral itself. For it is clear that the orebody cannot constitute a conducting unit unless its mineralized sheets and veins form in the main a connected network. The mechanical nature of the mineral deposit in place is thus a vital consideration, and for this reason the conductivities of small samples of the ore are of less significance than one might think. A misleading conclusion may readily be formed about the actual conductivity of a given type of ore from consideration of its composition only. For example, some deposits containing only a small percentage of sulfides are of such a nature that contact between particles exists, whereas in others containing much greater percentages the rock completely envelops the ore particles, and the mass, as a whole, exhibits only slight conductivity. Disseminated ores may be classed as generally unfavorable, and massive sulfide deposits as favorable, for electrical exploration. Actual measurements of fair average samples of the ore afford more information than tables of conductivity of similar ores. A surer and better criterion will be obtained from underground investigations of neighboring and similar deposits—or, best of all, from actual experiments upon a known neighboring body. It will, of course, often be impossible to obtain the latter type of test.

3. The last two factors of size and depth of burial must be considered together. However, in considering the inductive method, we must at times regard size in a rather novel way. Here the effective volume is not merely that filled with ore, but is more properly that around which the ore forms a complete conducting perimeter. For example, a thin ore vein in the form of a continuous closed loop (should one by chance exist) is nearly as effective as though the ore formed a continuous sheet over the area bounded by this loop. For the inductive method, then, the size of the ore may at times be considered the size of the picture frame, whether or not the picture be present.

Depth Frequently Overstated

4. The depth to which ores may be detected commonly tempts overstatement. We agree with Mr. Lundberg's statement of last May that "The greatest depth to which an orebody may be reached under favorable conditions cannot be generally given." This statement may be

somewhat developed. It is a law of electrodynamics that the direction and relative intensities of the electromagnetic quantities are preserved unaltered at corresponding points, when the scale of the geometry is

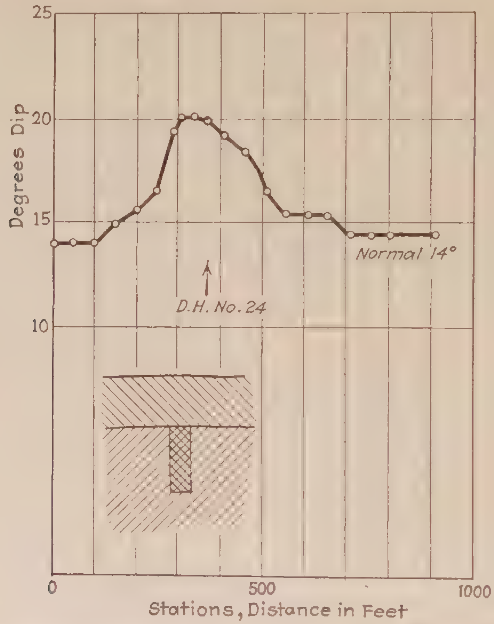

FIG. 14.—CURVE ILLUSTRATING THE DETECTION OF THE FALCONBRIDGE ORE BY THE MAGNETIC DIP NEEDLE.

uniformly changed. Therefore, a deep body is theoretically exactly as detectable as a shallow one, *provided* it occurs on the same relative scale. This is illustrated by Fig. 21. Here all the spheres are tangent to the same cone, with apex at the surface, and all are equally detectable at the

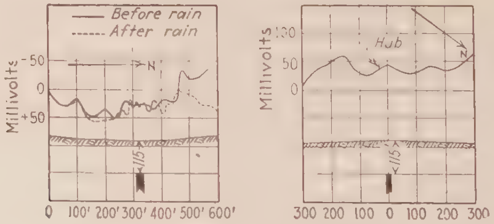

FIG. 15.—RESULTS OF TWO TRAVERSES OVER THE ORE AT DRILL HOLE NO. 24 TAKEN BY THE SELF-POTENTIAL METHOD.

surface. The law of detectability of ores with respect to distance is an inverse cube law, for the magnetic, gravitational (torsion balance), surface potential, and inductive methods. Some investigators seem to have been misled upon this topic, for two of the active operators of electrical

methods claim in their descriptive pamphlets that an inverse square law is applicable to their methods.

If you will carefully examine the list of successes with electrical methods you will find the vast majority have been obtained at covers of less than one hundred feet. One can see, by the following example, how slight is the chance of detecting ore at a depth of 500 ft.: Assume a solid pyrite body of a tonnage of 5,000,000. Let it be under 500 ft. of cover and roughly of a spherical form. Then the mean diameter of the mass will be nearly 400 ft. Now, if this mass is excited under the most favorable circumstances of a uniform inductive field, its own reaction at the most favorable point at the surface will be less than 2½ per cent. of the value of the exciting field. Such a small value will prove very difficult to recognize. It is true that the mass would be more easily found if in the form of a large plane sheet, but my general point still holds: that, for detection, deep bodies must be enormously large. I would, therefore, advise skepticism when working ranges of 500 ft. or more are claimed.

A Practical Rule

From our experience, with theory, models and practical field work, we should give the following rule: "It will, in general, not be economical

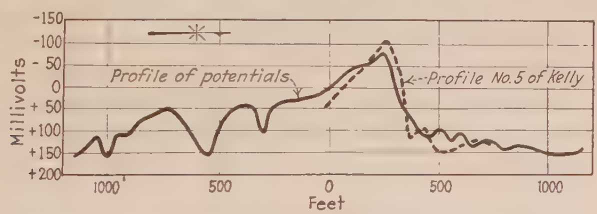

Fig. 16.—A profile, taken in the Edison Kettle hole by the self-potential method, compared with that obtained by S. F. Kelly.

to search for massive conducting ores unless two dimensions of the expected orebody approximate the depth of cover." In other words, if the body is lenticular, each of two perpendicular dimensions of the lens should roughly equal the depth of cover. Actual survey conditions are so largely unknown that mathematical refinements in such rules have no place. The statement made will, therefore, serve to give a satisfactory idea of what, in general, constitutes a detectable orebody. As a rougher but simpler rule, which has exceptions under certain conditions, we should advise that physical explorations be confined to regions where the orebodies are expected to approach within 200 ft. of the surface.

Difficulties of Interpreting Results the Great Obstacle

The difficulties in extending the practical limits to which ore may be detected are of a fundamental type. It is not that more sensitive or more accurate receiving apparatus is needed, nor that more powerful

sources are required. The difficulty arises from the existence of the extraneous efforts of the unknown ground conditions, and is really one of interpreting results. Of course, when the ore is relatively near, it responds in a loud voice, and drowns out all else. When it is further away the inevitable babble of all the other differences begins to become confusing, and finally the small voice of the ore is completely masked by other messages. It will do no good to use a loud speaker, for this will simply increase the static along with the message. The problem is one of selection rather than of sensitivity. In its basic feature it is like the sub-

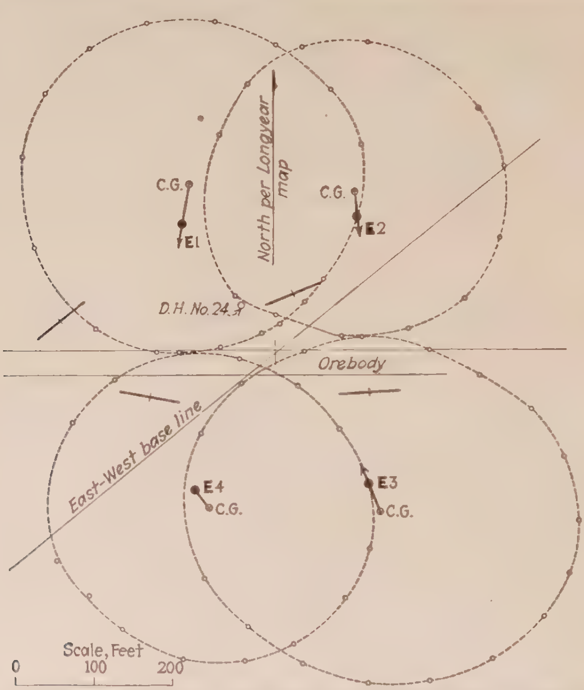

FIG. 17.—EQUIPOTENTIAL OVALS OBTAINED NEAR DRILL HOLE NO. 24 WITH SELF-POTENTIAL METHOD.

marine detection problem. There the game was to hear the submarine without hearing your own boat. Here, we must hear the ore without the response from near-by surface conditions. There are, then, natural limits in working range which offer increasing resistance to attempts toward their extension.

Economic Aspects

The business of mining has always involved its own peculiar risks, not found in other branches of commerce and engineering. Evaluation of such risks by intelligent analysis, and the reduction of uncertainties by engineering studies, have helped the industry to proceed along sound

lines. The history of the mining business has been the history of the replacement of the "risk all, gain all" attitude of the speculator by policies founded upon the discovery and analysis of facts.

The greatest uncertainties are those met in the exploration of virgin lands for new orebodies. This division of mining is obviously an essential

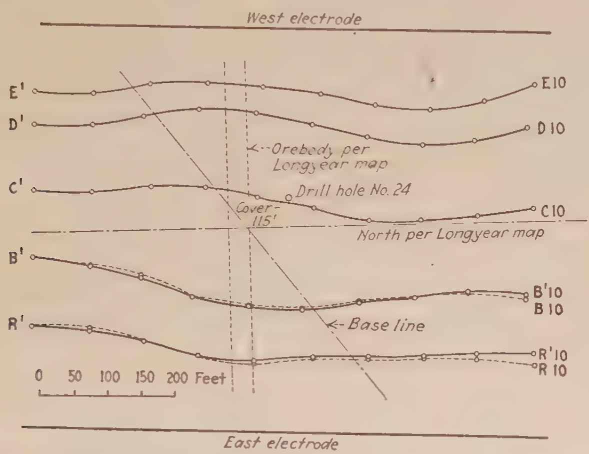

FIG. 18.—RESULTS OBTAINED BY APPLYING THE LUNDBERG EXTENDED ELECTRODE METHOD AT DRILL HOLE No. 24.

part of the business, and it is important for the industry to eliminate, as far as possible, the gambling aspects of its exploration expenditures, and to put exploration upon as sound a business basis as present circumstances

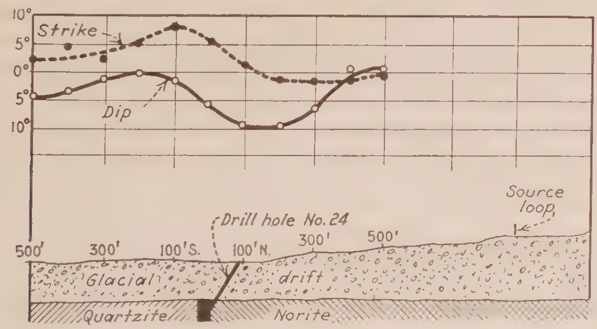

FIG. 19.—CURVE OBTAINED OVER THE LONGYEAR OREBODY WITH THE INDUCTIVE METHOD, USING A FREQUENCY OF 60,000 CYCLES.

will permit. Not only this, but foresight for the future demands that a fund of data be collected concerning the relation of the newer exploration methods to the possibilities and economics of exploration work. As in all questions in which probability and chance strongly enter, conclusions must be founded on a large fund of experience. It is, therefore, espe-

cially important to obtain a statistical view of the results of exploration with the physical methods, and to avoid being misled by success or failure in a few single instances.

The question is, "What is the cost of the failures in physical ore exploration, compared to the value of the successes?" We must be careful to get both sides of the picture. There is a natural tendency in this work to adopt the sun dial's motto, "I count only sunny hours." A satisfactory commercial averaging of both sides of the picture is unfortunately impossible at present. Partly because of the youth of the subject, sufficient data are not available. However, we may profitably discuss some aspects of the commercial side of exploration with physical methods.

The Cost of Prospecting

There is, first, the cost of prospecting with these methods. The costs of exploration are dependent upon the field conditions and location. They may be analyzed in several parts:

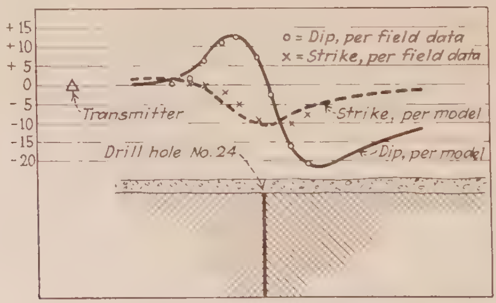

Fig. 20.—Results obtained with the inductive method over the Longyear ore-body compared with those obtained from a model.

1. The cost of transporting the party and equipment to the field location. This is variable, of course, but in a survey of a month or more it is usually relatively small.

2. The cost of maintaining camp, including commissary.

3. The cost of the transit survey, the clearing of brush, and the staking out of stations on traverses. This item will be fairly large in difficult, brush-covered country. To keep ahead of the field party we have had to maintain at times three axe crews of four men each, who cut through about 15,000 ft. of traverse per day. In the region near Sudbury, Ont., transit costs have averaged, in over 100 miles of traverse, about \$25 to \$35 per mile, including all costs for brush cutting.

4. The other costs are those directly involved in taking the data of the physical survey. In our usual type of exploration, they are divided between the inductive, the terrestrial magnetic, and the self-potential

methods, about as the ratios of 4 to 1 to 1. The inductive method is naturally the more elaborate and expensive.

In one survey of 22 working days near Sudbury, Ont., the total field costs for the self-potential, magnetic and inductive methods, apart from travel and shipping expenses, were under $5 per acre.

In this survey, the traverses were separated 500 ft. except where mineralization was discovered. In the most interesting areas, a separation of only 200 ft. was used, and 10 additional cross traverses were run.

The total over-all cost of about $5 per acre is roughly typical of the area, although we have completed similar surveys in the same region totaling 4000 acres, under somewhat more favorable topographic conditions, at the rate of just over $4 per acre.

At present, we would put the total cost for a three-method survey of the above type at from $5 to $7.50 per acre. These figures, of course, apply only to surveys of several square miles. In smaller ones, the fixed charges bring up the cost per acre.

No Physical Method Detects Ore

In considering exploration costs, a general fact should be borne in mind, which is sometimes neglected by enthusiasts: No physical method detects ore, but only some physical characteristics, usually, but not *exclusively*, associated with ore. There are conducting ores, but also conducting graphitic slates. There are heavy ores, but also heavy rock formations. There are ores showing chemical activity, and a resulting earth current, but many other ground conditions lead to current flows. No method can positively discover ore. By using several methods, it is sometimes possible to obtain independent confirmations of indications, but not absolute certainty.

Because of these facts, a certain percentage of failures must be expected, and this percentage will be dependent upon the intelligence, knowledge, and experience of the exploration group. As yet no adequate fund of data is available to show what percentage of success, under given operating conditions, will be attained. We ourselves have done research exclusively until last year and have thus far made practical surveys of only nine different mineral regions. Four of these were over known ore, used by us for experimentation. In each of these four cases our results agreed with ore known to exist. Of the remaining five areas, only one has thus far been drilled. Here a mineralized fault was discovered in the position indicated by us, with commercial ore in spots. It is not yet known whether a sufficient quality exists for a mine. One of the other locations was trenched, with the resulting discovery of non-commercial sulfides near the surface. In the course of time experience will permit a clear statistical representation of the probabilities of success in physical prospecting and of the degree of the risk involved. It will, however, be

impossible to predict the composition, or type, of the suspected orebody, and the drill and underground work must be considered as indispensable as ever. Physical prospecting, like geology, can simply indicate favorable positions for drill holes.

Underground Exploration an Attractive Field

It is obvious that returns from investments in physical prospecting will be highly dependent upon the success in choosing areas favorable for their use. Because of this fact, we believe underground explorations in producing mines will be an attractive field for future applications of physics. Since it is a well-known fact that about nine-tenths of all new discoveries of ores are near existing mines, one can feel sure that such

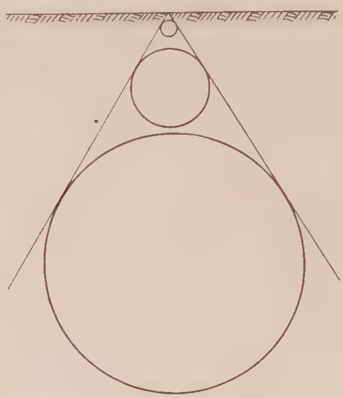

Fig. 21.—These spheres are all equally detectable at the surface.

areas are highly favorable for electrical prospecting. Furthermore, operations underground are likely to be much nearer the ore, hence the limitations of range are not so serious. In many instances, it would be most valuable to determine whether ore exists within a radius of 100 ft. of certain drifts and crosscuts.

The progress which the future will bring should be continuous and gratifying, but it will not be spectacular in the sense that any glorified witch-hazel wands will be discovered. Improved technique and increased knowledge will be attained through the same process of intelligent and energetic research to which all the important applications of physical science owe their birth. Although physical exploration for ore is a scientific development which can claim only a short history, its economic value has already been demonstrated in many areas. The present accomplishments and technique in this field warrant serious consideration by mining men in laying any sound policy and program of ore prospecting. When conservatively and intelligently applied, in regions suited to their nature, I believe that physical prospecting methods will amply repay investments.

DISCUSSION

A. M. Bateman, New Haven, Conn.—What is your opinion as regards the possible application of these methods to underground workings in existing mines?

M. Mason.—Directly as developed today there possibly is no application, but there are physical possibilities for obtaining much information from essentially the methods of today. There will be variations due to the immediate presence of the conducting masses within which you are working.

I am impressed by analogies with the things that one sees in physics. If 25 years ago even a well-trained physicist had seen the negative on which was traced a curve such as one gets in ionization and been told that out of it the structure of the atom would be discovered, he would have laughed. But by working from the simpler cases and with a number of thoroughly intelligent, scientific and honest-minded investigators in groups at this problem, I am convinced that as one goes from simple cases to the more complex ones, those minor variations which now disturb in our readings will be interpreted, and that we will be able to know structure as well as ore deposits and the application of all of these methods to the determining of structure to aid the geologist. Looking at the future of 25 or 30 years from now, I am completely optimistic over the enormous use to which this tool will be put when further knowledge of interpretation and formal processes of interpretation have been studied and carried out.

Member.—I was engaged during the war in the detection of submarines, using the method which is based upon electrolytic action. We found it possible to detect the immersion of metals in saline solutions such as sea water at very considerable distances. The method offers great possibilities and, I think, might be used in connection with orebodies.

Dr. Mason referred to the currents which occur between the upper portions of oxidized orebodies and the unoxidized portions below and I think it might be possible to develop methods along that line for detecting the electrical energy which is passing out of, or actually detecting the radio energy which comes from certain classes of minerals which we know, such as uranium. Has any work been done along these lines?

I must confess that when I talk about detecting pieces of iron at distances of 40 ft. in sea water with absolutely no connection between iron and the water and I show a deflection across the scale as wide as that screen which you have before you, the uninitiated would think I could also show the magnetic effect and superimposed upon that the electrochemical effect or the radiation effect.

M. Mason.—Radioactive minerals will show effect readily. An effect corresponding in distance to that you have spoken of, I should think would be quite hidden in the earth due to the great amount of earth current normally present and I should doubt the value of anything more delicate than the self-potential method which has been applied so widely by Mr. Kelly, particularly in this country.

Dr. von Buelow-Trummer, New York, N. Y.—Our company is working with all the different kinds of methods—with the magnetometer, with the seismograph and with the electrical methods. I agree with Dr. Mason that it remains to be found out before actual work is started what kind of a tool we have. It is not possible, generally speaking, to say, "Use a torsion balance." It is seldom that you have occasion to use a torsion balance. For practical use the torsion balance is too slow to detect.

We frequently cross-check electrical measurements with magnetic measurements and have thus had the opportunity to be more sure that the indications we get are in a certain way coming from the orebody. We do not claim success unless, at the places where we have had indications, actual drilling has proved the presence of the ore.

There are very many places where we have had this opportunity and have been quite successful.

The greatest depth that can be obtained with any actual electrical measurements is about 400 or 500 ft. We have had one opportunity of finding a very large orebody by starting in on a known orebody which was 650 ft. below the surface. It was a very large body and very good ore and even though the surface conditions are very bad the orebody showed up very nicely. We went ahead with these measurements and found other indications exactly similar to those on top of the orebody, but no drilling has been done and I do not claim it to be a success as yet.

A large amount of work has been done in Canada during the past month. Unfortunately some failures have taken place in the opinion of people not quite familiar with the work and who think that what we find is the ore. We find indications. We believe that the indications should be checked against other measurements and with geological features and then drilling resorted to.

The use of these measurements is that unnecessary drilling is spared and you know about what drill holes cost if they have to go down, say, 400 ft. The electrical measurements, as well as the geophysical measurements, will cover the entire area.

G. C. Riddell, New York, N. Y.—It would seem that one of the major ore districts of the country is ideally adapted to the use of these methods in accordance with the conditions described by Dr. Mason. I refer to the Missouri sheet deposits, which lie from 100 to 200 ft. deep, and are dense in character. I presume a great deal of work has been done there with these methods?

M. Mason.—I understand that the deposits are very poorly adapted to the method as being not highly conducting and not sufficiently continuous.

L. B. Slichter, Madison, Wis.—I think the reason the methods have not been successful in the Tri-State district is that the ore is nearly all sphalerite and that is the only sulfide which is an insulator. On that account the electrical methods have not yet proved successful. Since the orebodies are, moreover, of complicated shape and not of great density, they are not amenable to detection by the torsion balance.

A. H. Rogers, New York, N. Y.—Perhaps Mr. Riddell refers to the southern district where the lead is. I will say that we have done a great deal of testing on those ores in a laboratory way and we find the mineralization is not heavy enough. Furthermore, another objection is that ground water is a very good conductor.

H. A. Guess, New York, N. Y.—A year or so ago we used the Eötvös torsion balance in the Tri-State field, not with the idea that we would delimit orebodies particularly, but with the thought that we might broadly show the flint areas within which drilling might be profitable as compared with the surrounding areas. By the torsion balance we did get broadly a fair amount of information. To our surprise the flint gave a lower general specific gravity than did the surrounding rocks due to the presence generally throughout that area of so many vugs.

We have used in Newfoundland the surface potential method of Mr. Lundberg. We began in August, 1926, and, perhaps due to the fact that the ore we found was quite close to the surface, we have been quite successful up there in opening up (it was all done within a few months) a very considerable tonnage of complex sulfide ore containing lead, zinc and copper. We are continuing the work this year and plan to continue for some time, but have not been as fortunate this year as last, possibly because there may be no ore in the areas just now being prospected.

E. H. Guilford, Montreal, P. Q., Canada.—I am with the Radiore Co. and we use an electrical prospecting method that is based on the inductive system using high-frequency current. We use a vertical loop and direction-finding coils. Our

method has been worked out so that we not only locate the plan view position of the conductors but approximately the depth as well. During the past few months we have entered Canada, having sent two crews there in June. We now have five. In the past two months four different conductors not previously developed have been drilled and have all checked. In addition to that some seven or eight conductors have been trenched and all checked. At the present time the five crews are contracted through until some time in February.

I quite agree with Dr. Mason that one of the chief difficulties with electrical prospecting is teaching our customers what they can expect as well as what they cannot expect. I have great difficulty in preventing them from getting over-enthusiastic.

Basically, it is quite true that electrical prospecting will only tell you the position of the electroconductive areas or zones and you must resort to other measures to determine the character or the values of those areas. It has been our experience in the past three years with the conductors that have been proved up, after having been located by our method, that so far we have located only sulfide zones.

Some question has been raised from time to time as to whether we would not locate or indicate areas which were better conductors than adjacent areas, but which might not be sulfide. I think the answer probably lies in the ratio of conductivity. The ratio of the average sulfide body of any considerable extent is so much greater over the adjacent material, it is so enormous, you might say, that only in those cases will you positively locate them electrically, and where the difference in conductivity is very small or slight, the reaction that is obtained is so slight that it is not noted as a rule.

H. T. F. Lundberg, New York, N. Y.—In the laboratory work which we have carried on in Sweden and here also, we have found that in the ground we have more complicated conditions than we ever could imagine when we started. In fact the ground itself is sometimes a very good conductor, especially if we are in sedimentary regions. The ground is porous and carries solutions which often have a rather high conductivity. This will sometimes prevent the possibility of finding any ore in such regions. An example of that is the Mississippi Valley, previously talked about.

Therefore, we have found that it is necessary not only to investigate all data on the ore, but also on the rock and on the rock in different places of the area, the water conditions and the conductivity and electrical conditions of the water. Furthermore, it is necessary to investigate the petrographic conditions, the arrangement of the minerals, the arrangement of the conducting minerals as well as the arrangement of the non-conducting minerals. By doing so it seems possible to elaborate methods by means of which certain indications may be selected as being due to more homogeneous sulfides, for instance, as distinguished from shales, slates, graphitic slates and schists.

Thus by further research and further work in the laboratory and in the field, it will be possible and sometimes already is possible to eliminate many of these so-called indications from non-metallic mineralization, and from the knowledge of the locality to eliminate them as very certainly barren and of a character discouraging for drilling. Of course, there remain many doubtful cases and they have to be drilled, but that is the course that we think the methods will take so that they do not encourage too much useless drilling and too much useless exploration work.

Member.—Is the method suitable for prospecting anthracite coal?

M. Mason.—I should hesitate to make any such promise.

H. T. F. Lundberg.—Some of the anthracites are oxidizing and under certain conditions they are surrounded by electrical fields that can be detected, but they have to be rather close to the surface. Some anthracites and some coals are conductive

and if within reasonable reach it might be possible to trace them, but most of the electrical work in coal has failed.

E. G. Leonardon, New York, N. Y.—The only work I know of on coal is some that was done at Wilkes Barre, Pa. by S. F. Kelly and some by us in the French Alps. Some coals are good conductors and give rise to the phenomena of spontaneous polarization; some others are sufficiently insulating to be detected by potential studies, but as far as I know not a great deal of work has been done along this latter line. Much of this work has been done in the Saar coal basin by the Schlumberger method but this is out of the subject on account of the fact that it is stratigraphical work.

I quite agree with Dr. Mason in what he has said, especially upon the fact that what we need most in electrical prospecting is experience. Many discoveries are made every month, or every week, but not all of them prove to be orebodies or real discoveries.

My company made a discovery in Serbia in 1913 and was informed of the results by drilling in 1923. The orebody thus outlined is one of importance and is now one of the biggest copper mines in Europe. We located it under a mountain of andesite. In the case of another discovery made by us in Canada in 1924, drilling results were not obtained until two years later.

As explained by Dr. Mason, too many people are skeptical and some are over-enthusiastic. The difficulty is to give them a true idea of the results that can be obtained mathematically and physically.

K. Sundberg, Houston, Texas (written discussion).—Dr. Mason has outlined the work done by himself and other physicists in reviewing the question of the application of physics to ore detection since 1923. This general problem was taken up in Sweden about two years earlier.[4] As I have been fortunate enough to be technically in charge of this work and to cooperate with several experienced physicists and engineers, I might be justified in making some remarks supplementing Dr. Mason's paper.

The main results of our work up to the end of 1924 have been published in the Year Book of the Swedish Geological Survey for 1923 and later results are scattered in several papers, one having been presented by Mr. Lundberg before this meeting.[5]

The first development of the methods took place within areas in northern Sweden, comparatively favorable for electrical prospecting, and several commercial orebodies were located by our methods before these methods were exploited elsewhere.

We have several prospecting patents, one of the most important being the fundamental Conklin patent for electromagnetic prospecting. In the practical application, especially of the electromagnetic methods, we are, therefore, not restricted in the same way as others.

It might be considered an interesting coincidence that in 1921, when we for the first time made application for an electromagnetic patent[6] we gave almost word for word the advantage of the then new method over the other electrical methods, as Dr. Mason in his paper now describes the basic advantage of the inductive method over other electrical methods. Our method to which I refer was similar to the inductive method described by Dr. Mason, in that only direction of the electromagnetic field was determined. After one year's work with this direction method, we found it satisfactory only under simple conditions, however. Our work in the field now always comprises investigation both of direction and amplitude of the electromagnetic

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[4] At that date the Swedish magnetic methods of Thalén and Tiberg and the Lundberg electrical method had already found wide application.

[5] See page 87.

[6] Swedish patent application No. 2881.

field vector, and in many cases the phase must be determined too, according to our experience.

But even if the electromagnetic method comprises complete determinations of the field vectors, I do not think it justifiable to consider them capable of more general application in ore prospecting than other electrical methods, unless we include in the electromagnetic group also the methods in which current is supplied to the ground not inductively by a loop, but galvanically by electrodes. As a matter of fact, the difference between the methods of energizing—*i. e.*, by induction or by electrodes (galvanic method)—is more fundamental, in my opinion, than the difference between the methods of observation; *i. e.*, electromagnetic or potential. One basic advantage of the galvanic method is the fact that in areas of steep-dipping, conducting orebodies, enclosed within rocks of poor conductivity beneath a soil cover of average conductivity, the contrast on the surface between the area above the orebody and a barren area is highly increased because of the poor conductivity of the rocks, if electrodes are used. Ore types of the kind referred to are common within glaciated Pre-Cambrian areas, and in such cases galvanic methods often give much more pronounced indications than inductive, especially with linear electrodes. The case might be exemplified in the following manner:

Assuming a point electrode on the surface, the potential of the electric field on a point at a distance a from the electrode (point 1) if no differences in conductivity exist beneath the surface will be $\frac{1}{a}$ in an arbitrary scale, assuming direct or low-frequency current. In the case of rocks of high resistance beneath the earth cover, the potential will be much larger.

If we now consider the case of the point electrode above an orebody under the conditions mentioned, the potential in a point above the orebody at the distance a from the electrode (point 3) will be less than $\frac{1}{a}\cdot$ The difference in potential between points 2 and 3 therefore is much larger than between 1 and 3, the presence of the highly resistant rocks increasing the difference and thus the strength of the indications. Similar conditions apply for the electromagnetic field. In the case of an inductive method, on the other hand, the presence of the earth cover acts in the opposite direction, at least if the conductivity of the earth cover is good, if the earth cover is thick, or if high frequency is used, because strong secondary currents are then induced in the earth cover, making the indication of an orebody beneath the cover less pronounced, sometimes not even noticeable. This is also a case where the phase displacement plays an important role. We know from our experience cases where the specific resistance less than one meter beneath the surface is of the magnitude of 100 ohms per cu. cm. and it has happened that the difference in phase between two points 300 ft. from each other has run as high as 30°.

Dr. Mason stresses the fact that it is not always possible both to outline a conducting body and determine its conductivity, because dimensions and conductivity jointly determine the electromagnetic field. In cases, however, where the geology gives us some information regarding the shape of the conducting body, it is often possible to solve the problem.

In the case of deep, steep-dipping orebodies of lenticular form, which are common, we have often predicted with good, even remarkable, accuracy the outlines of unknown bodies before they were drilled. The principle for the corresponding electrical measurements is, broadly speaking, to use several set-ups of the sending equipment or electrodes and force the main part of the currents in the conducting bodies as near the hanging and foot walls respectively as possible.

R. D. Hoffman, Swastika, Ontario (written discussion).—It was with a great deal of interest and pleasure that I read the paper by Dr. Max Mason. It is not for me

here to go into the physical aspects of the various methods, that must be taken up by physicists and geophysicists. What I shall try to do is to present my observations as a practical miner who is interested in the finding of new orebodies.

Electrical prospecting lends itself to exploitation of the public by unscrupulous mining brokers, especially in Canada, where a "boom" spirit in mining exists, unless due care is taken by men in the field, who are reporting the results. We have had one case, the Area and the Amulet episode, in the Rouyn area, Quebec. Here a zealous electrical prospector allowed the results to be given out prematurely and a wide orgy of speculative buying resulted.

As to the cost of electrical prospecting, I have a few figures which I think may be of interest. During the past summer we had occasion to employ the Swedish American Prospecting Co. With lines cut out at 150-ft. intervals through fairly dense bush, we found that we could cover approximately 25 acres a day. The labor cost for this work was $2 per acre for electrical preparation and $4 for the cost of the electrical survey, a total cost of $6 per acre. This $4 cost of electrical survey was based on a contract price of $100 per day for the electrical engineers. During the latter part of the season, with 200-ft. intervals, we could complete 40 acres per day with an approximate cost of $4 per acre.

We had made geological maps of the areas examined and in several cases we had noted some slight mineralization. The engineer of the Swedish American Prospecting Co. reported some small "kicks," but stated they were not strong enough to indicate orebodies. The electrical engineer was quite unaware of the existence of these areas of mineralization and those of us connected with the survey were very much impressed with this phase of the work.

We also noted that where there was an outcrop in a swamp small readings were observed approaching this outcrop; that is, the difference in overburden and moisture contained in the ground had an appreciable effect on the readings.

The electromagnetic method was employed in this work. We did not succeed in finding a commercial orebody and on the completion of 2000 acres felt that the odds were too great to continue the work.

Through the courtesy of Messrs. Norrie and Tower, who had employed the Schlumberger method for several months, I can give a few figures on costs of this method. It was found that eight men could prepare 200 acres a day for electrical prospecting at the cost of 50 c. per acre. With the Schlumberger method it was not necessary to cut out such careful lines as with the Swedish American method, where each line had to be carefully cut out as on a survey. The two men in the Schlumberger party covered 100 acres per day. These men were working on a monthly basis. The total cost per acre was approximately somewhat less than $1.

I have no figures on the Radiore method. From what I could see of their work in the field I believe that the cost per acre by this method would be more expensive than that of the Swedish American or the Schlumberger methods.

Let us consider how electrical prospecting works out in a new mineral area where caere are a few mines known. In the Rouyn area, Messrs. Norrie, Tower and I had threfully chosen some 10,000 acres of ground out of 30,000 acres available. We felt that orebodies similar to the Horne, Waite and Amulet might be found. All things being equal, we thought the best areas to explore would be those around the peripheries of the granodiorite intrusives. We did not succeed in finding a single orebody on these 10,000 acres. In addition, some 30,000 to 50,000 acres owned by other individuals and companies were explored "electrically" and no orebody found. From $200,000 to $350,000 was spent on this type of work with no tangible result. Surely this does not show economic returns!

Let us consider a mineral area where some worth-while mineralization is known to occur, but no mines, for instance an area like Chabougaman, Quebec. Here there is

an intrusive gabbro contact of 60 miles with some mineralization, in one place a small lens 100 ft. long and 10 ft. wide of $10 gold-copper ore. The country is very much overburdened and the only way it can be thoroughly explored is by some electrical method. This 60-mile gabbro contact has 60 × 640 acres, or 38,400 acres. This area might be surveyed at a minimum cost of $100,000. If any indication is found, it would be necessary to thoroughly test it to see first if there were any valuable ore minerals and secondly if these were present in economic amounts. I hardly think any large exploration company would undertake such a scheme. It would be far more advantageous to wait until a real "find" was made by a prospector in the field and then pay up to $10,000 down to secure an option on it.

The real practical use for electrical prospecting, I believe, will be to extend the limits of known sulfide orebodies, and will be confined to ores such as chalcopyrite and galena, which give definite positive reactions. In a word, electrical prospecting will supplement diamond-drilling and in that way will be a great aid to the mining industry.

Geophysics and the Mining Engineer

By Allen H. Rogers,* New York, N. Y.

(Boston Meeting, August, 1928)

It has always seemed to me that there is a certain similarity between the work of the mining engineer and that of the doctor of medicine—each has very often to be governed in his actions by conditions which he cannot see. It is not necessary to detail the unseen conditions with which the mining engineer or geologist has to deal—the further extension of ore-bodies, the existence of new ones, the throw of a fault, the probable change in grade or character of the ore with advance of the workings, are a few of the conditions which the mining engineer has to determine as well as he can from the deductions he is able to make from what he can observe. Just so the physician is often confronted with conditions that he can not observe. Probably less than half of human ills make a manifestation at the surface of the body from which definite and certain conclusions can be drawn. The diagnostician must observe various things and, from these observations in combination, conclude what his course should be to bring about the cure of the trouble. Both professions would be assisted immensely by any agency that would permit them to look below the surface, something that would make more certain the diagnosis which each is called upon to make.

The physician has had such an agency since 1895, when Röntgen discovered the X-ray; which, by making possible shadowgraphs of the osseous portion of the human body, advanced immensely the diagnosis of bone lesions. Later, with improvement in the technique and more understanding in the interpretation of what the photographic plate shows, other portions of the human anatomy could be examined and at the present time there is hardly an instance of internal disorder to which the aid of the X-ray is not called. Some organs are made opaque by administering mineral substances; in other cases, conclusions are drawn from barely perceptible shadows on the photographic plate. Generally the surgeon's knife is directed with exactitude to the seat of the trouble and so not only is the difficulty revealed but the necessity for an exploratory operation is averted.

What the X-ray is to medicine and surgery, geophysical prospecting methods are to the mineral industry. They give the mining engineer and geologist information regarding subsurface conditions which otherwise

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could be obtained only by excavating. They differ, of course, from the X-ray in that no concrete picture is the result; it has been only by long and painstaking experiment in the light of physical laws that the message received can be interpreted. And while this interpretation can be taught, it can be made with the greatest degree of certainty only by those experienced in the practice of the art. Even then, just as with many instances in X-ray work, the interpretation is not infallible. There is still much to learn.

Help in Finding the Ore

But with all their limitations, geophysical prospecting methods have become an important factor of assistance to the mining engineer in solving his most difficult problem—that of finding the ore. Suppose, in driving along a vein, you encounter a fault, and crossing it, find that the vein is not there. Which way will you turn? You remember the rule that "if you encounter the fault on its hanging-wall side, turn to the hanging wall of the vein; if on the foot-wall, then towards the foot wall of the vein." But this rule applies to normal faults, and the fault may be reverse. Furthermore, even if normal, the horizontal component of the movement may be so great as to invalidate the principle on which the foregoing rule is based. Francis T. Freeland[1] gives eight rules for picking up the faulted portion of a vein. With each rule he cites the many exceptions where the rule will not work and concludes the paper by saying:

"The study of faults is often complicated by the circumstance that the vein or the fault, or both, are not planes, and hence give crooked lines of intersection which are not parallel; or by the plasticity of the rocks and the strong folding and crushing of the strata; or by multiple and radiating faults; or by simultaneous, secondary and successive mineralization of vein and fault; or by the vagaries of eruptive rocks, requiring more extensive developments than are economically permissible, for their complete determination."

All of us have had the experience of trying to pick up a faulted vein and, unless surface observations have given the clue, we know that the chances of failure are very great. When, as is often the case, the clue cannot be obtained from the surface, due to the masking of the vein outcrop by drift, electrical prospecting will often show the location of the faulted portion of the vein. A case in point was described by Moore and Ebbutt,[2] who gave the details of an electrical survey and the underground work which confirmed the results.

One of the most important features in working a mine is to explore the walls of the vein. A mine is generally opened on the strength of an

[1] F. T. Freeland: Fault Rules. *Trans.* (1892) **21,** 491.

[2] J. I. Moore and F. Ebbutt: Electrical Prospecting at the Britannia Mine. *Trans. Can. Inst. of Min. and Met.* (1926) **29,** 84.

outcropping vein or other type of orebody, which either contains metal values or shows evidence of containing such values with some depth. During the course of development, the walls are crosscut or drilled at intervals to see whether, by chance, there is any parallel orebody. The physical conditions of the rocks inclosing a vein frequently permit the deposition of ore in more than one vein or body and generally near and parallel to it. Such an orebody may not manifest itself at the surface because the outcrop is covered. An electrical survey will reveal whether such a condition is present and will indicate the portions of the ground that promise to contain ore and so warrant prospecting underground. In other words, the portions of the neighboring ground that hold out no hope of containing ore are determined and thus the expense of prospecting them by crosscutting is saved.

Symptoms of Presence of Ore

But it is in the search for orebodies in virgin ground that the geophysical methods of prospecting stand preeminent. Of course, we must have "symptoms" of the presence of ore. In this case, the "symptoms" would be favorable geological conditions. It would be foolish to spend any time in searching for copper ore in Ohio, for example, for even though there is a possibility that there are copper orebodies in Ohio, the chance is extremely remote. But in such a region as the Pre-Cambrian of Canada, almost any portion offers possibilities. In examining a property in such a region, a preliminary study of the geology would be the first essential step. It is fairly well established that, even in this favorable region, the intrusive granites are not favorable for finding ore. The rock formation is generally pretty well covered with glacial till, therefore it is not usually possible to get much detailed information regarding the geology, but the work is worth while because sometimes large areas can be excluded from favorable consideration.

With such a knowledge of the geology as can be acquired, an electrical reconnaissance survey will speedily determine the portions that hold out possibilities of containing ore. Detailed surveying over those portions will generally determine whether the expense of trenching or drilling is warranted, under favorable conditions actually delimiting the horizontal projection of the top of the orebody.

Prospecting of this character has been very successful in finding orebodies in the glaciated regions of Canada, Newfoundland and Europe. The earliest work was incited by finding boulders of ore, evidently ice-transported. The course of the glaciers was determined and prospecting up the icestream started with the result of finding such orebodies as Boliden, Menstraesk and others in Sweden and Buchans in Newfoundland. Others have been encountered without the guidance of boulders.

Much of the glaciated area in the north is water or swamp but these conditions are not deterrents to prospecting the ground beneath by geophysical methods, as the work can be conducted on the ice during the winter with, in some respects, greater ease than on dry land.

A favorable field to consider in this class of prospecting is the search for additional orebodies in old camps. The favorable geological conditions under which the originally discovered orebodies were formed are often continued along the strike of the formation but t̨he presence of orebodies, if any, is concealed by drift. Work of this character has resulted in some success and there are still a great many localities where study of the conditions might indicate the possibility of a favorable outcome to such a campaign.

Principles of Geophysics

The science of geophysics treats of the physical properties of the earth and its component parts. It is not of recent birth, for the systematic study of the earth's magnetism was begun by Gauss as early as 1834 and the study of gravity began even earlier (XVIIIth Century). The force of gravity was generally studied by pendulums although as early as 1797 Cavendish experimented with a form of torsion balance. The present very useful form of torsion balance we owe to the Hungarian Baron Lorand von Eötvös. Since 1891, this instrument has been used for the study of subsurface structure, but much earlier the discovery was made that iron ores attracted the magnetic needle, and deposits of these ores were located by its use. The dip needle is said to have been used for locating iron ores in New Jersey as early as 1760. This seems a little doubtful but there is no doubt that it was used for this purp se in Sweden about the middle of the last century. As for the use of electricity in prospecting, as far back as 1830, Robert W. Fox published a paper[3] on electromagnetic prospecting of metalliferous veins, and in 1881, Carl Barus experimented with the self-potential method on the Comstock Lode. A very good short historical review of the development of geophysical prospecting methods is given in a paper by Donald C. Barton.[4] There was little general knowledge of these early attempts, however, and little progress was made until after the war, but since that time many have been devoting their time to the study of the application of geophysical methods to prospecting.

The underlying principles of geophysical prospecting have been expounded by a number of writers, usually in terms too highly technical for the average mining engineer who has forgotten his higher mathe-

[3] R. W. Fox: On the Electromagnetic Properties of Metalliferous Veins in the Mines of Cornwall. *Phil. Trans.* (1830) **3**, 399.

[4] D. C. Barton: Applied Geophysical Methods in America. *Econ. Geol.* (1927), **22**, 649.

matics. Generally speaking, however, they depend on some difference in physical quality of the body sought from the rock inclosing it. Thus the torsion balance, which gives no response when the ground within its range is of uniform density, gives a big "kick" when in the neighborhood of a salt dome, the specific gravity of which is considerably lower than that of the rock surrounding it. So, too, when an electric current passes through a block of ground containing a body of sulfides, the current, due to the fact that the sulfides conduct electricity more readily than the inclosing rock, concentrates in the sulfide body and if means can be found to trace its distribution this concentration will be determined and so also the location of the sulfide body. The distribution of the current in the ground can be traced by shunting a portion of it out of the ground through appropriate apparatus or by examining the magnetic field which it creates.

Application of Geophysical Methods

The widest application of geophysical methods for prospecting, so far, has been in the petroleum industry. Here the problem has been different from that in the metal field. No one, as yet, has succeeded in showing the actual presence of oil but there has been great success in determining the next best thing, *i. e.*, the favorable structure for the accumulation of oil. The seismic and torsion balance methods have been mostly employed in the oil-field work. The seismic method has been used where it was desired to indicate the broad structural features of a territory and the details of the features so determined have been worked out by the torsion balance. These methods have little application in the search for metallic ores, however. The usual orebody is too small to be picked up easily by these methods and, furthermore, the topography in mining districts is usually rough, which causes serious complications in interpreting the results of the torsion balance. The electrical methods take advantage of an outstanding characteristic of most metallic ores—*i. e.*, their high degree of electrical conductivity—and aside from the few cases where magnetic surveying is preeminent, electrical prospecting will probably continue to have preference where metallic ores are sought.

Two Groups of Prospecting Systems

Electrical prospecting systems fall into two groups, one depending on the natural currents generated by the oxidation of the sulfides in the ore, the other on passing an electric current through the ground. In the latter, there are a variety of methods, depending on whether direct or alternating current is used, the frequency of the latter, the method of introducing the current into the ground (whether galvanically, *i. e.*, by contact, or by induction), and finally the methods of observation. An opinion by the writer as to which method is best would probably be con-

sidered prejudiced, and as a generality no one method can be said to be the best. It can be said, however, that some of the claims made for some of the methods in use cannot be substantiated in practice, as they transgress natural physical laws. For certain types of ore occurrence, one method would be better than another; for other types, another method would be the best. Part of the skill in the use of geophysical methods consists in determining which method to use under a given combination of conditions, topographical and geological. The principal requirement for the execution of a successful electrical survey, however, is the ability to interpret the results. This ability is acquired by an extended practical experience based on a thorough knowledge of the fundamental electrical physics involved, combined with an adequate knowledge of geology. We believe that a knowledge of geology is quite as essential to a correct interpretation of the electrical results as is a knowledge of the electrical principles involved. Often, it is only by bringing to bear a knowledge of the geology of the region that indications, seemingly of one or more orebodies, can be properly classified as valueless. This is not done, of course, without testing the indications by the various electrical and magnetic means available. An example that might be cited is where graphite-bearing schists are encountered. Such rocks frequently show very superior conductivity in the direction of the schistosity whereas at right angles to this direction the conductivity is apt to be poor. This would not be true of a lens of pyrite, for the conductivity would probably be approximately the same in both directions. The conductivity of the indication in both directions is determined and if it is known, from a knowledge of the geology, that graphitic schists exist, a pretty safe conclusion as to what the indication means can be reached. Means exist for determining the width, in some cases with a high degree of accuracy, of the conductor giving the indication and this again affords a means of classifying the conducting body, *i. e.*, as to whether it is a narrow worthless mineralized seam or wide enough to be worth while testing by trenching or drilling. It is also possible to determine the depth to the conducting surface of the orebody and an idea of the direction and amount of its dip can be gained. Thus, in spite of a widely advertised statement to the contrary, it is frequently possible to determine the character of a mineralized area by electrical prospecting. These details are not possible of determination in all cases and the usual practice is to test by trenching or drilling all indications that cannot be conclusively classified by electrical and geological considerations.

Cost of Electrical Prospecting

The question of cost is always raised by the operating man, frequently before the question of possible results is mentioned. Whatever the cost of an electrical survey by competent operation, it is certain to be much

cheaper than any other method of prospecting the same area with the same detailed knowledge resulting. Consider a square mile which it may be desired to prospect. If this were trenched along lines 200 ft. apart, 137,000 ft. of trenching would be required. In the very remote probability that such an area could be found over which the overburden averaged not more than 5 ft., the cost probably would be not less than $0.25 per foot, or a total of $34,250. Suppose the ground were prospected by drilling; if a hole 200 ft. deep were put down on every acre, which means an approximate interval of 200 ft., the total footage would be 128,000 and the cost could hardly be less than $300,000. Of course, the drilling would not be undertaken without some encouragement from surface indications but campaigns of trenching not very different from the foregoing have been undertaken in Canada.

An electric survey over this plot of ground would probably cost less than $5000 and if no indications of an orebody were obtained, it could be safely assumed that no ore (unless pure zincblende) existed. Thus the cost of determining this fact would be very moderate compared with the cost of drilling or trenching.

The costs of an electrical survey run from $4 an acre up. They are determined by many conditions. The necessity for prospecting at close intervals, due to the expectation of small orebodies, rough topography, heavy brush or timber or other conditions which impede progress over the surface, and the necessity for using more than one method are factors which increase costs. I have known the cost to run as high as $30 per acre but in no case has it been possible to figure that the same result could have been attained more cheaply by any other method. Furthermore, the subsequent underground work has been directed with certainty, just as the X-ray aids the surgeon.

The preparation for an electrical survey consists in providing maps of the property on which the geology is plotted if possible and in submitting rock and ore specimens to laboratory test for their electrical properties. These tests are generally made free of charge by the electrical prospecting companies. A reputable organization desires to study the problem from all data available before undertaking the survey because there are cases where electrical prospecting, in its present state of development, can accomplish little or nothing. As the development of the art goes on, however, these cases are constantly becoming fewer and probably the time is not far distant when almost any problem in the search for ore or the mapping of geological structure can be solved.

Earth-resistivity Measurements in the Lake Superior Copper Country

By W. O. Hotchkiss,* Houghton, Mich., W. J. Rooney,† Washington, D. C., and James Fisher,‡ Houghton, Mich.

(New York Meeting, February, 1928)

During the summer of 1927, the Department of Terrestrial Magnetism of the Carnegie Institution of Washington joined with the Michigan College of Mining and Technology in conducting a series of earth-resistivity measurements in the Michigan copper country. The geological structure in this section is quite varied and fairly well known. The purpose of the investigation was to discover the correlation between the variations in electrical resistivity of large masses of soil and rock in place and the geological structure below the surface. The work done may be roughly divided into two parts: (1) Measurement and study of the variations in resistivity with depth, or with volume, to establish such general relations as exist between the two; and (2) the determination of the specific resistivity of various formations in order to make available more fundamental data which may aid in the interpretation of the results of the former type of measurement.

A report by Mr. Rooney, which gives in detail the measurements made and the results obtained, forms the basis for this article.

Measurement and Study of Variations in Resistivity

The possibility of applying data from measurements of the physical properties of earth material to the study of the geological structure has long been recognized, and a number of methods for determining the position and character of concealed rock strata by means of seismic, gravitation, magnetic and electrical measurements made on the surface are being used with varying degrees of success. One physical property which varies widely for different materials, and hence affords an opportunity for distinguishing between them, is their electrical resistance. Locations were selected where the geological structures were well known and where the depths to the water level and bed rock had been determined by drilling. The problem was to determine the value of measurements of resistivity and its variations as indications of the geological structure below the surface.

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Method of Measurement

The scheme of measurement is based on a special case of the four terminal conductor. If an electric current is passed between any two points in a homogeneous conducting material, the current lines will distribute themselves according to well established laws. Near the points where the current is introduced the lines of flow are nearly radial and the cross-section of the path of current flow restricted. Hence, most of the resistance of the complete circuit is found in the region adjacent to these points. For this reason any determination of the specific resistance, or resistivity, of the material which depends upon a measurement of the potential difference between the same points at which the current is introduced is liable to serious error because of the effect of contact resistance at the current terminals. Midway between the two points, however, the lines of flow will have spread so that they are practically parallel and uniformly distributed throughout a definite cross-sectional area. Determination of the potential difference between two points in the neighborhood of the central section, together with the total current flowing through the section, leads rather simply to the value of the resistivity. It has been shown by Wenner[1] that if four points are taken equally spaced in a straight line and the current introduced at the outer pair, the ratio of the difference of potential between the inner pair to the total current passed through the material is proportional to the resistivity of the material. If the four points are on the plane surface of the conductor, the resistivity is given by the formula

$$\rho = 2\pi a \frac{V}{I}$$

where ρ is the resistivity, V the difference of potential between the inner pair of electrodes, I the true current through the ground and a the distance between adjacent terminals.

If the four points, instead of being at the surface, are taken within the body of the material so that the distance between them is small compared to the distance to the nearest surface, the formula for the resistivity becomes

$$\rho = 4\pi a \frac{V}{I}$$

The mathematical treatment of the subject is based on the assumption of a homogeneous material. Determinations of resistivity made by this method on earth materials which are not homogeneous must be considered as giving a general average value in which the resistivity of the material near the line of terminals is more heavily weighted, the

[1] F. Wenner: A Method of Measuring Earth Resistivity. U. S. Bureau of Standards *Bull.* (1916) **12**, No. 4, 469.

weighting diminishing with distance from this line. Roughly speaking, the effect of material at distances equal to the distance between adjacent terminals is found to be so small that the effect of materials beyond this distance is negligible. Thus, the body of earth involved in a single determination has linear dimensions of the same order as the distance between terminals. By increasing the distance between terminals greater depths of earth may be included so that from a series of such measurements a fairly satisfactory knowledge of the variation of resistivity with depth can be obtained.

As an illustration of the method, suppose a series of observations are taken on the surface of a lake where the water is approximately 25 ft. deep. The terminals are spaced for the first determination of ρ at 5 ft.; that is 15 ft. between the outside or current terminals: V and I are measured and the value of ρ computed from the relation $\rho = 2\pi a \frac{V}{I}$. This value of ρ will be the average resistivity for the water contained in a semicylinder 5 ft. long and 5 ft. in radius. The terminals are then changed to a 10-ft. interval, that is 30 ft. between extreme terminals. The value of ρ should be the same as before, except for a slight possible change in resistivity due to the temperature gradient. Increasing the terminal intervals by 5-ft. steps, no change will be found in the value of ρ until the distance between the outer terminals exceeds 75 ft. As soon, however, as the terminal interval exceeds 25 ft., the material at the bottom will begin to be included in the measured volume and a change in the average resistivity will be noted, the change becoming more marked as larger intervals are taken and greater amounts of the bottom material included. When the distance-resistivity curve is plotted with distances as abscissas and resistivities as ordinates, the curve will be a horizontal straight line for the first 25 ft. and then will have a turning point, the curve rising or falling depending on whether the resistivity of the material of the bottom is more or less than that of water. The location of this turning point will give the depth of the water and the change in slope of the curve will be an indication of the relative resistivities of the material of the bottom and water. When the terminal interval becomes large enough to include in the measured volume a bed of still different resistivity, another turning point will be obtained and the depth to this bed determined. In a similar manner, other concealed changes in structure may be revealed by resistivity measurements made at the surface, provided the lateral distribution is fairly uniform and the resistivities of the materials making up the formation are sufficiently different one from the other.

Apparatus Employed

The apparatus used was developed by the Department of Terrestrial Magnetism of the Carnegie Institution and given a fairly complete trial

by Mr. Rooney near the vicinity of Washington, D. C., at Watheroo, Western Australia, and at Ebro, Spain, before its use in the Lake Superior copper country. The measuring circuits, with instruments, and the arrangement of the contact points are shown diagrammatically in Fig. 1. The contact to earth at points C_1, P_1, P_2 and C_2 is made in ordinary soil by means of steel rods about ⅝ in. in diameter and 2 ft. long. These rods are usually driven into the ground to a depth of about one foot, but the depth should never exceed 5 per cent. of the distance between electrodes. For work on bare rock surfaces, where it would be impracticable to drive in electrodes, satisfactory contact may be secured by the

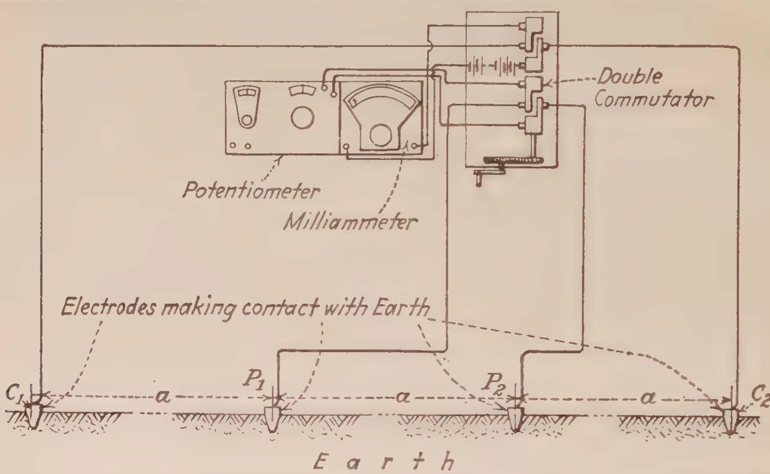

FIG. 1.—DIAGRAM OF EARTH-RESISTIVITY METER, SHOWING CONNECTIONS TO EARTH.

use of flat spiral coils of copper wire about 2 in. in diameter embedded in cotton waste soaked with a solution of copper sulfate. Electrodes of this latter type may be weighted down with a piece of stone to insure good contact on a horizontal surface or wedged against the surface with wooden rods when the surface is not horizontal. Connections are made to the electrodes with large battery clips and the circuit completed to the measuring instruments with No. 14 double-braided, rubber-covered copper wire in suitable lengths. It was sometimes found necessary to reduce the contact resistance at the electrodes by using more than one steel rod at the "contact points," or by increasing the size of the coil used on bare rock surfaces.

Although the method used gives a determination of the resistivity which is independent of the contact resistance it is desirable that the current flowing in the circuit, for a given battery voltage, be large enough to be read on the ammeter to a fair degree of accuracy. Special precautions in making contact with the ground or rock are usually required only when the interval between electrodes is large and does not affect the

results, since the area over which the rods forming a single electrode are distributed can be kept sufficiently small to be considered as a point in comparison to the distance between electrodes as required by the underlying theory.

The measuring instruments consist of a potentiometer and a milliammeter. The former is a Leeds & Northrup potentiometer indicator with ranges 0 to 100 and 0 to 1000 millivolts, and the latter a Weston, Model 1, direct-current milliammeter with four ranges: 0 to 5, 0 to 50, 0 to 200 and 0 to 500 milliamperes. By using various combinations it has been found possible to obtain accurate results on materials of resistivities from less than 100 to more than 4,000,000 ohms per centimeter cube.

In order to minimize the effect of polarization at the electrodes and to eliminate the effect of currents present in the earth from other sources, a special double commutator built in the shops of the Department of Terrestrial Magnetism was inserted in the lines between the meters and the electrodes. One section of the commutator, as may be seen in Fig. 1, reverses the voltage applied across the extreme electrodes and consequently the current flowing through the ground. With the other the potential difference between the intermediate electrodes is rectified so that it can be measured with the direct-current potentiometer, while any extraneous potentials are reversed so rapidly that they do not register. The commutator is turned at a speed between 10 and 15 r. p. s. and hence the current through the milliammeter is interrupted from 20 to 30 times a second for a brief interval. This causes it to register as a steady current but one which is somewhat lower than the true current through the ground. The ratio of the true current to the registered value may be calculated from the lengths of the conducting and insulating sectors of the commutator and that of the bearing surfaces of the brushes, or even more readily determined experimentally by using the instrument to measure the drop across a known resistance with the commutator alternately running and stationary. The ratio, or "commutator factor," is a constant as long as the adjustment of the brushes is not changed.

The current is supplied by radio "B" batteries. Two in series, giving 90 volts, are generally used although a greater or lesser voltage may sometimes be used to advantage. Connections from the batteries and commutator to the meters and lines are made with suitable leads, plugs and jacks.

General Determinations

To demonstrate the possibility of determining concealed changes in geological structure, a series of measurements was made in the vicinity of two diamond-drill holes (Nos. 24 and 25) on the property of the Copper Range Consolidated Mining Co., west of Trimountain in Houghton County. At this location the rocks of the Keweenawan series are

covered with glacial drift to a considerable depth. The depth of the overburden is given by the drill records as approximately 64 ft. at drill hole No. 24 and 76 ft. at drill hole No. 25.

At hole No. 24, two lines were laid out spanning the drill hole, one approximately along the strike and the other across it. Determinations of resistivity were made with electrode separations from 10 to 100 ft. in 10-ft. intervals to include successively corresponding depths of earth. Additional measurements were made at electrode separations of 150 and 200 ft. and intermediate measurements were made at separations of 25 and 35 ft. along the strike.

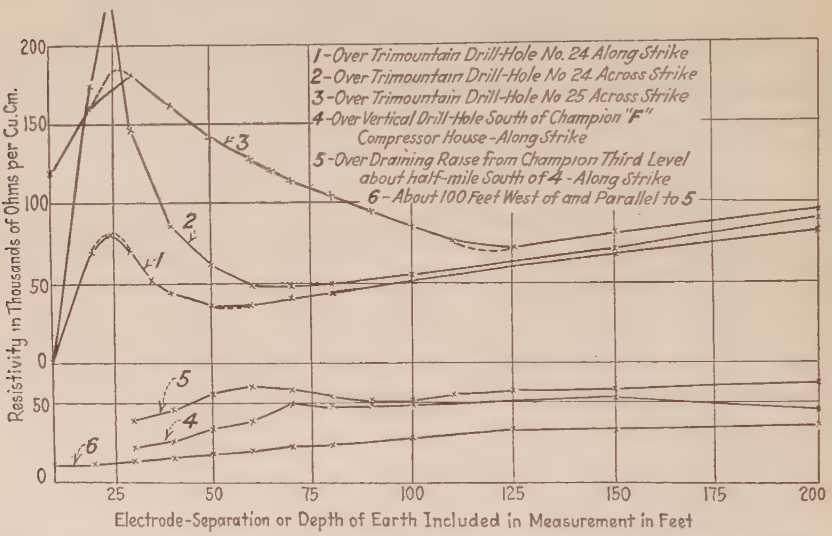

FIG. 2.—RESISTIVITY DEPTH CURVES FOR DETERMINATION OF OVERBURDEN DEPTH AND UNDERGROUND WATER LEVEL NEAR PAINESDALE, HOUGHTON COUNTY, MICH.

The results of these determinations are shown in curves 1 and 2 of Fig. 2. The curves are typical for a region of this character, the average resistivity first increasing with included depth and then falling off sharply to a minimum beyond which there is a slow and fairly uniform increase in the resistivity value as the depth included in the measurement is made greater. The significance of the curves from the standpoint of prospecting lies in the variations in the recorded resistivity rather than in the absolute values, and the important factor is the electrode separation or included depth at which a marked change occurs. On both curves, it will be noted, the resistivity at the smallest electrode separation is low. These determinations represent little more than the thin layer of moist, vegetation-supporting soil at the surface, which is generally a good conductor. Beneath this lies the dry coarse material of the drift with the result that an increase to 20 ft. in the electrode separation is accompanied

by the sharp rise in the average resistivity. At 30 ft. electrode separation, this increase is found to have ceased, and including depths of 40 ft. caused a pronounced drop in the recorded value. This means that at some place between 20 and 30 ft. the electrical character of the material changed, a less resistant material coming into play. This change is undoubtedly due to the presence of water. To fix the level of this underground water with more certainty, the determinations at separations of 25 and 35 ft. were made, with results which will be seen to be quite consistent with the general trend of the curve. Both curves are interpreted as indicating the depth to underground water as 25 ft.

As the electrode separation is still further increased, another decided change in the trend of the curves is observed, both having minima between 50 and 70 ft. It is obvious that beyond these separations some material of higher resistivity than the moist overburden is being included in successively larger volume to cause the average resistivity to increase. The electrode separation at which this increase commences is taken as approximately equal to the depth to rock. It should be borne in mind in this connection that the depth indicated by the resistivity records is not that at a single point but along a line between the intermediate electrodes, since all the material between them to a distance equal to their spacing has an effect on the result. In the present instance, the lines were so laid out that curve 1 indicates the average depth for some 50 or 60 ft. northeast of the drill hole and curve 2 that for the same distance northwest of it. The depth to rock as determined from curve 1 would be between 50 and 55 ft. and from curve 2 about 70 ft. The average for the two, then, is between 60 and 65 ft., which is not far from the depth of 64 ft. given by the drill records at hole No. 24.

It will be noted that the absolute values of resistivity are quite different on the two curves up to about 70 ft. This is probably due to non-uniformity in the overburden and is capable of explanation on the assumption either that the moist surface layer is shallower or the underlying drift material is of coarser structure on the line across the strike than on that along it. At greater separations, where proportionally larger volumes of the rock are included and local differences in the overburden are less apt to predominate, the absolute values differ only by a few per cent. A similar series of observations was made at drill hole No. 25, some 800 ft. northwest of No. 24, with results as shown in curve 3 of Fig. 2. Here only a single line across the strike was surveyed. The resistivity data here give the underground water level as some 28 to 30 ft. below the surface and indicate the depth to rock to be in the neighborhood of 120 ft. The drill records give the depth to rock at hole No. 25 as 76 ft.; hence the agreement is not nearly as good as at hole No. 24.

The conditions for measurement were not nearly so good at this site because of the irregular topography, the line running mostly along the

bottom of a ravine with steep slopes upward on either side. This may account for much of the difference between curve 3 and the previous two since the lateral spreading of the current lines in measurements made in the ravine tends to exaggerate the effect of the drift material in the adjacent hillsides, but such an effect is hardly enough to account for all the difference between the two depth determinations. A possible explanation of the difference between the depth indicated by the resistivity determinations and that measured in the drilling operation is that the drill may have struck a high point on an irregular rock surface. If this were the case, the average depth along a line through it might be more nearly equal to the value determined from the curve. Irregularities in rock contour sufficiently great to make such a supposition tenable are not uncommon in the outcrops of the region.

Another series of observations was taken in the neighborhood of a vertical drill hole about one-half mile south of Champion mine. Here the depth to the water line could be obtained by direct measurement with a plumb line. The average resistivity, as shown in curve 4 of Fig. 2, was found to increase with included depth until a separation of 70 ft. was reached and then to decrease for a time, indicating the depth to water to be approximately 70 ft. Direct measurement in the casing checked this value to within one foot.

Curves 5 and 6 of Fig. 2 show the results of measurements made along two lines at a point about halfway between the Champion and Globe mines. The lines were about 100 ft. apart and along the strike. Just beneath one of these lines is a "raise" from the workings of the Champion mine to the ledge top, constructed to drain off as much as possible of the underground water and prevent it from seeping into the lower levels of the mine. It will be noted that both curves are comparatively flat and lack the pronounced maxima which characterize water-level points in the upper group of curves. Here again the topography was very irregular, and most of the differences between curves 5 and 6 are undoubtedly to be attributed to the fact that the control third of the former line runs along a hillside while the latter was in a gully. Neither here nor at the "water check" drill hole were the determinations continued to sufficient depths to determine the distance to rock.

A series of measurements was taken to determine depth of overburden along a line running slightly east of north between Allouez and Ahmeek in Keweenaw County. The results of the determinations at this site are shown in the curves in Fig. 3. Here the dry overburden near the surface had a very high resistivity, a value of nearly 400,000 ohms per centimeter cube being recorded at a separation of 25 ft. Since a knowledge of depth to rock only was desired here, sufficient short separation measurements to fix definitely the water level were not attempted, but it is probably not far from 25 ft., since the resistivity dropped rapidly from that separation

to 100 ft. Between 100 and 150 ft., the decrease in resistivity was sharply checked, indicating the inclusion of material of higher resistivity than the moist portion of the overburden, although probably less than that of the material near the surface. It would appear, therefore, that the average depth of the overburden is somewhat less than 150 ft. In undertaking this portion of the work, it was planned to investigate the possible exist-

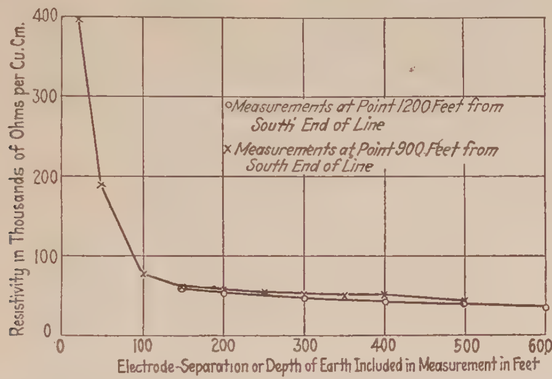

FIG. 3.—RESISTIVITY DEPTH CURVES FOR DETERMINATION OF OVERBURDEN DEPTH WEST OF STATE ROAD BETWEEN ALLOUEZ AND AHMEEK, KEWEENAW COUNTY, MICH.

ence of a narrow transverse gully in the bed rock of the region, but because of the masking effect of the high-resistivity layer at the surface and the difficulty of checking small differences in resistivity under the unfavorable weather conditions existing at the time, that part of the program was abandoned.

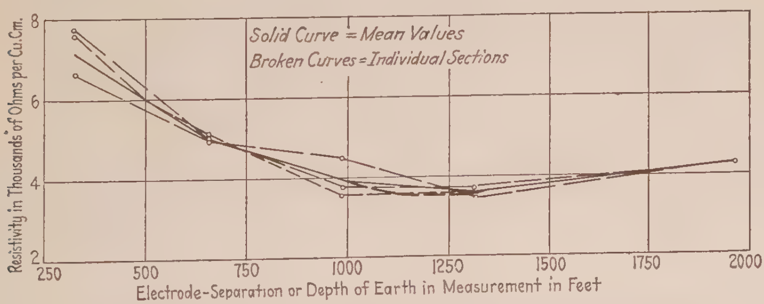

FIG. 4.—RESISTIVITY DEPTH CURVES OVER EASTERN SANDSTONE BEDS, NEAR HOUGHTON-BARAGA COUNTY LINE, MICHIGAN.

An example of the variation of resistivity with depth on a somewhat larger scale is shown in Fig. 4. The curves here show the results of measurements along a line some 6000 ft. long over the flat-bedded eastern sandstone near the Houghton-Baraga county line. The resistivity of the sandstone near the surface here was found to be very uniform and not far from 10,000 ohms per centimeter cube. To depths of 200 ft. only

small and random changes were noted in the recorded values. When further determinations were made with electrode separations greater than 200 ft., the resistivity dropped gradually to a minimum reached when the depth included was about 1200 ft. Including rock to greater depths caused a small increase in the recorded values. The results obtained at three different positions were quite similar, as will be seen in the curves for the individual sections. The sandstone beds in this section are known to be thick, but exact data on their depths are not available. The resistivity of porous rock of this kind is to a great extent dependent on the amount and character of the solutions it contains, and the decrease in the average resistivity when materials between depths of 200 and 1200

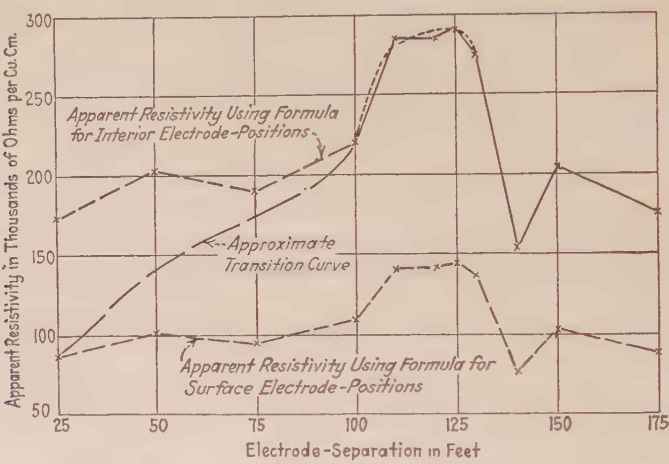

FIG. 5.—RESISTIVITY MEASUREMENTS IN STOPE ABOVE THIRD LEVEL OF CHAMPION MINE, TO DETERMINE THICKNESS OF LEDGE ABOVE.

ft. are included is doubtless due to the presence of more moisture or more concentrated solutions in the rock at those depths.

As a further demonstration of the possibility of determining the location of discontinuities by this method, an unusual and interesting series of observations were made underground in a stope above the third level of the Champion mine, with a view to checking the thickness of the ledge above the stope. Measurements were made at a number of different electrode separations on a line along the stope, the electrodes being set on the center line of the back of the stope. The results obtained are shown in the curves in Fig. 5. Here the conditions were abnormal, in that the electrodes were so situated that they can be considered neither as placed on a plane surface of the conductor nor entirely within its mass. Hence neither of the formulas given in the discussion of the method exactly represents the relation between the resistivity and the measured current and potential. For this reason the values shown in Fig. 5 are termed "apparent resistivity," the true value lying in general somewhere

between the two curves. For the smallest separations used (25 ft.), in which the distance between electrodes was about equal to the width of the stope, the conditions under which the surface formula $\left(2\pi a \frac{V}{I}\right)$ holds were approximated, while for separations of 100 ft. or more, in which the cross-sectional area of the effective conducting volume was great in comparison to that of the open stope, the true values may be taken as corresponding quite closely to those shown on the upper of the curves, particularly if the stope itself is considered merely as an area of very high resistivity. The approximate transition curve shown in Fig. 5 gives an indication of the relative values at least for intermediate separations.

It should be borne in mind in this connection that the variation rather than the absolute value of the resistivity is the determining factor in this type of measurement, and hence the inability to fix the latter definitely is relatively unimportant provided that the relations between the resistivity and the measured current and potential do not change rapidly with electrode separation at the critical points on the curve. In the present case, for instance, any inferences drawn from the portion of the curves between separations of 25 and 100 ft. would be of little value because, as indicated by the rough transition curve, the relation is changing rapidly in that region. On the other hand, when electrode separations greater than 100 ft. are considered, the area of the open stope has become so small in comparison to that of the effective conducting volume that the effect of the change in the ratio of the two is only a minor factor in causing changes in the measured values of resistivity.

Examining the curves with these facts in mind, the most striking feature is the abrupt drop in resistivity which starts shortly after an electrode separation of 120 ft. is reached. This is taken to mean that when separations beyond that were used and the measurements began to include material beyond that radial distance from the electrodes, the moist overburden just above the ledge began to come into play with the consequent lowering of the apparent average resistivity of the measured volume. The thickness of the ledge then, as determined from the resistivity data, is given by the electrode separation at which the curve began to drop sharply, the best assignable value being 125 ft. This value is found to check closely with the records of the mine, a drill hole about 800 ft. northeast of the place at which the test was conducted showing a thickness of ledge of 120 ft. and one approximately the same distance southwest a thickness of 130 ft.

If the cross-sectional area of the mine in this region were compared with the cross-section of the effective conducting volume for each electrode separation, further correlations between the resistivity variations and the structure would undoubtedly be found. The increase in the apparent resistivity from that at 25-ft. separation, which represents a

fairly uniform volume of the lode, to that at 100-ft. separation is unquestionably connected with the inclusion in the measured volume, first, of the open-stope area and then of increasing volumes of the loose waste rock and stamp sand between the stope and the third level. Likewise the second increase in resistivity following the drop due to the moist overburden may be attributed to the effect of the dry portion of the overburden above the water line.

Resistivity of Specific Earth Materials

So far we have been concerned only with the variations in the determined resistivity values as indications of changes in structure. To interpret the results of such determinations to best advantage, a knowledge of the absolute resistivity of different earth materials is essential. For nearly all such materials, water being the exception, laboratory results are apt to be misleading, because the resistivity depends not only on the composition of the material but on its physical condition, which is liable to great variations in different samples. Cracks, position in flow, impregnating solutions, etc., all have an influence on the resistivity of the rock mass taken as a whole. Hence to obtain information suitable for the purpose, the resistivity must be determined with the material *in situ*.

To secure such fundamental data for some of the characteristic rocks of the copper country, measurements were made on a number of outcrops, the conditions being so chosen that the entire volume included in the measurement should consist of a single type of rock. A great many measurements were made at different locations, on typical formations, and under a variety of conditions as to volume, position in the flow and direction of current flow. Details are given here for two typical cases only.

Quincy Trap

Measurements were made at electrode separations ranging from 5 to 100 ft. on a fairly extensive outcrop. The mean value for 12 sets of observations was 236,000 ohms per centimeter cube, minimum 185,000 and maximum 390,000. Eight determinations along the strike near the middle of the outcrop, at electrode separations from 10 to 75 ft., gave values between 185,000 and 250,000, the average being 214,000. Two series at 10-ft. separations across the strike showed a mean of 279,000. The variation in the results obtained at different positions on the outcrop was negligible. One measurement along the strike at electrode separation of 5 ft. gave 284,000 ,and one at 100-ft. separation a value nearly twice the average for the outcrop. The high value obtained at the 5-ft. separation may be due to surface cracks and that at 100-ft. separation to the possible inclusion of other material, since the volume included in this measurement extended laterally well beyond the visible rock surface.

TABLE 1.—*Resistivity of Trap Rock in Outcrop at Quincy Mine*
¼ Mile West of No. 2 Shaft

Location	Electrode Separation, Feet	Direction of Current Flow	Resistivity, Ohms per Cm. Cube
Near center of outcrop northeast of highest point, on flat nearly horizontal portion.	5	Along strike	284,000
	10	Along strike	212,000
	15	Along strike	221,700
On line including highest point of outcrop.	25	Along strike	185,000
	50	North, along strike	200,500
	50	South, along strike	254,000
	75	Along strike	221,000
	100	Along strike	390,000
At point as near foot of flow as possible to get about 200 ft. south of highest point of outcrop.	10	Along strike	211,000
	10	Across strike	252,500
At point as near hanging side of flow as possible to get about 100 ft. west of highest point of outcrop.	10	Along strike	209,000
	10	Across strike	306,000

Great Conglomerate Outcrop at Eagle Harbor

The results obtained on an outcrop of the great conglomerate along the shore of Eagle Harbor were remarkably uniform. The outcrop was small and the largest volume it was considered advisable to measure was one with linear dimensions of 20 ft. Seven determinations at separations from 4 to 20 ft. gave a mean value of 109,200 ohms per centimeter cube, the extreme values differing by less than 10 per cent. A somewhat higher value (141,500) was obtained with a separation of 2 ft., but this determination is more likely to be erroneous by reason of inaccurate spacing of electrodes or to be affected by local inhomogeneity and should not be regarded as being as representative as the other values.

TABLE 2.—*Resistivity in Outcrop of Great Conglomerate at Eagle Harbor*

Location	Electrode Separation, Feet	Resistivity, Ohms per Cm. Cube
On top of narrow ridge formed by outcrop, opposite entrance to harbor, near range light cut.	2	(141,500)
	4	117,000
On sloping side of outcrop toward the harbor.	5 east	104,200
	5 west	103,400
	10 east	108,700
	10 west	111,800
	20	109,800
On ledge at harbor shore	5	110,000

An interesting series of determinations was made on the main Calumet and Hecla copper-bearing conglomerate. A cave-in from surface,

near Calumet and Hecla shaft No. 12, exposed a block of the conglomerate with face about 20 ft. long and 12 ft. high. To the eye and hand, some parts of the face were apparently barren of copper while others were well impregnated. Only small volumes, 2 and 3-ft. separations, were measured because the block has no plane surface greater than about 60 sq. ft.

Measurements on the barren part of the block gave fairly uniform results comparable to those obtained on the great conglomerate outcrop at Eagle Harbor. The mean value from the three measurements made was 126,000 ohms per centimeter cube. On the part of the block where copper was apparent, the resulting values were not only much lower but differed widely with position and direction of current flow. The largest value registered in this part of the block was about 20,000 ohms per centimeter cube and results all the way from that value to practically zero were obtained in the other measurements. A probable explanation of the seemingly anomalous results is that the amount and distribution of the copper in the different parts of the block are so variable as to vitiate the assumption of homogeneity on which the method is based. The presence of an irregular mass of copper, or loosely connected masses, confined to a part of the block and running transversely through it, could account for all the differences noted in the results. While the results obtained here have probably little meaning as far as the actual resistivity is itself concerned, their bearing on the problem of locating concealed copper is of interest. It would seem well worth while to follow them up by a determination of the copper content of the two sections of the block.

Plans for Further Investigation

The character of geological discontinuities which may be detected by the method described is little known. The experiments described indicate much promise, but a large amount of work must be done before it will be possible to ascertain how much we may be able to understand of the "language" in which the rocks speak to us through this method.

It is apparent that both character and condition of the materials investigated control the resistivity. Since these each vary so widely, there is an infinite variation possible. It is therefore obvious that only certain ranges of differing conditions can be detected by this method, so it will be unwise to expect too much until further tests give us a better basis of judgment.

In the tests of the past season we have found resistivities per centimeter cube varying as follows:

Glacial drift	825 to 396,000
Keweenawan lavas	11,900 to 4,355,000
Nonesuch shale	12,300 to 18,100
Great conglomerate	104,000 to 141,500
Eastern sandstone	3,500 to 11,780
Western sandstone	17,900 to 41,400

These figures show that various geologic formations cannot be expected to have characteristic numerical resistivity values which without further aid will distinguish them. The resistivity method should be looked upon as a means of securing certain geological data to be studied and interpreted in connection with all other geological data. It will not be conclusive of itself in general.

With these considerations in mind, plans are being made for the work of a field party in the summer of 1928 in the iron districts of Michigan. Tests of the resistivity of the various rock formations in various known conditions are contemplated. For this purpose information is being gathered as to locations where drill records are well kept and give as nearly complete information as can be found.

In outcrop, it is proposed to measure the resistivity of the iron formation in its different phases: the salts, greenstones, granites and quartzites. Glacial drift of known character and with known water conditions, and of varying thickness will be tested. Tests will be made about some of the mines having special water problems. The resistivity relation between iron ore and the adjacent iron formation will be tried out through known thickness of drift covers to see if the greater porosity and water content of the ore will produce a sufficiently different resistivity to permit measurement. Another item that will be studied is the black slate foot wall. It is possible that the carbon in this may cause resistivity differences between it and adjacent beds that will permit it to be followed.

In summary, we may say that work has proceeded far enough to assure us that in many instances depth to ground-water level and depth to rock can be measured with sufficient accuracy to give information of value. There is reason to hope that further study will permit us to tell many things of importance about the character and condition of the rocks below the glacial drift. It is still too early in the investigation to express an opinion as to whether or not the method will have definite utility as a means of locating ore, but it is entirely reasonable to think that it may have.

DISCUSSION

W. Weaver, Madison, Wis.—Professor Fisher, you spoke of the turning point in curve No. 3 indicating a depth to ledge which was somewhat greater than actually obtained.

J. Fisher.—The curve shows a depth of about 1200 feet.

W. Weaver.—I did not get the actual measurements.

J. Fisher.—Except that measurement, we do not know anything about the actual depth of the sandstone.

W. Weaver.—I believe we are talking about a different curve. I mean one that read approximately 120. Am I mistaken in believing that you said you moved about 600 or 800 ft., and I understood you to say you had measured values at that point.

J. Fisher.—About 800 ft., measured depth 120 ft. Actual depth 76 ft.

W. Weaver.—There is one thing that seems to me worth noting; that is the intimate connection between this experimental work and the theoretical work on which I reported in my paper. The paper now under discussion is phrased entirely in terms of resistivity, whereas I happened to have traced my work in terms of potential differences and conductivity. As resistivity is simply the reciprocal of conductivity, it is merely a matter of words. This is experimental work of exactly the kind of which I have discussed the theory.

There is one question relative to the application which has been given in this paper, and in preceding papers by Rooney and Gish, that bothers me considerably. As I understand it, your interpretation of depth on these curves is largely based on the turning point, which amounts to saying that when the electrical spacing reaches a certain distance, say 200 ft., you begin to experience resistivity that occurs at a depth of 200 feet.

J. Fisher.—Yes.

W. Weaver.—It would be a distinct advantage if that turns out to be a sufficiently accurate criterion for all future work.

J. Fisher.—The results check very well. We assume we measure the resistivity of a semi-cylinder, the distance between the electrodes being the radius of the cylinder .

W. Weaver.—I realize that making this point places me in the position of standing before a giraffe and saying, "There ain't no such animal," for you make your measurements on the basis of that assumption, and the measurements turn out surprisingly well. However, in the second curve of my paper, on the basis of an assumption of a homogeneous earth, approximately 30 per cent. of the current must penetrate deeper than the distance between the electrodes. So in the statement that your currents are penetrating a depth equal to the electrode spacing you are apparently making a statement only very roughly true, say seven-tenths true. However, our computation is based on a homogeneous earth, and we do not actually have a homogeneous earth.

One further remark: Your curves showing the reversal are curves of exactly the type of which I gave theoretical examples toward the end of my paper, and show the effect of lifting to the surface one layer after another and getting the consecutive effects of the various resistances.

In regard to the question as to how the slope of that curve may depend upon the conductivity, I may say we have the answer to that question. Our complete solution for any number of layers and any conductivity contains that answer. It does not contain it in simple enough fashion so that one can read it out of the formula. It requires laborious computation to produce curves from which the facts may be read, rather than from the formula, but the relationship does exist and when sufficiently analyzed to merit interpretation it will be a very useful thing.

J. Fisher.—It looks as though the position on the turning point of the curve is entirely independent of whether 30 per cent. or 60 per cent. of the current is in the second layer.

K. Sundberg, Houston, Texas.—We also have a similar system, and we have come to the conclusion that this method of determining the depth by taking this turning point is correct, not only by experiments on a small scale, and measurements, but also by theory. I hope that within a short time we will be in position to publish these conclusions.

J. Fisher.—The method has been tried out by Mr. Rooney. He started about four years ago. It was tried out at Washington, and also in Australia and Spain.

There is no hope of discovering whether or not a certain copper-bearing lode lies 100 ft. below the surface, or anything of that kind, no matter how much work is done on the method, but it is of interest to the miner in the Lake Superior section to know where his water line is and also where his ledge is.

J. Janney, Pioche, Nev.—Will you explain what would be the limits of the use of this method in finding what local veins are applicable to deposit in vertical strata?

J. Fisher.—I do not think it has any use. It is not a question of limits. It would probably be possible to determine whether the bed rock was a sandstone or a lava flow, but whether or not a vein could be located is a different question. I do not see how it could.

W. Weaver.—I think Professor Fisher's remark that the method has no use is a conservative statement of the type the industry needs. I think also that it goes a little too far. I would like to point out that one of my curves shows that a region of high conductivity only 10 ft. thick buried at considerable depth has a large effect on the surface.

All the work we have presented here was on horizontal layered structure and does not apply directly to the question of vertical layers, but a vertical structure is merely a horizontal structure turned over. So far as the mathematician is concerned, he rotates his paper 90° and the horizontal becomes the vertical, and there is no great difference between the analyses of the two cases; the analysis of one is, with proper modification, applicable to the other.

F. W. Lee, Washington, D. C. (written discussion).—We are grateful to the writers for developing and interpreting conductivity methods of measuring resistance of material when covered with overburden. Attention should be called to the fact that instruments capable of directly measuring the resistance between equipotential lines, as illustrated in the papers, are now available.

The adaptation of this method to the measurements of resistance of various strata in drill holes, especially in the oil district, would prove of interest in interpreting electrical induction methods in that field.

Certain Applications of the Surface Potential Method

By Warren Weaver,* Madison, Wis.

(New York Meeting, February, 1928)

Some of the advantages of the inductive method of electrical prospecting were emphasized in a paper by Dr. Max Mason.[1] Since this emphasis was misunderstood by some to indicate a too exclusive interest in inductive methods, we are especially happy to help correct this impression by reporting on some of our work on surface potential methods.

Earth Currents and Surface Potentials

When two electrodes, A and B, are placed on the surface of the ground a distance d apart, and a 100-volt battery is connected across the elec-

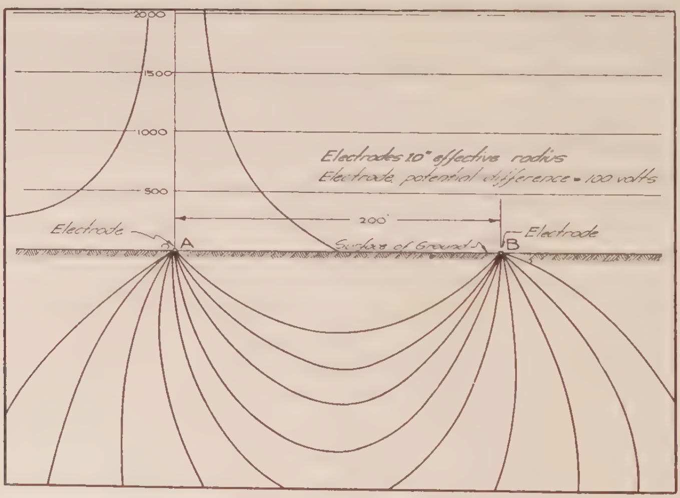

Fig. 1.—Lines of current flow and surface potential in millivolts in the case of a homogeneous earth.

trodes, current flows from the positive electrode A to the negative electrode B. The current does not pass directly from A to B along the straight line joining these two points. In fact, a negligible amount of the total current chooses this direct path, and the flow actually takes place

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* Physical Exploration Corpn.

[1] See page 9.

along the curved lines shown in Fig. 1. The curved current paths of this figure indicate that some of the current penetrates to depths equal to and greater than the distance between electrodes. It may seem, at first sight, strange that any of the current should go so deep, rather than to choose the shorter paths near the surface of the ground. The nature of such steady currents, however, is to repel one another, somewhat as two bodies with like electrification repel one another. Each current element thus seeks a path that enables it to pass from A to B and, at the same time, stay as far as possible from all the other current elements. This mutual repulsion causes a spreading out of the current paths and forces some of the current to penetrate to great depths.

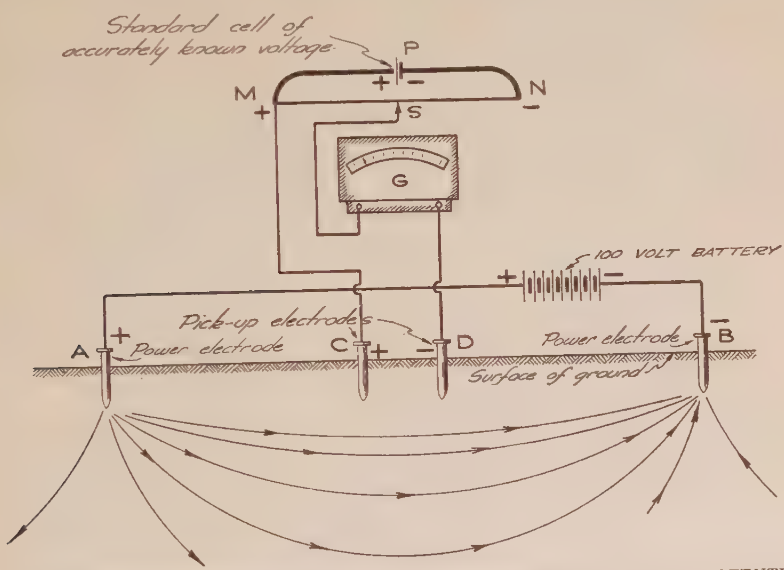

FIG. 2.—SCHEMATIC DIAGRAM SHOWING METHOD OF MEASURING SURFACE POTENTIALS.

A quantitative measure of the depth of penetration is given in Fig. 3, which gives the fraction I_D/I of the total current I which penetrates deeper than a depth D. From this figure, for example, one reads that three-tenths of the total current penetrates deeper than the distance between electrodes, and seventeen-hundredths deeper than twice the distance between electrodes.

In Fig. 1, the curve plotted above the surface of the ground shows the variation in electrical potential at points on the surface and along the straight line joining A and B. Electrical potential is a quantity which bears the same relationship to a flow of electric current that hydraulic pressure, or "head," does to the flow of the liquid. The electric current that flows between any two points of a conducting material is directly

proportional to the difference of potential, or "electrical pressure" between these two points. If the potential difference is decreased the current will decrease; and conversely, if the current decreases we can conclude that the potential difference has decreased. Thus to any current distribution there corresponds a definite distribution of potential.

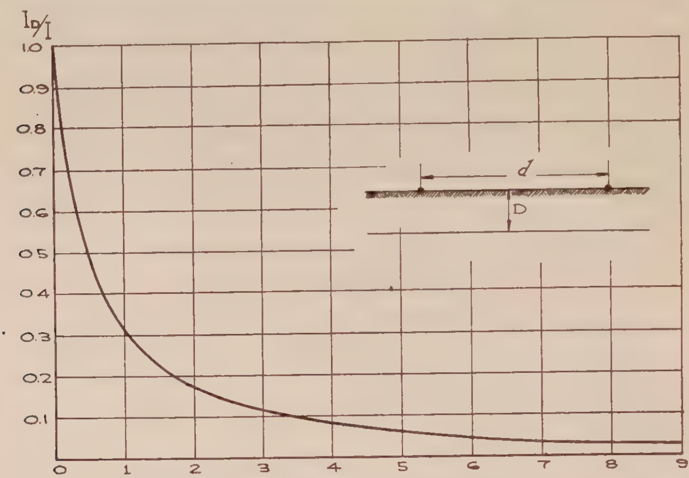

FIG. 3.—THE FRACTION I_D/I OF THE TOTAL CURRENT WHICH PENETRATES BELOW A DEPTH D.

MEASUREMENT OF SURFACE POTENTIALS

Since all applications of the surface potential method depend on the possibility of experimentally measuring the potential difference between two points on the ground surface, it will be well to indicate the principle of such measurements. In Fig. 2 (which refers to the situation depicted in Fig. 1), suppose we wish to measure the potential difference between the points C and D, the power electrodes at which the current enters and leaves still being located at A and B. Metallic stakes are driven into the ground at C and D. One of these stakes, or electrodes, namely C, is connected directly to a potentiometer P. This potentiometer consists, in essence, of a resistance wire MN across which is impressed the accurately known voltage of a standard cell. A slider S may be moved into contact with any point of the wire MN, and since the potential drops at a constant rate along this uniform wire, the potential difference between M and S is known for any position of S. The other pickup electrode D is connected, through the galvanometer G, to the slider S. Now the pickup electrode C is directly connected to M by a low-resistance wire along which there is a negligible potential drop. Therefore, C and M are at same potential. The slider S is now moved into such a position that no current flows from S to D, or vice versa; this fact being revealed by a zero

reading of the galvanometer. When no current flows between S and D, they must be at the same potential. The two pairs of points, C and M, D and S, are now at the same potential, so that the potential between C and D, which we wish to measure, is equal to the known potential between M and S.

In the schematic diagram, Fig. 2, the situation has been reduced to its simplest possible form. The problem of designing refined apparatus for obtaining the necessary high accuracy in the field involves many important considerations which do not properly fall under the present discussion.

The potentials of Fig. 1, as in all the figures of this paper, are expressed in millivolts, the power electrodes being assumed to have effective radii of 10 in. and to be maintained at a potential difference of 100 volts.

Surface Potentials When Layer is Present

The current and potential distributions shown in Fig. 1 are calculated on the basis of a perfectly homogeneous earth. It is convenient to refer to these as *normal* distributions. If the earth is not homogeneous as regards its electrical conductivity, this normal current flow is distorted

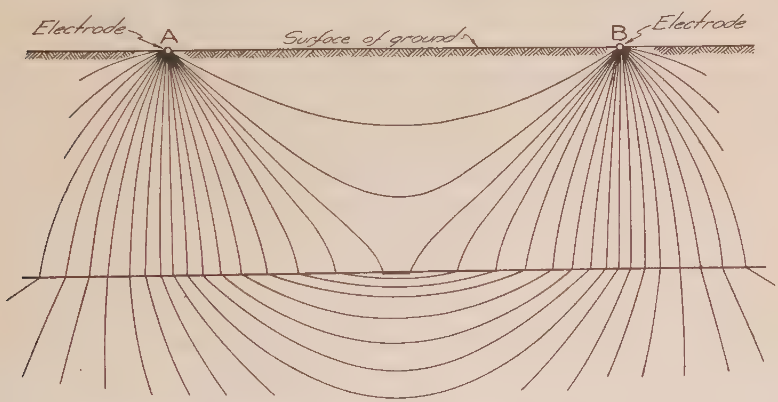

Fig. 4.—Cross-section showing lines of current flow between electrodes. Lower material fifty times as conducting as upper layer.

and there is a resultant alteration in the potentials which exist at points on the surface of the ground. If, for example, a layer of fair conducting material is underlaid by a very good conducting material, the current tends to go from the electrode A down to the good conducting material, pass over through it, and then come back up to the output electrode B. The current paths for such a case are shown in Fig. 4, based on an exact solution of this current flow problem. This greatly altered current distribution is not directly observable, but accompanying it is an alteration of the surface potentials which is observable. In fact, it is clear

from Fig. 4 that more current flows in the lower region and less near the surface of the ground than would flow, in each case, if the earth were homogeneous. If less current flows near the ground surface, this means that the potential difference between two points on the surface of the ground must be less than it would be if the layer were not present. Fig. 5 shows the actual surface potentials which exist when a layer, 40 ft. deep and of one conductivity σ_1, is underlaid by material of a second conductivity σ_2. The various potential curves are drawn for different ratios of these conductivities. The heavier curve labeled $\sigma_1 = \sigma_2$ shows the normal potential that would exist if the layer were absent. When the

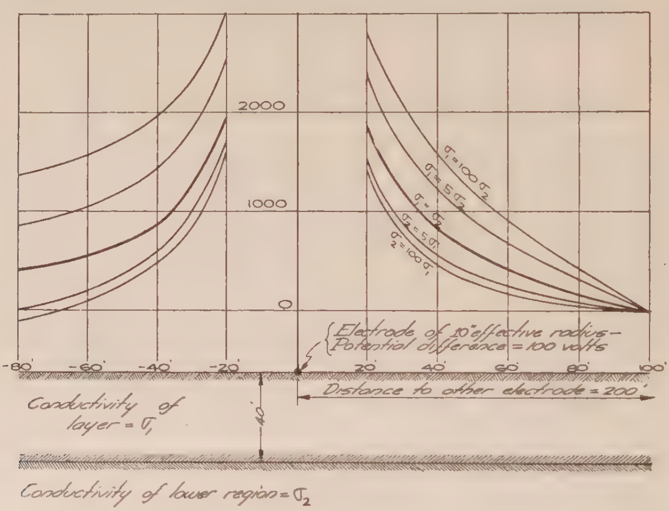

Fig. 5.—Surface potentials in millivolts.

conductivity of the upper layer is 100 times that of the underlying material there is a marked increase in the potential from its normal value. When the lower material is 100 times as conducting as the upper layer, there is a decrease in the potential. It should be remarked that the curves for much higher ratios of conductivities—say 1000 to 10,000 to 1—would practically coincide with those drawn for a 100 to 1 ratio. The curves for a 5 to 1 ratio of conductivities is seen from the figure to give departures from the normal potential curve roughly one-half as large as the 100 to 1 ratio. This is a general characteristic of such problems and was pointed out in Dr. Mason's paper. If one calls the effect realized for an exceedingly great (practically infinite) ratio of conductivities the "saturation effect," it is somewhat surprising how large a fraction of this saturation effect is produced by small ratios of conductivities. It is not possible to give one set of figures that holds for all points and for all layer depths, but this general statement is quantitatively illustrated by Table

1, constructed for this case of a 40-ft. layer and 200-ft. electrode spacing, and for a point 60 ft. from the input electrode A.

TABLE 1.—*Per Cent. of Saturation Effect for Various Ratios of Conductivities*

Ratio of conductivities....	$\sigma_1 = 100\sigma_2$	$\sigma_1 = 10\sigma_2$	$\sigma_1 = 5\sigma_2$	$\sigma_1 = 3\sigma_2$	$\sigma_1 = 2\sigma_2$	$\sigma_2 = 2\sigma_1$	$\sigma_2 = 3\sigma_1$	$\sigma_2 = 5\sigma_1$	$\sigma_2 = 10\sigma_1$	$\sigma_2 = 100\sigma_1$
Per cent. of saturation effect realized..........	96	73	55	39	24	38	56	71	85	97

The fact illustrated by this table is of special significance, inasmuch as it indicates the possibility of differentiating between layers of materials whose conductivities differ by only a small ratio. Such a possibility is of obvious interest in connection with interpretation of structure, quite apart from the immediate question of the location of ore.

Fig. 5 indicates that the departures from the normal potential curve are somewhat larger when the better conducting material forms the upper layer. This holds in general, so that measurements down through a good conducting layer to a poor conductor can be made somewhat more accurately than measurements down through a poor conductor to a good one. Fig. 5 also shows that the departures from the normal potentials caused by the presence of the layer are sufficiently large to permit accurate measurement. When σ_1 is considerably larger than σ_2, for example, the departures from the normal potential are, in general, of the same order as the normal potentials themselves.

The question of the magnitude of the departures from normal potentials deserves further consideration. As the electrode spacing is increased the depth of effective penetration of the currents also increases. Thus the effect of the buried region of higher or lower conductivity can be made very large by increasing the electrode spacing until a large fraction of the total current passes through the lower region and thus feels the effect, so to speak, of the difference in conductivity. These remarks are illustrated by Fig. 6, which, for variety and to show the applicability of this method to deep structures, is drawn for a layer 400 ft. deep. This figure shows not the total potential, but rather the departure from the normal potential. Any interpretation of nonhomogeneity must be based on this departure from the normal potential; so that its magnitude, or, if one chooses, the ratio of this departure to the normal potential, constitutes a proper criterion for judging the accuracy with which this method can be applied. It should also be noted that this figure shows the potential at a point P, whose relationship to the electrodes is kept fixed as the electrode spacing increases. As the electrode spacing d is increased, the normal potential at P drops off, being, in fact, inversely proportional to d when the electrode potential is kept the same; but the departure from the normal

potential, as the figure shows, actually increases in magnitude from less than 10 mv. at 400 ft. spacing, to 60 mv. at 2800 ft. spacing. On account of the falling off of the normal potential, the ratio of the departure from normal to the normal potential increases more rapidly than does the departure itself. At 400 ft. spacing, for example, the normal potential is 200 mv., and the departure 7 mv., so that the layer, at this spacing, causes an effect of only 3.5 per cent. At 1200 ft. spacing the normal potential is 67 mv. and the departure 31 mv., giving a 46 per cent. effect. At 2800 ft. and 3600 ft. spacings, the effect has increased to 210 and 300 per cent. respectively.

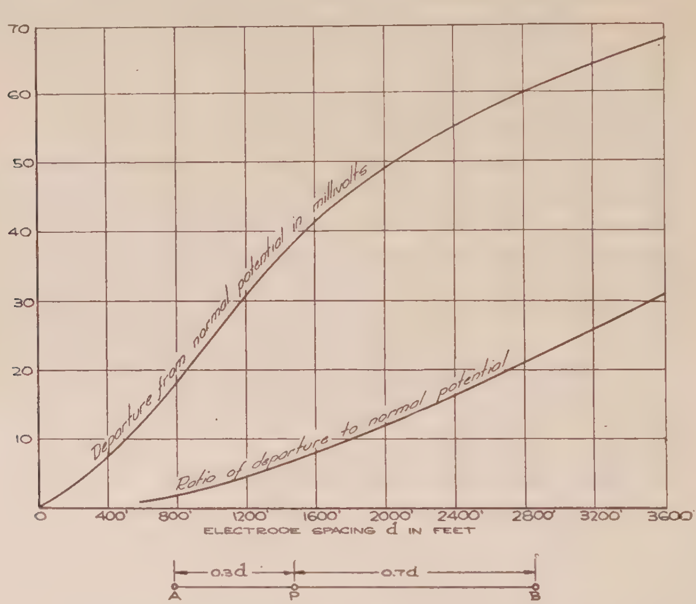

FIG. 6.—CURVE SHOWING DEPARTURE FROM NORMAL POTENTIAL AS ELECTRODE SPACING IS INCREASED. LAYER 400 FT. DEEP. CONDUCTIVITY RELATIONSHIP: $\sigma_1 = 100\sigma_2$, WHERE σ_1 AND σ_2 ARE CONDUCTIVITIES OF LAYER AND LOWER REGION. ALL POTENTIALS SHOWN ARE MEASURED AT THE POINT P.

The curves of Fig. 6 are typical of those which we have calculated for all layer depths and all conductivities, so that it appears that the increase in the effect, and hence the possible accuracy of the method, is limited only by experimental accuracy and by the longitudinal extent of the structure under investigation. It has been suggested that the effect of a change in conductivity which exists at a depth h first makes itself felt in surface measurements when the electrode spacing is equal to the layer depth. Fig. 6 shows that this is a very rough criterion indeed. There is no marked or sudden change in any of the surface effects as the electrode spacing reaches and passes a value equal to the layer depth. The layer

has some effect for all electrode spacings, and produces a continuously larger effect as the spacing is increased.

Interpretation of Field Curves

In all of the discussion thus far, we have considered the effect of a layer of known depth and conductivity. From a practical point of view, the converse is the important problem. Namely, given a curve showing the potentials on the surface, with a fixed or variable electrode spacing as the case may be, do these data enable us to deduce unambiguously the depth of layer and the ratio of conductivities? This problem must be approached with caution and judgment, as must any problem in which one is asked to draw conclusions from incomplete data. In this particu-

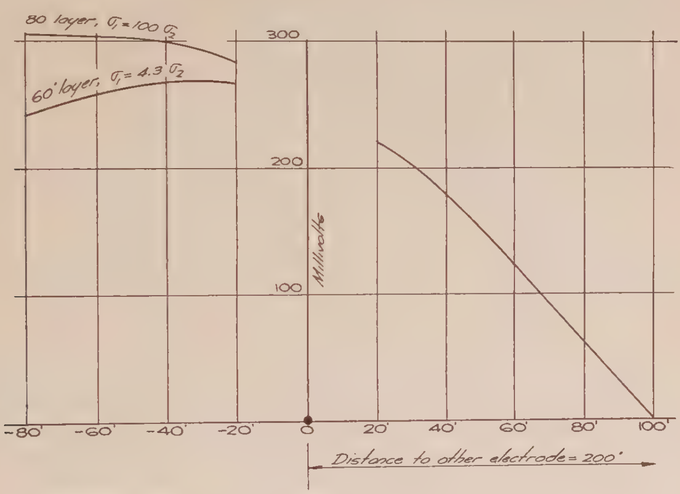

Fig. 7.—Departure from normal surface potentials due to two different layers.

lar instance, however, a complete knowledge of the potential distribution at all points, and for all layer depths and conductivities, makes it possible for one to interpret the evidence in one and only one way. There must remain, of course, a small range of uncertainty due to errors in measurement, etc. As an example of how one may obtain unique interpretations, and to illustrate the necessity of complete information, consider the two curves of Fig. 7. These curves show the departure from the normal potential due to two different layers, one of 60-ft. depth, the other of 80.-ft. depth, the conductivities also being different. If one were to measure the potentials only at points between the electrodes, where the two curves coincide to a high approximation, it would obviously be impossible to decide which of the two layers was actually producing the observed effect. At points outside the electrodes, however, the two curves split,

and it is easy to distinguish between them. Fig. 8 refers to this same case, and illustrates further how one can distinguish between these two particular layers. Fig. 8 shows the increase in the departure from normal potential at the point P, three-tenths of the way from one electrode toward the other, as the electrode spacing is increased. While Fig. 7 shows that the two layers chosen for this example produce the same potential at points between the electrodes for an electrode spacing of 200 ft., Fig. 8 shows that this ceases to be true as the electrode spacing is increased. By increasing the electrode spacing to 600 ft., the two different layers produce effects, at the point P, of 180 and 300 mv. respectively.

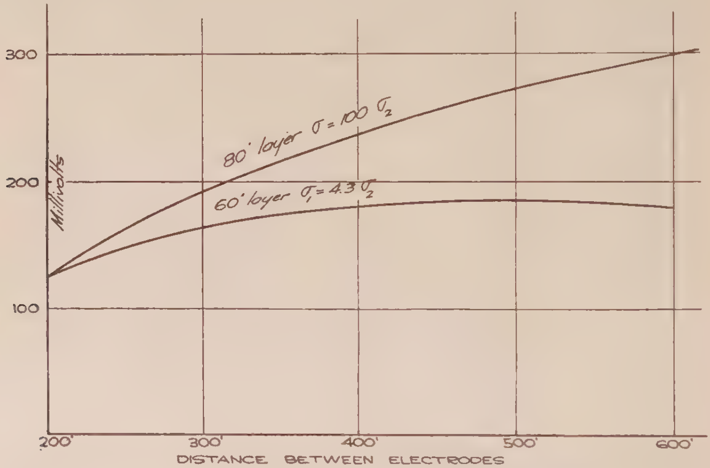

FIG. 8.—DEPARTURE FROM NORMAL POTENTIAL AT POINT P (SEE FIG. 6) AS ELECTRODE SPACING IS INCREASED.

We have considered, in this example, three types of evidence, namely, that coming from points between the electrodes in the curve of Fig. 7; that coming from points outside the electrodes on Fig. 7; and that obtained at a fixed point as the electrode spacing is increased, as in Fig. 8. In this particular example, the first type of evidence fails in that it can not decide between the two possible layers under consideration. This type of evidence should, however, have credit for selecting, out of the infinite number of possibilities, a relatively small number for further consideration. Once the possibilities are cut down to the two cases mentioned, either of the other two types of evidence is capable of making the final choice.

The foregoing discussion has been made more simple than is the actual interpretation of a field curve. There would be, in point of fact, a continuous range of layer depths and conductivities (instead of two) which would give approximately the same effect at points between

the electrodes. The method of procedure, however, is accurately illustrated by this example, all types of evidence being dovetailed together to obtain a unique interpretation.

This discussion of the problem of interpretation makes clear the necessity of obtaining as much evidence as possible. Only points along the line joining the electrodes are mentioned above, but it is sometimes advisable to measure the potentials at points off this line. Fig. 9, for example, shows the results of some four electrode surface potential model measurements made in the early spring of 1925. The two power

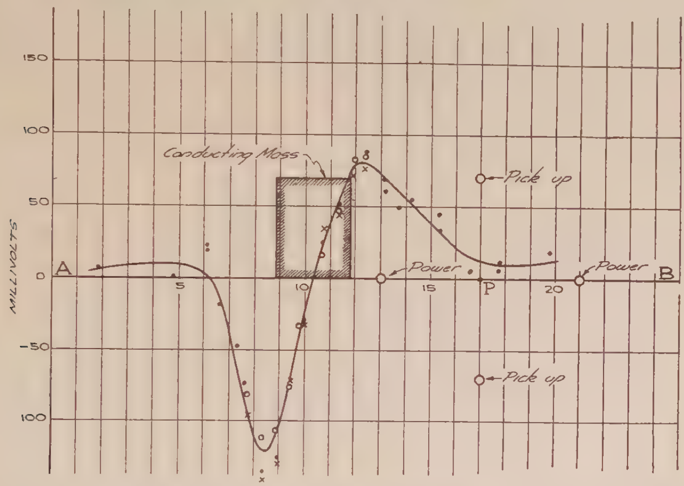

Fig. 9.—Plan view: Surface potentials caused by buried conducting mass. Curve shows potential difference between pickup electrodes as power electrodes are moved along line A-B.

electrodes and the two pickup electrodes were located at the corners of a square of which the diagonal was 8 ft. This whole configuration of four electrodes was held relatively fixed, and moved, along the line joining the power electrodes, over a buried mass of high conductivity. This was an almost perfectly conducting mass, 4 by 3 ft. in its horizontal dimensions, 8 in. thick, and buried 4 ft. deep. The ordinates of the curve are the differences of the potentials at the pickup electrodes. To measure the potential difference plotted at the point P, the electrodes were placed as shown, with their center points at P.

Surface Potentials Due to Tilted Layer

In the case of a single horizontal layer, the solutions on which the foregoing discussion is based form a complete and exact theoretical basis for field procedure, and it appears that it should be possible to make accurate and unambiguous determinations of depth and conduc-

tivity ratio. The way in which the above results are affected by a tilt of the layer surface is illustrated by Fig. 10. A sound interpretation of experimental evidence can be made only after many cases have been studied, and it should be realized that a single illustration, such as that here shown, is, of itself, of restricted significance. The most striking feature of this figure, as contrasted with those for a horizontal layer, is the lack of symmetry about the midpoint, and the fact that the potential at the midpoint between electrodes is no longer equal to the average of the two electrode potentials, but exceeds this voltage by an amount which is many times the accuracy of such voltage measurements. If the electrodes were in a direction parallel to the strike of the slanting

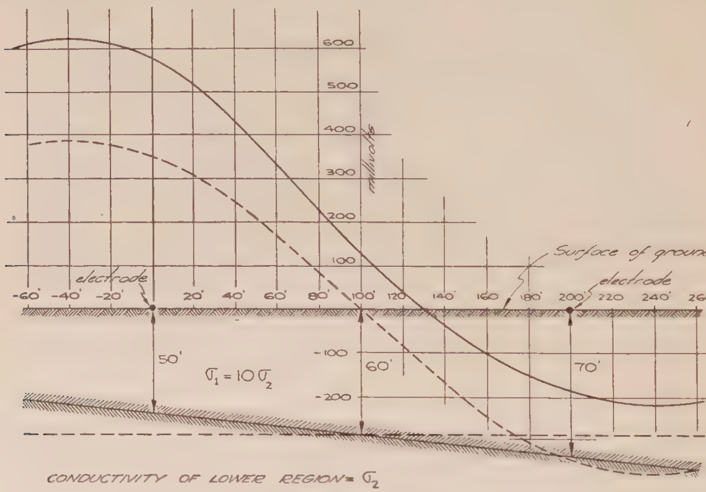

FIG. 10.—DEPARTURE FROM NORMAL POTENTIALS CAUSED BY SLANTING LAYER.

layer, it is easy to see that the potential at the midpoint between electrodes would again be the average electrode potential. Hence, when field measurements indicate, through their lack of symmetry, a slanting layer, the strike can be determined by rotating the power electrodes about their midpoint until the potential at the midpoint is the mean potential. The electrodes can then be set perpendicular to the strike, and measurements taken to determine the dip.

The dotted curve of Fig. 10 shows, for comparison, the departure from normal potentials, which would be caused by a horizontal layer whose depth is the depth, 60 ft., of the slanting layer at the midpoint. The layer shown has a slope of 1 to 10 (or slightly less than 6°). Thus such a tilt is seen to be sufficient to cause considerable departure from the potentials that would be caused by the same layer if it were horizontal.

SURFACE POTENTIALS DUE TO SEVERAL HORIZONTAL LAYERS

When more than one layer exists, the situation is necessarily more complicated. The exact solution of the current flow and potential distribution problem has been obtained for any number of layers of any thickness and any conductivities. We will illustrate the nature of the results with a case of two layers. Fig. 11 shows the case in question. An upper layer of depth 40 ft. and conductivity σ_1 is underlaid by a layer that is 10 times as conducting. This, in turn, is underlaid by a very deep region of poor conductivity. Two double-layer cases are shown on the figure; namely, those for which the thickness h_2 of region 2 is 10 ft. and 20 ft. respectively. If region 2 is of zero thickness, the double-layer case reduces to a single-layer case, the curve for which is

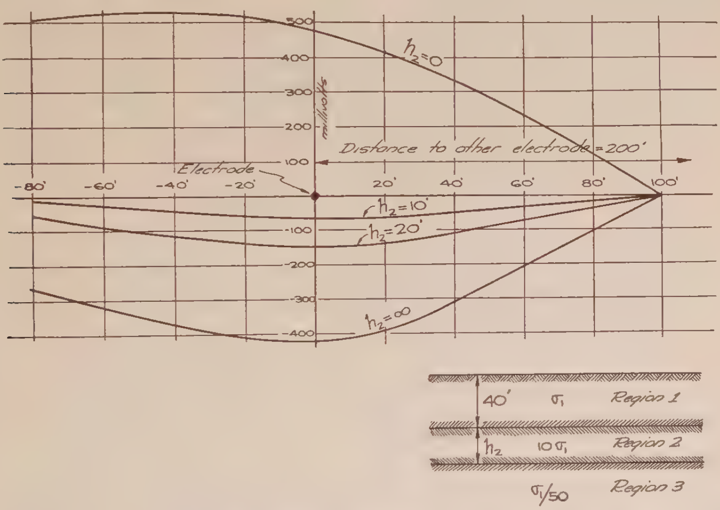

FIG. 11.—DEPARTURE FROM NORMAL POTENTIALS DUE TO TWO LAYERS OF THE DEPTHS AND CONDUCTIVITIES INDICATED IN THE SKETCH IN LOWER RIGHT-HAND CORNER.

the upper one of Fig. 10. If region 2 is infinitely thick, region 3 is removed from consideration and there are again but two regions, the curve for this single-layer case being the lowest curve. Region 2, being of high conductivity, tends to produce subnormal values of the surface potential. Region 3, being a poor conductor, tends to produce values above the normal. When region 2 is only 10 ft. thick, its tendency to produce low potentials outweighs the tendency of the infinitely deep region 3 to produce high potentials.

The question of interpretation of field results should now be reexamined, to see whether it is possible, given a measured set of potentials, to tell whether these potentials were caused by one or more layers. This problem, although more complicated, is similar to the case discussed

earlier. Guided by the theoretical solution of all such conceivable problems, one must painstakingly consider one possibility after another, using all types of evidence to help in the process of elimination.

The two central curves of Fig. 11, both of which are "two-layer" curves, can be almost exactly duplicated by single-layer curves, the lower surface of this single layer lying in region 2, and the conductivity being only slightly higher than that of region 1. On the basis of this evidence alone, therefore, it would be impossible to say whether such a field curve was produced by such a single layer, or by the double layer for which the curve was originally drawn. In this case, and in general for all cases of multilayered structures, evidence obtained by increasing the electrode spacing is very valuable. Fig. 12, for instance,

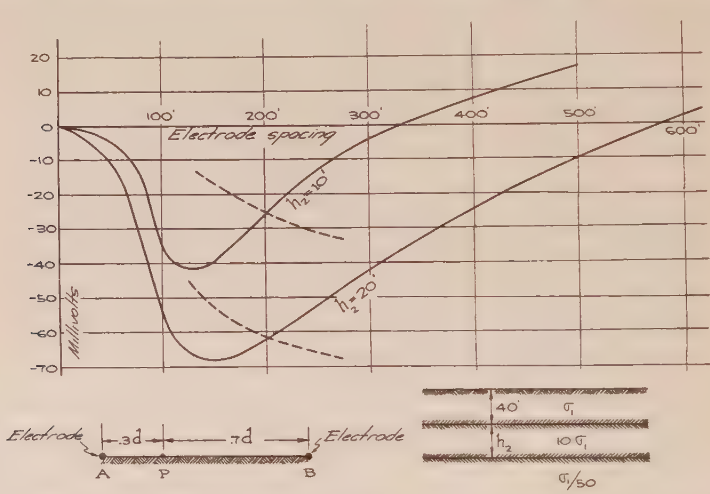

FIG. 12.—DEPARTURE FROM NORMAL POTENTIALS AT POINT P AS ELECTRODE SPACING IS INCREASED.

shows the potential at the point P, three-tenths of the way from one power electrode towards the other, as the electrode spacing is increased. The curves drawn full are the potential for the two-layered structures considered in Fig. 11. The dotted curves are the curves for those single-layered structures which, on Fig. 12, would be indistinguishable from the two layers. These curves intersect for an electrode spacing of 200 ft., but diverge widely for other electrode spacings, and it is clear that Fig. 12 makes it easy to discard the single-layer possibility.

There is a simple physical reason for the behavior of the full curves in Fig. 12. Consider the case of the two-layered structure formed by the 40-ft. layer over a 20-ft. layer. When the electrode spacing is 200 ft., the second layer has a thickness which is one-tenth the electrode spacing. For this ratio of one-tenth, the highly conducting layer has sufficient effect on the surface to pull the potentials down below their normal

values—that is to say, the layer effect outweighs the effect of the underlying poor conductor, whose tendency is to produce high potentials. When the electrode spacing is increased to 400 ft., the thickness of the layer is then only one-twentieth the electrode spacing. The infinitely deep underlying poor conductor is, however, just as deep as ever. Thus increasing the electrode spacing increases the relative importance of the very deep underlying poor conductor, and its tendency to produce high surface potentials now predominates. The departure from the normal potential thus actually reverses in sign as the electrode spacing is increased. Such an effect as this is a total impossibility in the case of a single layer.

Although this discussion refers to one definite case, it accurately suggests the general procedure. Increase of the electrode spacing effectively lifts the layers nearer and nearer the surface, so that their surface effects are successively increased. Having obtained, in this way, a general idea of the nature of the structure, evidence of all possible sorts must be correlated to help determine the various depths and conductivities.

Conductors Located in Strata

The discussion to this point has shown how a knowledge of the potential distribution in the case of stratified structures furnishes methods by which these strata may be discovered and measured. There are, however, other interesting applications. Suppose a mass of highly conducting material has roughly the form of a sphere. If this sphere is located in an otherwise homogeneous earth, and the surface potentials

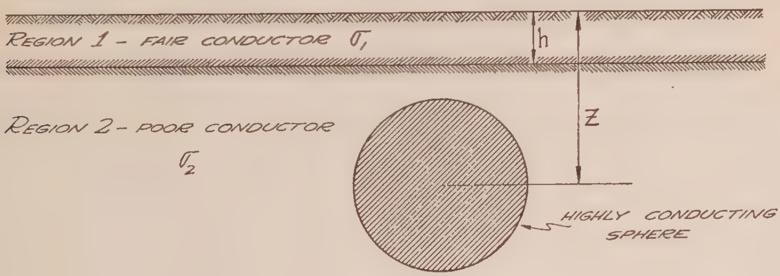

FIG. 13.—HIGHLY CONDUCTING MASS LOCATED BELOW CONDUCTING LAYER, SO THAT SHIELDING EFFECTS OCCUR.

due to two power electrodes are observed, these will depart from normal potentials in a known way. We do not intend to discuss this familiar situation; we wish to consider a slightly different case. In discussing some of the limitations of the surface potential method, Dr. Mason pointed out in a brief remark that it is not always possible to detect ore-bodies by the surface potential method when these bodies are located as in Fig. 13. The cause of the difficulty of detection, under such circum-

stances, lies in the unfavorable shielding effects caused by the upper layer. We wish to enlarge somewhat upon the remark made by Dr. Mason, and give a quantitative illustration of the shielding effect to which he referred.

To detect the sphere we must first excite it, and the disturbance caused by the sphere must then emerge to the surface. Shielding affects both of these processes, although we have found that the second effect is much the more serious. Fig. 14 shows the combination of both effects, for the special case where the depth of the center of the sphere is twice the thickness of the upper layer. The ordinates are the products of the two different shielding ratios, and are to be interpreted in the following manner: If the upper layer is 10 times as conducting as the lower region, and if the electrodes are spaced a distance equal to five times the layer

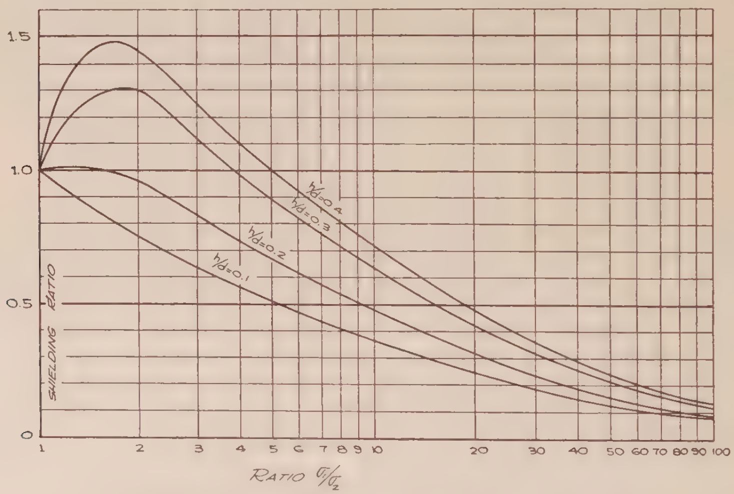

FIG. 14.—TOTAL SHIELDING EFFECT OF LAYER.

depth, a sphere whose center is located at twice the layer depth is practically half as detectable as though the layer were not present, the curve for $h/d = 0.2$ reading 0.49 at $\sigma_1 = 10\sigma_2$.

The curves drawn take into account one consideration not mentioned above; namely, that the detectability of the sphere depends not only on the disturbance it can produce at the surface, but also on the normal potential gradient upon which this disturbance is superposed. This normal potential gradient (normal meaning, in this instance, in the absence of the sphere) is different whether or not the layer is present; and this difference must be allowed for.

One observes from these curves that it is indeed impractical to hope to detect such a sphere if the ratio of conductivities is too high. For a 100 to 1 ratio, the detectability is cut down by an adverse factor of one-tenth. For milder ratios of conductivity, however, the disadvantage is

not so great as to shut out hope for success. It is particularly interesting to note that for very small ratios of conductivity, and suitable placing of electrodes, the disadvantage may be turned into an actual advantage. It is also noteworthy that the saturation effects due to an infinite ratio of conductivities—namely, perfect shielding—is not realized so rapidly, as the ratio of conductivities increases, as was the case with the data presented in Table 1. In fact, if we consider the curve of Fig. 14 for $h/d = 0.2$, we see that the per cent. of total shielding realized for ratios of 2:1, 3:1, 5:1, and 10:1 are, respectively, 5, 16, 35 and 50 per cent. The corresponding figures from Table 1 are 38, 56, 71, and 85 per cent.

DISCUSSION

W. Weaver.—Most of the theoretical problems in geophysics are too difficult for solution unless one introduces simplifying assumptions. In many instances these assumptions are of such doubtful character that one has correspondingly small confidence in the results obtained. It is, therefore, perhaps worth noting that the problem on which I have just reported is a satisfactory one because it is one of the few problems in geophysics that can be solved without simplifying assumptions. The curves I have shown are based on exact solutions.

S. F. Kelly, New York, N. Y.—In Fig. 8, Mr. Weaver gave an illustration of the potential differences being measured across the line joining the power electrodes, the pickup electrodes being moved in successive steps across the conducting body. Were the power electrodes moved or were they stationary?

W. Weaver.—The configuration of power electrodes and pickup electrodes were moved as a fixed configuration across the whole body.

H. Hedstrom, Houston, Tex. (written discussion).—In Mr. Weaver's discussion of the curves of Fig. 6, he says that there is no marked or sudden change in any of the surface effects as the electrode spacing reaches and passes a value equal to the layer depth. This is plain, but it does not therefore follow that the "practical rule" criticized by Mr. Weaver "is a very rough criterion indeed."

As Mr. Weaver points out, the possible accuracy of the method is limited by experimental accuracy. In practical field work this accuracy is decreased by several causes, one of them usually being the inhomogeneity and irregularity of the very surface over which the potential differences are measured. This tends to obscure the effect of a deeper layer, until this effect reaches a certain minimum value, which, for practical purposes (and at moderate electrode spacings), can be put at some 2 to 3 per cent. of the "normal potential."

If this point is considered, Mr. Weaver's own figures, as well as his "ratio curve" in Fig. 6, show that the "practical rule" holds very well. That is, the effect of the deeper layer does not make itself felt until the power electrode spacing in this arrangement is about equal to the layer depth, but from then on the effect increases rapidly with increased distance between the electrodes.

In our organization, the Swedish American Prospecting Corpn., we have found that this rule may well be applied for our "two-electrodes" system; that is, when the four electrodes are placed at points along a straight line, with equal spacing r and with the two pickup electrodes between the two power electrodes. But another wording of the rule must be applied for our "one-electrode" system. This arrangement uses a large and *fixed* distance between the power electrodes A and B and keeps the two pickup

electrodes C and D at distances r and $2r$ from the one power electrode A, on a straight line passing through A at right angles to AB (theoretically on a circle passing through A and with its center at B).

In both these cases our "practical rule" says that the effect of a deeper layer makes itself first felt when the electrode spacing r is about equal to one-third of the depth to the layer. In the first case this is the same as saying that the power-electrode distance must be at least equal to the depth. In the second case, however, the distance between the power electrodes has no effect whatever on the readings, provided that the distance between them is large enough to allow the pickup electrodes always to remain on a circle through A with its center at B.

From this it is evident that it is only the pickup-electrode distance r that has to do with the effect of a deeper layer on the potential readings at the surface.

Therefore it is not correct to say, as Mr. Weaver does, that "the effect of the buried region of higher or lower conductivity can be made very large by increasing the (power) electrode spacing until a large fraction of the total cŭrrent passes through the lower region and thus feels the effect, so to speak, of the difference in conductivity."

The following example will make this more clear. Two power electrodes, A and B, are put 30 ft. apart on the ground; one pickup electrode C is placed 30 ft. distant from both A and B, and the other pickup electode D is placed on the line AB 60 ft. from A and 30 ft. from B. The conductivity of the ground is constant to a depth of 60 ft., where the conductivity changes to a new value, which remains constant from 60 ft. down.

If these two conductivities were equal, the potential difference between C and D would evidently be equal to the total potential at D. From this "normal" potential there will be a departure of +7.5 per cent. if the "lower" conductivity is 10 times smaller, and −6.0 per cent. if it is 10 times higher than the "upper" conductivity. The "saturation effect" in the former case is +9.0 per cent. and in the latter case −6.8 per cent.

If the power electrode B is shifted away from the power electrode A along a line perpendicular to CD, the potential difference between C and D evidently remains exactly the same as before, however large the distance between the power electrodes is taken.

Thus an increase of the power electrode spacing, in this case, from one-half the depth of the buried region of different conductivity to any larger distance does not at all change the effect of the buried region on the measured potential difference, even though the "fraction of total current (that) passes through the lower region" increases enormously.

These figures are taken from curves in diagrams for potential work, used by the Swedish American Prospecting Corpn. These curves are calculated for the potential difference between two pickup electrodes at distances r and $2r$ from one power electrode, the other power electrode assumed kept at equal distance from both pickup electrodes (in practice usually far away, as described). They apply equally well for the two-electrode system described above, as the potential differences measured with this system are simply twice as large as those obtained with our one-electrode system. The curves show the ratio between measured potential difference and normal potential difference for all ratios between electrode distance r and depth to "the buried region," and for all ratios between the respective conductivities. Since the end of last year we have these curves plotted in diagrams in such a way that curves obtained in the field work can be fitted in on the curves in the diagram, and thus the depth to the buried region as well as the ratio between the conductivities can be read directly from the diagram.

These curves show for the "saturation effect," caused by comparatively very poor and very good conductivity of the buried region, the following figures: At an

electrode spacing r equal to one-third of the depth to the deeper layer 1.03 and 0.98 respectively (that is, practically "normal" potential difference); with r equal to half the depth, the corresponding figures are 1.09 and 0.93, and with r equal to the depth they are 1.50 and 0.69.

As shown by the figures quoted, and as pointed out by Mr. Weaver, it is remarkable how large a fraction of this "saturation effect" (obtained by practically infinite ratio of conductivities) is produced by comparatively small ratios of conductivities.

This is true especially for moderate spacings r between pickup electrodes and power electrode. At a spacing r equal to twice the layer depth, a ratio between upper and lower conductivity of 0.10 produces about 70 per cent. of the effect for a ratio 0, and a ratio 10 produces some 85 per cent. of the effect of an infinite ratio. The curves of ratio between measured and normal potential difference for the conductivity ratios 0.02 and 50 coincide, for practical purposes, with the 0 and ∞ curves.

At larger electrode spacings, these curves approach the same values as the respective conductivity ratios, and therefore split; but as one would very seldom be able to take advantage of this fact in practical field work, one must agree with Mr. Weaver that only when the conductivity of the buried region differs by a rather small ratio from the conductivity above it is it possible to determine this ratio by potential measurements on the surface.

W. Weaver (written discussion).—Mr. Hedstrom's discussion presents two points of disagreement: (1) Mr. Hedstrom says, "it does not therefore follow that the practical rule criticized by Mr. Weaver is a very rough criterion indeed;" and remarks further, "Mr. Weaver's own figures as well as his ratio curve in Fig. 6 show that the practical rule works very well." (2) Mr. Hedstrom says, "therefore it is not correct to say, as Mr. Weaver does, that the effect of the buried region of higher or lower conductivity can be made very large by increasing the power electrode spacing."

Answer to Criticism 1.—Before curves such as those of Fig. 6 were calculated and drawn it might have been anticipated that these curves would show a sudden and marked increase when the power electrode spacing equaled the layer depth. Mr. Hedstrom agrees that the curves of Fig. 6 do not show any such sudden change. So far, then, he agrees with my criticism of the "practical rule." But Mr. Hedstrom offers a new and unusual argument for the rule. He points out that beside the "normal potential" and the "departure from normal" (due to the presence of the layer) there is often present a third potential, a "disturbance potential" due to inhomogeneities and irregularities of the ground surface. These disturbance potentials, he says, are from 2 to 3 per cent. of the normal potential. It is clear that it would be most unwise to assign significance to departures from normal potentials unless these departures are of sufficient size to guarantee that they are actually due to a layer, and are not spurious effects due to surface irregularities. That is to say, departures from normal potentials are certainly not significant and interpretable until they well exceed the disturbance potentials, Mr. Hedstrom's argument for this "practical rule" hence completely collapses on his own figures. He admits that disturbance potentials may be 2 to 3 per cent. (they are often considerably larger than this); while the electrode spacing indicated by his own rule furnishes for interpretation a departure potential due to the layer of only 3.5 per cent.

The practical rule to which I referred in my paper is that developed by Rooney and Gish.[2] This rule, which has been widely used in subsequent publications by various authors, states that the layer depth is equal to that spacing *between adjacent electrodes* (of an equally spaced, linear, four-electrode system) for which the resistivity first

[2] W. J. Rooney and O. H. Gish: Measurement of Resistivity of Large Masses of Undisturbed Earth. *Terrestrial Magnetism and Atmospheric Electricity* (1925) 161.

shows a marked change. It is surprising to find Mr. Hedstrom advocating a quite different rule. Instead of using the distance between adjacent electrodes, he uses the distance between the power electrodes—a distance which is three times as great. Our work clearly shows that Mr. Hedstrom's rule is not as serviceable as the Rooney-Gish rule. The latter rule, to be sure, serves a useful purpose; we maintain that it is, however, only roughly approximate and in general inadequate.

Reply to Criticism 2.—In discussing the possibility of locating deep layers, I showed that even for very deep layers the effect at the surface can be made large by using a suitable arrangement of electrodes and by increasing the electrode spacing. Mr. Hedstrom does not offer evidence that my solution is not sound. In fact, he admits the correctness of Fig. 6, which shows the increase of response with electrode spacing. Mr. Hedstrom criticizes my solution only by showing that this important problem is *not* solvable if one uses a *totally different arrangement of electrodes.* The electrode configuration which he uses is different from any that I suggested, mentioned, or would consider using. In a word, he criticizes a solution of an important problem by merely pointing out one of the innumerable ways in which this problem cannot be solved.

Recent Results in Electrical Prospecting for Ore

By Hans Lundberg,* New York, N. Y.

(New York Meeting, February, 1928)

In order to comprehend the help and information that may be expected from electrical prospecting, it is necessary to have at least a general knowledge of the methods and principles involved in preparing and interpreting results obtained in the field. Electrical prospecting is rather complicated, both in theory and practice, and in order to make a brief description clear to the practical mining man it has been necessary to idealize and generalize the problems, at the risk of having such simplification criticized by the theoretical man. This simplification must not be misinterpreted as showing a lack of theoretical knowledge.

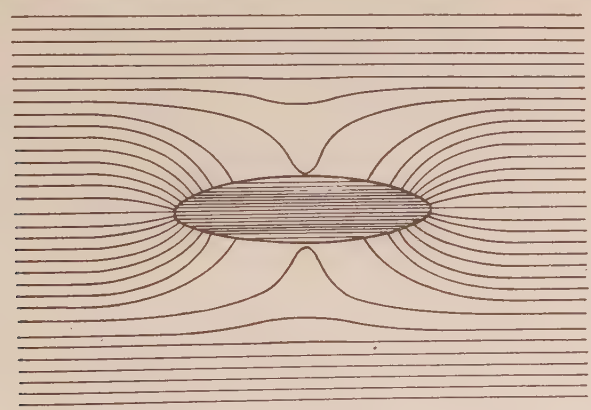

Fig. 1.—Flow of current in and around a conducting body.

For its operation, electrical prospecting mainly depends on the fact that most metallic ores conduct an electric current much more readily than barren rocks. If an electric current flows through a block of ground within which a body of chalcopyrite is enclosed in country rock of schist, the current will find less resistance to its passage through the chalcopyrite than through the schist, even though the passage through the latter be much shorter. This is because the longer path through the chalcopyrite offers less resistance, owing to the superior electrical conductivity of the chalcopyrite (Fig. 1).

The good conductor, chalcopyrite, in the poor conductor, schist, thus attracts the current and deflects the normal flow far outside the conduct-

* Mining Engineer and Geologist, Swedish American Prospecting Corpn.

ing body itself. The density of the current in the schist will be very small, while that within the chalcopyrite will be relatively high. Advantage is taken of this condition in electrical prospecting.

Disturbances or anomalies in the current flow are located by mapping on the surface the course of the current flow or by studying directly or indirectly the current density in a sufficient number of arbitrary points. The presence in the ground of the body of superior conductivity is thus revealed.

Method of Electrical Prospecting

Faint terrestrial current is always flowing through the ground (so-called earth current), but this is generally too weak and unsteady to be useful for the purpose of electrical prospecting. Current is, therefore, directly impressed by means of a generator. Alternating current is preferred, as it is more applicable, possible to amplify, and difficulties caused by chemical polarization, etc., are avoided.

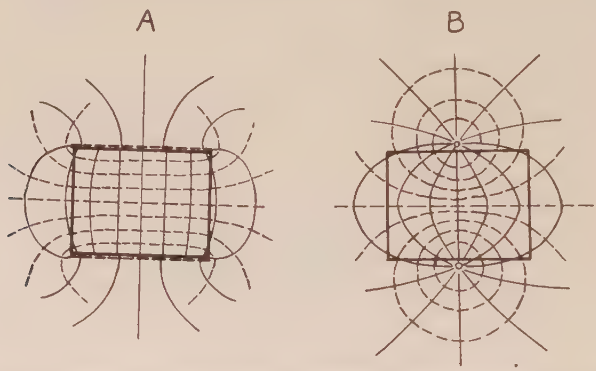

Fig. 2.—Block of ground flooded by current by means of: *A*, linear electrodes; *B*, point electrodes.

The current is sometimes supplied to the ground between two points on the surface (electrodes) connected to the terminals of the generator. In order to cause the current to flow uniformly through a certain block of ground, the electrodes, instead of being merely two points on the ground, are often given the shape of two parallel lines (Fig. 2). Disturbances caused by an orebody become much stronger and easier to interpret if linear electrodes are used.

The electrodes are made of stranded, bare wire of copper or bronze and are brought into contact with the ground at close intervals by means of iron pins, one to two feet long. Insulated cables connect the electrodes to the generator (Fig. 3*a*). Single-phase alternating current of audio-frequency (50 to 10,000 cycles) is used. Currents of very high frequency generally penetrate only a few feet into the ground and therefore are seldom used for electrical prospecting.

The generator creates a tension of audio frequency between the electrodes, causing alternating currents to flow in the ground, which may be studied by investigating either the electrical field or the electromagnetic field.

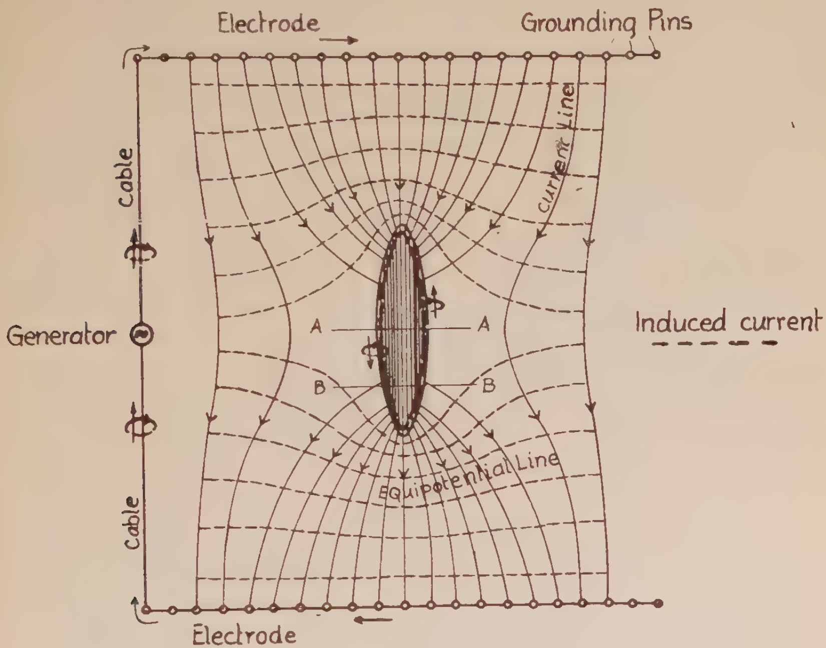

FIG. 3a.—OREBODY WITH CURRENT FLOW BETWEEN ELECTRODES AND FLOW INDUCED.

Investigating the Electrical Field

The electrical field may be investigated by determining the difference in potential and phase between arbitrary points. As a rule, however, this field is most easily studied by locating on the surface points of the same electrical tension or potential. These points are then surveyed and plotted on a map, points of equal potential being connected by curves similar to level curves. These equipotential lines[1] are at right angles to the lines representing the current flow and thus the shape of the equipotential lines shows the distribution of the current in the underlying ground (Fig. 3a). Where an orebody contracts the current lines, the equipotential lines are thrust apart.

From the lines of equipotential it is sometimes possible to approximately locate the conducting body. Boundaries, dip and strike, how-

[1] Theoretically, no equipotential lines exist in alternating electrical fields, as these fields vary in phase from point to point. In practice, however, the phase differences are often very small over large areas and equipotential lines can be located.

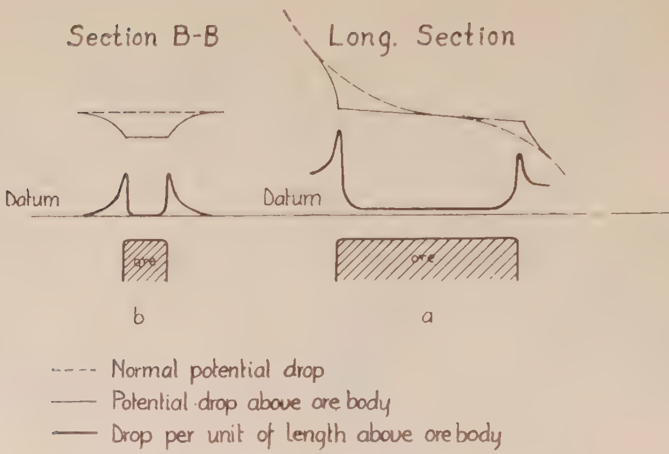

FIG. 3b.—OREBODY WITH CURRENT FLOW.

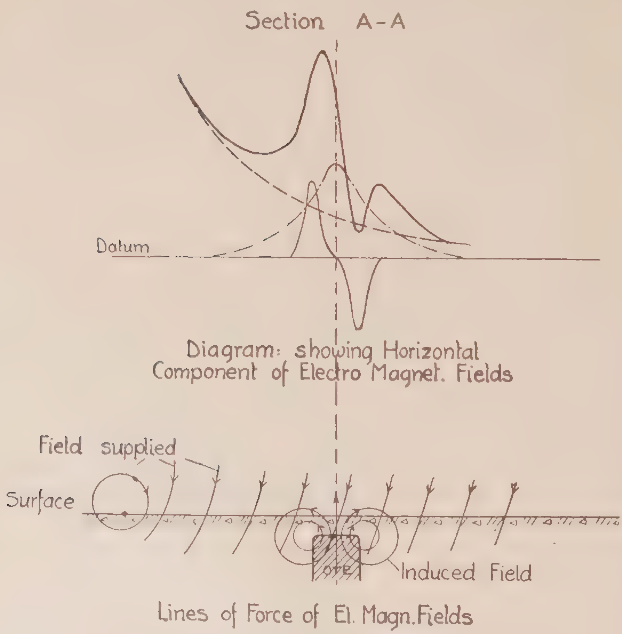

FIG. 3c.—OREBODY WITH CURRENT FLOW.

ever, may be more closely determined by investigating the potential and phase of the electrical field directly. From the fall of the potential, or, what is the same, the drop in voltage, it is as a rule possible to determine the boundaries closely.

In barren ground the current flows uniformly and with a normal density. The fall of potential is constant but fairly large, owing to the low conductivity of the ground.

In the ore the current density is much higher, but on account of the superior conductivity of the ore, the fall of potential becomes very small, compared to the fall on barren ground.

The converging of the current to the orebody causes increased density in the wall rock just before the current passes into the orebody proper. Above an orebody and on the sides the density will be decreased (see Fig. 1). The wall rock is a poor conductor and increased current density means an increased drop in voltage, and the drop, therefore, increases toward the ore boundary. A decreased density, on the other hand, means decreasing drop of potential. The fall of potential in passing over an orebody thus becomes very characteristic.

Fig. 3*b* shows the normal drop of potential (dotted line) as compared to that above an orebody (thin solid line). The diagram *a* is taken along the body and *b* is taken across the body at section line *BB* in Fig. 3*a*. The heavy solid line represents the fall of potential per unit of length.

This curve can be directly computed from observations of the potential taken at points at equal intervals, say 5 or 10 ft. apart. Pronounced maxima are obtained above the boundaries of the orebody. Thus by observing the fall of potential at a sufficient number of sections, the conducting body can be outlined sometimes within a few feet.

The apparatus used for finding points of equal potential is a simple, movable circuit which consists of two electrodes or searching rods of iron connected to a telephone receiver. With this searching circuit, points of equal potential are obtained by locating on the ground points between which no sound is heard in the telephone. These points are then marked out on the ground, surveyed and recorded on a map.

Studying the Electromagnetic Field

The current flow in the ground may also be studied by investigating the electromagnetic field. Any flow of alternating current is surrounded by an alternating electromagnetic field.

Fig. 4 is a section across a long conductor *a* buried in the ground and carrying alternating current. This current is surrounded by an electromagnetic field, which may be represented by concentric circles around *a*. The intensity of the electromagnetic field is proportional directly to the strength of the current and inversely to the distance. At points on the

surface, direction and intensity of the electromagnetic field is represented by arrows, whose length is in proportion to the intensity. The total intensity may be divided into components; for instance, one vertical and one horizontal. The upper part of Fig. 4 shows graphs of the two components as well as of the total field intensity. The total and the horizontal intensities show maxima above the current flow. The vertical

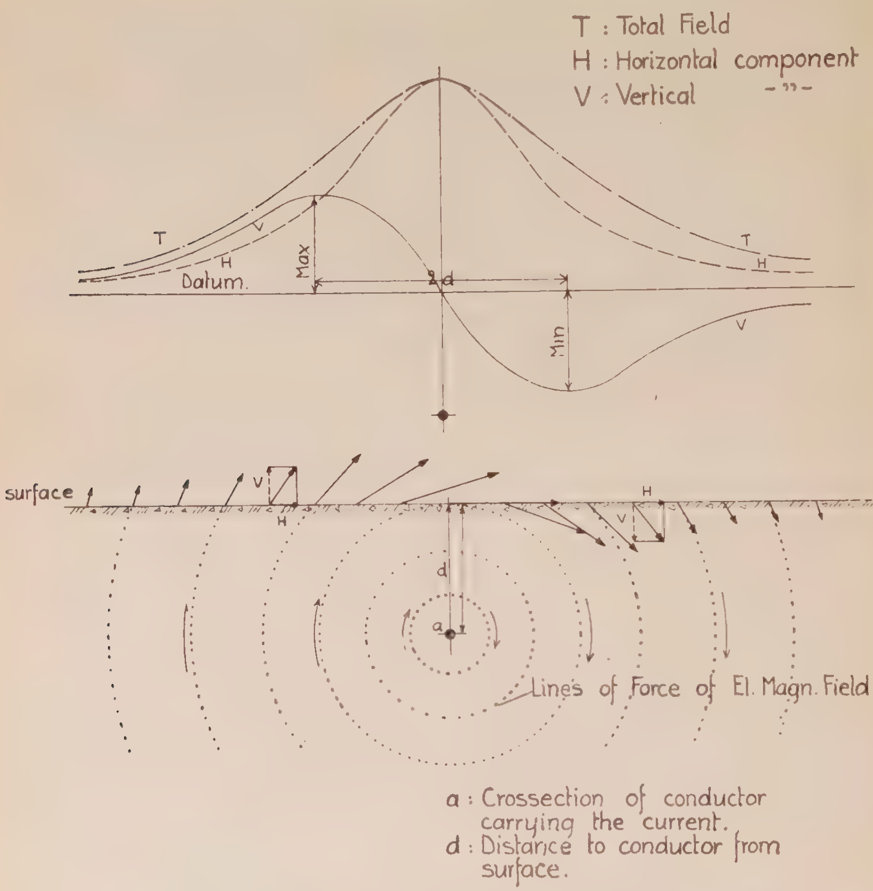

FIG. 4.—ELECTROMAGNETIC FIELD AROUND A FLOW OF CURRENT.

intensity shows a maximum on one side and a minimum on the other. Half the distance between the maximum and minimum of the vertical intensity is a measure of the distance from the surface to the current flow.

Now we turn back to the orebody of Fig. 3. The current density in the orebody is much higher than in the surrounding ground and the intensity of the electromagnetic field would consequently show a maxi-

mum above the orebody. Fig. 5*a* shows schematically the horizontal component of this field along section *AA* Fig. 3*a*.

An electromagnetic field also surrounds the current flow in the cables connecting the electrodes and the current flowing in the ground. The horizontal component of this field, when no orebody is present, is shown in Fig. 5*b*.

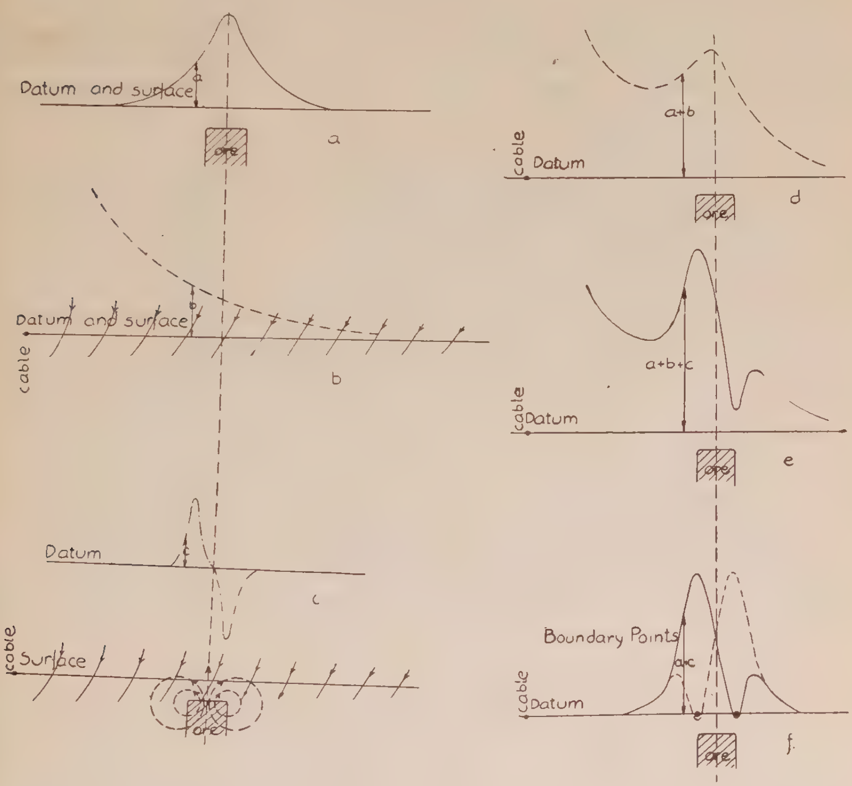

FIG. 5.—DIAGRAMS OF HORIZONTAL COMPONENTS OF ELECTROMAGNETIC FIELD.
a. Around concentrated current flowing through orebody.
b. Around current in cable and barren ground.
c. Around current induced in orebody.
d. Around current directly supplied.
e. Actually observed while surveying.
f. Purely caused by orebody.

Our orebody is now exposed to the electromagnetic field and eddy currents are induced in this body, as in any conducting body exposed to an electromagnetic field. The eddies combine to form a resultant eddy, or secondary current, the main part of which flows in a closed circuit along the edges of the orebody (heavy dotted line in orebody, Fig. 3*a*). Other eddy currents not shown in the figures flow in the walls

of the orebody. Also these secondary currents have an electromagnetic field of their own, the horizontal component of which is shown schematically in Fig. 5*c*. These three fields, the field from the current concentrated in the orebody, the field from the current in the cables and the barren ground, and finally the field induced in the orebody, all combine to form one electromagnetic field, which may be obtained by addition of the three fields. Adding *a* and *b* (Fig. 5) together will give the curve *d* which represents the horizontal component of the electromagnetic field surrounding all current directly furnished by the generator. If *c* is also added, the curve *e* is obtained, which is the horizontal component of the field that we actually observe when investigating along the section line *AA* (Fig. 3*a*).

In order to obtain a curve that shows only fields caused by the orebody, the field *b*, from current in cables and barren ground, is subtracted. The curve *f* thus obtained shows a distinct maximum above the edge of the orebody nearest to the cable, and a less pronounced minimum above the other edge. The minimum sometimes does not show very plainly, especially if the orebody is narrow. Therefore, the far edge of the orebody may be determined better if the cables connecting the electrodes are laid out on the other side of the body. The field represented by the dotted line (Fig. 5*f*) will then, with a distinct maximum, indicate this edge of the body.

Thus by investigating the electromagnetic field along a number of sections, it will be possible to determine the boundaries of the orebody, sometimes within a few feet. Generally both vertical and horizontal components of the field are determined, though only the latter has been mentioned above.

Inductive Methods

Often it is difficult to obtain good grounding for the electrodes; for instance, in very dry, arid regions or during the winter, when ice and snow cover the ground. In such cases current is supplied to the ground inductively, by means of a loop of insulated cable laid out on the surface and connected to the poles of the generator. The alternating current thus caused to flow in the closed loop, the primary current, is surrounded by an electromagnetic field, the primary field (Fig. 6). Conditions, with this arrangement, become simpler and less complicated than those previously described. No current is supplied directly to the ground and only eddy currents flow in the conducting body, situated in the electromagnetic field (inside and outside the loop). The eddy currents combine to form eddy or secondary currents flowing along the edge of the body, and down the dip as previously described. The secondary field surrounding the secondary current combines with the primary field to form a

third field (Fig. 7*a*). The direction and strength of this resultant field may be observed at arbitrary points, but as a rule observations are taken along section lines staked across the strike of the body.

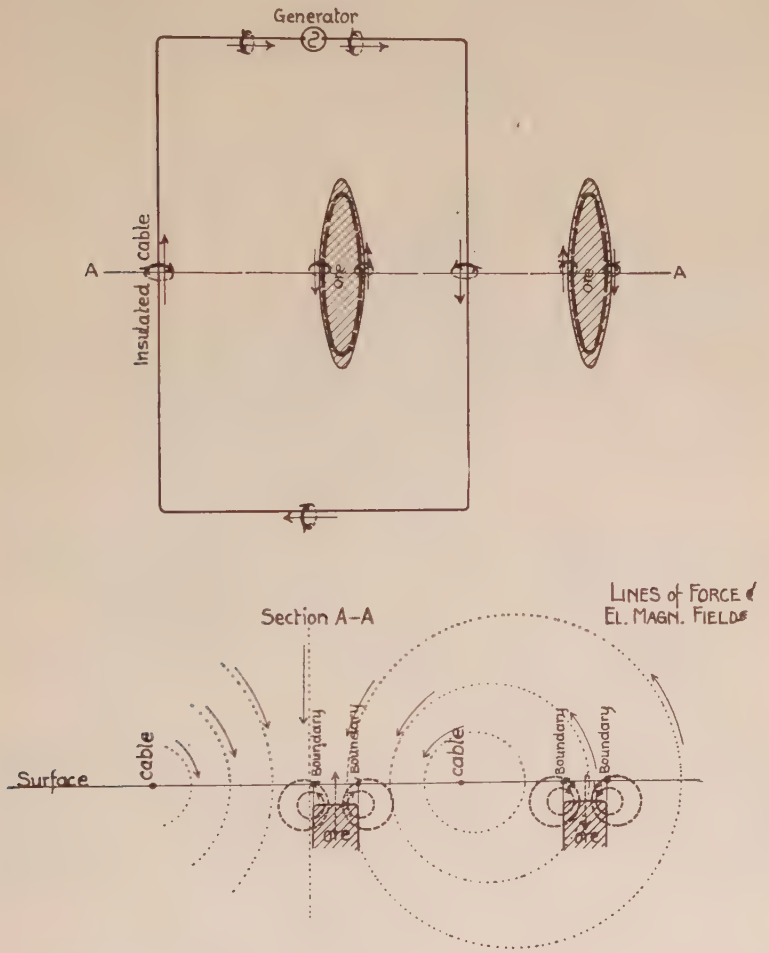

FIG. 6.—SECONDARY CURRENT FLOW INDUCED IN CONDUCTING BODIES BY PRIMARY CURRENT IN CABLE.

The primary electromagnetic field is easily calculated from the size and shape of the loop laid out on the ground, and then by subtraction of the calculated primary field from the resultant field as actually observed, the secondary field from the current in the orebody will be obtained (Fig. 7*b*). Fig. 7*c* shows horizontal and vertical components of the secondary fields above an orebody.

The foregoing is an elementary explanation only. Actually, the secondary currents differ in phase in different sections of the orebody and differ in phase from the primary field as well. This complicates conditions considerably but to enter into these problems here would carry us far into abstruse technical features of the subject. It may be said, however, that, among other factors, the phase difference depends on the specific conductivity of the orebody; thus determining the phase often makes it possible to derive the conductivity.

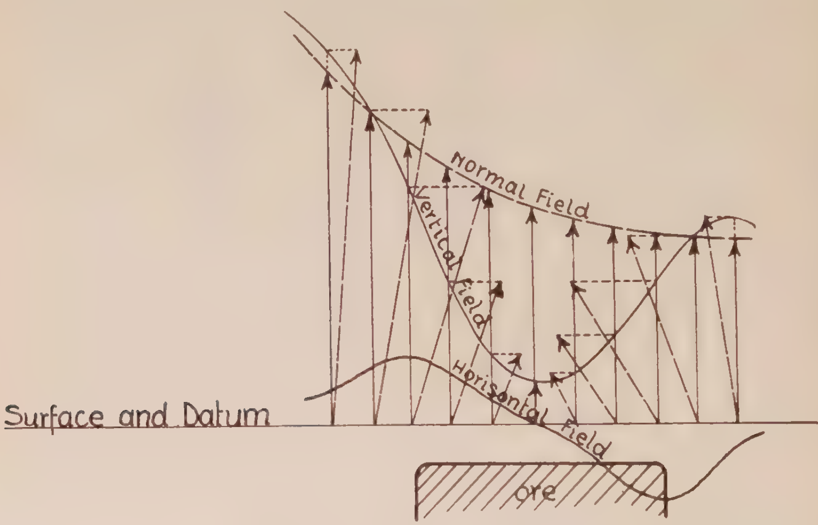

FIG. 7*a*.—FIELD COMPONENTS AS OBSERVED AND COMPARED TO NORMAL FIELD.

Under favorable conditions, it is thus possible to locate an orebody existing under the surface, determine the shape and lateral extension, and its depth from the surface, and finally obtain an idea as to the conductivity, which is often a good measure of the grade and character of the mineralization.

Apparatus and Operations

The apparatus used for investigating the electromagnetic field is simple and easy to carry around in the field. Movable coils of insulated wire, so-called frames, are used. If exposed to an electromagnetic field, a tension or voltage is induced in the turns of the frame, being proportional to the strength of the electromagnetic field passing the frame. The voltage may be observed on a voltmeter.

If a telephone receiver is connected to the frame, a sound will be heard corresponding to the frequency of the generator. The axis of

the field is found where no sound is heard in the telephone; that is, when the plane of the frame is parallel to the axis of the field. No voltage is then induced, as no lines of force cut through the plane of the windings in the frame.

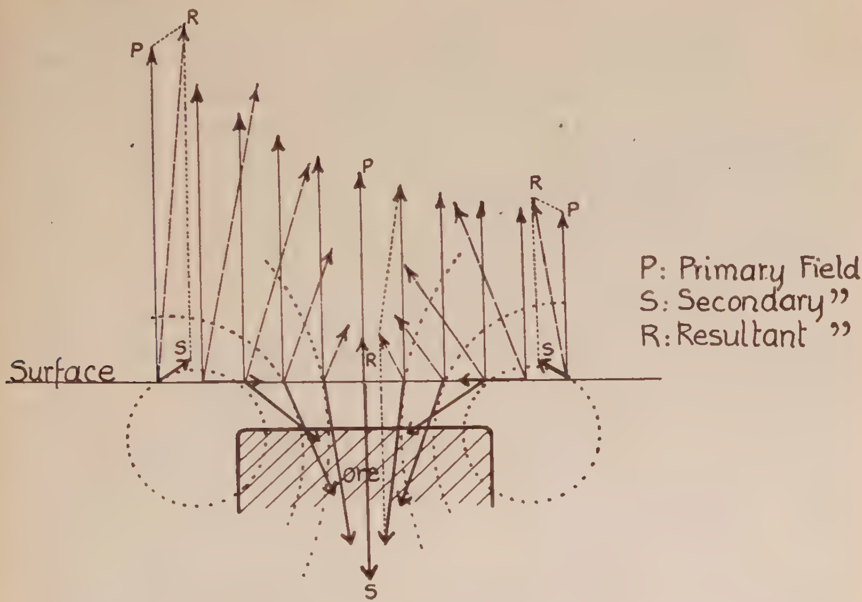

Fig. 7b.—Deriving secondary field by geometric subtraction of primary field (calculated) from resultant field (observed).

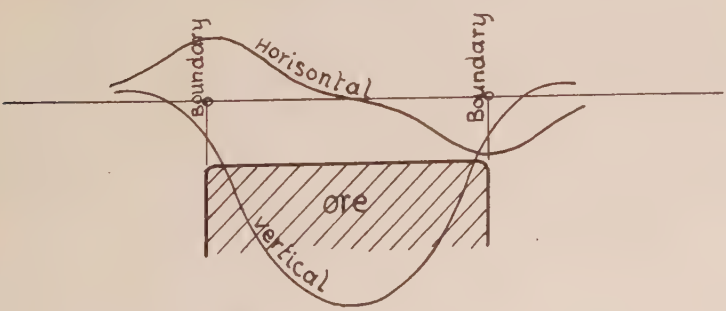

Fig. 7c.—Field components of secondary field (caused by orebody).

Many electrical prospectors determine directions only. It is impossible, however, to interpret the general character and strength and also the outline of the disturbing cause, unless the field is more completely investigated.

The method of direct observation and measurement of the field has several disadvantages. In order to obtain the correct values by direct

reading, knowledge of intensity and frequency of the current from the generator is required at each moment of observation. Further, as no exploration of the field is possible without amplifying devices and as the amplifying factor changes very easily, direct observations become of doubtful accuracy.

Therefore, methods of comparative readings have been developed which are independent of the intensity and frequency of the current source, and where the amplification is used for the purpose of detecting only and not for measuring. These methods have been patented. If, for instance, two frames are connected with a balancing arrangement, the phase and intensity can be compared at two different points by balancing the voltage induced in the two frames.

Different balancing devices are used; the turning of one frame for balancing is the simplest. By turning one of the frames a certain angle from the axis of the electromagnetic field, the voltage in the other frame is balanced. The angle then turned is a measure of the ratio of the intensity of the two field components which are in phase.

In using any of the methods described, the procedure is first to carry out an electrical reconnaissance. This is done by making observations along lines 100 to 250 ft. apart. If indications of the presence of a mass of good conductivity are obtained, more detailed work at such points is desirable. This consists mainly of taking observations at closer intervals. The detail work results in accurate definition of the outline of the conducting mass, its approximate dip, its depth from the surface, etc.

Essentials of Practice

Essentials of the success of electrical methods in serving the mining industry are: (1) Methods developed to such a degree that the properties of electrical and electromagnetic fields can be correctly determined; (2) knowledge of the strength and character of the influences thereon of orebodies; (3) experience in interpreting the results; and (4) last but not least, knowledge of geology.

Especially on virgin ground, geological reconnaissance should precede electrical prospecting, with the object of outlining carefully the ore-bearing rock, and determining the possibilities of electrical prospecting. No electrical survey will then be carried out on ground which, for geological reasons, may be considered barren or where the electrical methods are inapplicable. Furthermore, if the geological examination can determine direction of strike or fracturing, the appropriate field may be employed without delay of various set-ups to find the most favorable course.

As the success of electrical prospecting depends on the difference in conductivity between the ore and the country rock within which the

orebody is enclosed, the conductivity of the two should be determined, if possible.

The conductivity of an orebody depends on the distribution of conductive ore minerals and, in addition, on its size (length, width and depth). Most minerals with metallic luster show good electrical conductivity; for instance, the sulfides. Zincblende is an exception but as it is often associated with ore minerals of good conductivity—for instance, pyrite or galena—there is likelihood also of finding zinc.

As a rule the conductivity is increased in proportion to the content of metallic minerals, but the texture of the ore is of great importance. For instance, ore composed of isolated metallic grains in a nonconducting gangue may not be as good a conductor as ore having the metallic minerals arranged as streaks or stringers. In the latter case the conductivity of the ore in one direction may be much superior to that at right angles.

By macroscopic or microscopic examination it often can be ascertained whether a rock is a good conductor or not, but the direct determination of the conductivity is safest. Therefore it is always desirable, when any of the ore is available, to test specimens of it, as well as specimens of the wall rock and of the covering soil.

However, the figures of conductivity obtained by testing samples of rock and soil often become misleading as to the actual conditions in the ground. The conductivity is generally measured on more or less dry samples, and it is almost impossible to obtain them in the condition in which they occur in nature. The conductivity of rock and soil depends mainly on the porosity, content of moisture and on the conductivity of this moisture. Therefore, the conductivity of rock and soil varies considerably in different places, and is generally much higher than a sample test would indicate.

While igneous and Pre-Cambrian rocks are generally rather poor conductors, younger sedimentary rocks are apt to be comparatively good conductors. Even in Pre-Cambrian regions, the ground cannot be considered as nonconducting and the orebodies the only possible conductors present. In younger formations, conditions may be more complicated still, where we may have conducting or semiconducting ground enclosing orebodies of only slightly higher conductivity.

While in Pre-Cambrian regions the conductivity of the orebodies may be from 10 to 1000 times better than the surrounding ground, in younger sedimentary formations, the conductivity of an ore might be equal to or only slightly higher.

Pre-Cambrian formations, therefore, present more favorable conditions than younger sediments. An example of the latter type is found in the lead-zinc districts in the Mississippi Valley, where, therefore, it is necessary to apply sensitive and more elaborate methods.

But even if the conditions are favorable, a mass indicated as having good conductivity might not prove to be commercial ore. There are other conducting materials in the ground which cause disturbances. Pyrite and pyrrhotite, the most common sulfides in nature, and also graphite, are highly conductive and abundantly distributed in ore-bearing areas. Owing to their texture, graphitic slates are good conductors, at least in one direction, and sometimes cause stronger reactions than the orebodies. Occasionally basic dikes containing magnetite and pyrrhotite are good conductors. Therefore, methods of identifying noncommercial conductors are of great importance. It is not sufficient to confine the electrical work to a mere location of an electrical disturbance and leave the interpretation to the geologist or mining engineer. He may put down drill holes and find nothing but a valueless mineralization or conductive rock which it might have been possible to identify electrically and avoid the expenditure required for drilling.

There are a number of methods that permit only of locating (sometimes roughly) a conducting mass, but so far as I am informed, the Swedish methods are the only ones giving an "outline," and not only the "axis" of the conductor. By determining the width, for instance, a narrow conductor of no appreciable thickness (a mineralized contact or fault plane) will be easily eliminated in spite of its causing very strong disturbance. Further, if the geology of the locality is known, it is often possible to determine from the shape and extension of the conductor whether they are caused by ores, disseminations, graphitic slates or basic dikes.

A geological examination, therefore, is of importance even for the interpretation of the result. This examination is carried out in the usual manner, however, with special attention given to such features as are of importance for the electrical investigation. The occurrence and character of noncommercial mineralizations in the area should be included. Such mineralizations are quite common in an ore area, and being of no commercial value, often escape the attention of the practical mining man. Sometimes such mineralizations are of great help in leading to the discovery of bodies of commercial ore.

Graphitic and pyritic slate formations generally occupy distinct, wide horizons, following the general strike and dip. Usually indications caused by such slates are long, continuous and regular and of larger dimensions than are likely to be caused by the orebodies being sought. There are occasions, however, especially in strongly folded strata, where the slate is torn and divided into series of short lenses which cause indications very like orebodies, and may easily lead to useless exploration work, without avail, if the slate character cannot be determined electrically. As a rule, it is possible to identify them and, by means of a qualitative survey, obtain an idea as to the specific conductivity and physical nature

of the body. In a qualitative investigation, dielectric and magnetic constants of the conductors often have to be taken into account. With the technique now developed, it is usually possible to determine whether it is a solid or uniformly disseminated conductor, whether it has a banded or schistose structure, or is magnetic.

Indications often are caused by magnetic masses. By analyzing such an indication, it is possible to distinguish between a body that is both highly magnetic and conductive (a body of magnetite or pyrrhotite) and a body strongly magnetic but only slightly conductive, which is the general reaction from basic dikes.

It is, as a rule, possible to determine electrically whether a magnetic attraction (obtained by a dip needle or magnetometric survey) is caused by a sulfide body or by a basic dike.

By comparing all the data secured electrically and geologically, the indications obtained may be classed in two groups: (1) indications probably caused by noncommercial mineralization or conductive rocks, and (2) indications that ought to be more closely explored. In the latter case, the exploration is recommended either because the indication is likely to be caused by ore, or because knowledge of the conducting body cannot be obtained in any other way. Even if there thus remain doubtful cases, their number is reduced to a minimum by means of the detail and qualitative methods. Of course, conditions sometimes may be rather complicated and difficulties encountered. For instance, the location of small narrow orebodies, or of the shape of steep-dipping chimneys or pipes, is sometimes rather difficult and requires minute detail work.

In working on the surface, the main reaction is obtained from the top of the body; thus, excepting with bodies of low dip, generally little information is obtained as to the extension in depth of the body itself. At present, it is not possible to reach electrically an orebody situated beneath a layer having higher or equal conductivity.

The greatest depth to which an orebody may be reached cannot be given in general terms; it depends on the size of the body and the electrical properties of the ore and surrounding rocks. A great depth from the surface and too complicated conditions sometimes prevent the application of qualitative investigations.

It has proved rather difficult to convince claim owners that strong indications can be caused by conductive rock of no commercial value, but results from drill holes put down in spite of our recommendations have fully confirmed our decision not to drill.

Under certain conditions, it might prove quite sufficient to locate the indications in a reconnaissance survey only; for instance, it is cheaper to dig a trench through shallow overburden than to spend time in having a detail survey made. However, a detail survey often is justified, econom-

ical, and rather necessary in districts where conditions are complicated and exploration work is expensive.

Cost of Electrical Prospecting

Compared to other methods of exploring, the cost of electrical prospecting generally must be considered very cheap. The cost, of course, depends on the time required, which in turn depends partly on topographical, geological and climatic conditions.

Observations are taken at intervals of 30 ft. or closer along lines across the formation. As an average, one observer is able to make 7000 ft. per day with the aid of three unskilled helpers.

In a district with orebodies of large size, the sections may be laid out fairly far apart without danger of missing any orebody; on the other hand, if small orebodies are expected, the lines are laid much closer. As a rule, lines for a reconnaissance survey are laid 150 ft. apart, but in certain cases an interval of 100 ft. may be necessary; in others, 300 ft. may be permissible. Thus the ground covered in one day may be three times as large in one case as the other. The rapidity with which the lines may be traversed in making observations varies with topography, or other obstacles, and the weather. Our experience is that one team of two engineers with the necessary unskilled help has covered from 5 acres to 100 acres per day. Without taking into account the cost of transportation, the cost will therefore be from $2 to $40 per acre. I would place the average at about $6 for the primary or reconnaissance survey. When indications are encountered, extra time must be spent in making observations at closer intervals, and perhaps testing the conductivity in different directions.

Results of Various Surveys

A new art like electrical prospecting is likely to be received with either extreme skepticism or overwhelming faith. Often too much has been expected and disappointment has resulted. Often, too, when large areas are surveyed without finding orebodies, the methods are blamed for the failure, when the real reason is that no orebodies exist. It is frequently forgotten that a comparatively small sum of money has been spent in indicating that the ground has no ore, thereby saving a larger sum which might have been spent on useless exploration by excavation. Even if the result thus is negative, it certainly has a considerable economic value. The practical value of the methods, therefore, cannot be measured by the number of orebodies found.

As for positive results, only in a few places, for instance Sweden and Newfoundland, has the electrical work been checked up to such an extent that a true comparison can be made between conditions predicted and

those actually found in later exploration. It is, therefore, a little early to discuss in general the practical result in electrical prospecting.

The viewpoint of the physicist, the geologist and the practical mining man must be satisfied and often all of them have quite opposite views in the matter. A result might be a technical success, but considered a failure by the practical man. Therefore, I have chosen to discuss a few results from different districts in the light of local geology and conditions.

The organization with which I am associated has carried out electrical prospecting in several countries in Europe, Africa, the United States, Canada and Newfoundland. Work has been executed under quite different conditions, in tropic as well as arctic climate and under most varying geological conditions and consequently has yielded considerable experience. Also, numerous orebodies of commercial size and grade have been discovered. Some of the orebodies thus found electrically are of rather outstanding value. Earlier results, especially in Europe, have already been published, therefore I will confine the discussion to results on this continent. Failures and difficulties are mentioned as well as successes as all contribute to progress.

Survey in the Zinc-lead District in Upper Mississippi Valley

The ore-bearing area extends over large portions of Wisconsin, Illinois and Iowa along the Mississippi River.

Geology

The rocks are sedimentary with rather undisturbed beds. The absence of igneous rocks and tectonic disturbances are characteristic of the district. Rocks of interest in this connection are, counted downward: Niagara dolomite, Maquoketa shale and Galena dolomite of the Silurian age and then Ordovician Platteville limestone. The series dip slightly to the southwest and are in places gently folded.

The Niagara dolomite is almost completely eroded, remaining only on the highest hills. The Maquoketa beds form a dense bluish shale, which also is largely eroded, remaining only in higher parts. The Galena dolomite is the general ore-bearing rock. It is rather porous and easily weathered. The weathering extends from 60 to 100 ft. downward from the surface, often altering the rocks to a brownish clay. The present level of the ground water is rather deep, generally from 100 to 250 ft. Between the Galena and Platteville formation is often found a thin bed of bituminous shale called "oil rock," which seems to be connected with the formation of the ore.

Orebodies in the district are deposited by the ground water. Originally limestone, which has been altered to dolomite, the Galena dolomite has decreased its volume and crevices, openings and also brecciated

porous zones were formed, which became natural channels and loci for ore deposition.

The ore minerals are galena, zincblende and marcasite. Before the circulating water made its way deeper down, galena and some marcasite was deposited. Therefore the main lead deposits generally are found near the surface. Later, zincblende and marcasite were deposited. Marcasite often occurs in thin sheets along walls of crevices and openings, covering the galena higher up, and interbedded with zincblende deeper down.

From a practical standpoint the ore deposits are divided into four different types:

1. *Crevices and Openings.*—This type of deposit as a rule is found not deeper down than 100 ft. from the surface, and is mainly galena with smaller amounts of marcasite and zincblende. The crevices often occur in groups or systems. The dip is nearly vertical and the width only a few inches. Open spaces often are found at the intersection of two systems of crevices. These openings sometimes reach rather large dimensions, and galena, often coated with marcasite, is plastered to walls, roof and bottom.

2. *Honeycomb runs*, which occur as extensive, horizontal, rod-shaped orebodies of triangular or irregular cross-section, probably formed at the intersection of vertical and horizontal planes of circulating solutions. Deposits of both galena and zincblende are found of this type.

3. *Pitches and Flats.*—These deposits occur at different levels, and constitute a system of steep-dipping and horizontal crevices filled with ore. Such openings are probably formed by the sinking of large, irregular blocks of dolomite into a shrinking part of the underlying oil rock. At the present time, deposits of this type are commercially most important. They occur at greater depth, generally 180 to 300 ft. from the surface. The chief ore mineral here is zincblende.

4. *Disseminated Ores.*—Deposits of this type occur mainly as flat disseminations near the oil rock. The ore mineral is zincblende.

The shallow lead deposits now seem to be mined out thoroughly. Lead mining in the district started in the eighteenth century. The early lead prospecting has proved very efficient indeed. Only a few discoveries have been made during the last 25 years.

The fissures occur in groups and systems running parallel or at about right angles to each other and could easily be followed for miles when once discovered. A fissure was followed and investigated by test pits, to find the lead. As a rule, there seems to have been a relation between the topography and the mineralization of the fissure, slopes being more favorable places than valleys and ridges. However, in spite of the thorough prospecting already done, there is no reason to believe that the lead resources are entirely exhausted.

The zinc ore has been mined on a large scale for only about 50 years. The first discoveries were made when sinking shafts for lead to greater depths. The zinc is found most frequently in the vicinity of old lead workings, although there are lead-bearing areas where no zinc has been found and zinc deposits exist without apparent connection with any lead. There also seems to be some relation between the zinc deposits and the tectonic structure in the district. Quite a number of deposits occur along the edges of shallow depressions in the Galena dolomite and there is a marked trend of deposits along synclines of the shallow foldings of the rocks. Generally the zinc flats are to be expected in or near an area indicated by surface lead. It is difficult, however, to follow the geology in detail because outcrops are scarce.

Prospecting today is done mostly by churn drilling, the first holes in a new place being put down in a haphazard way. During the drilling, geological information is often obtained which makes it possible to proceed systematically towards a favorable area, but many holes are drilled without any useful information resulting. Prospecting for zinc is laborious and slow and, carried out systematically, the expense of prospecting the immense areas still remaining unprospected stands in no proportion to the value of the orebodies.

Electrical Prospecting

To ascertain whether electrical prospecting could be of any use in prospecting the district, several tests and surveys have been made. As conditions are rather complicated, different methods were tried out. In all, about one year has been devoted to the solving of the various problems. The first investigations were made in the winter 1923–24. At that time, conditions for electrical prospecting in this district were very little known and the electrical methods were not as highly developed as they are today.

First, a series of tests was made on known deposits with the equipotential method. In one case an indication was obtained on a long flat body of zinc and iron sulfides, situated at a depth of 200 ft. The indication extended beyond the body known by drilling and a hole put down on this part of the indication struck rather good lead ore, somewhat closer to the surface than had been anticipated from the geological conditions. Three or four more holes directed by the electrical survey were put down and most of them showed pyrite with little or no zinc at the ordinary level. Another test gave a good indication on a flat zinc-lead body about 100 ft. from the surface.

Although no definite proof as to the ability of the method of locating zinc ore was obtained, a few investigations were made on virgin ground. Indications were found, but most of them were rather unsatisfactory and

uncertain and influenced by conditions at the surface, or by conditions of the ground. For instance, the brownish clay occurring as a weathering product of the Galena dolomite was found to be a rather good conductor. Drilling carried on in the investigated area did not show much ore.

Though no definite proof had been obtained, the experiments were not discontinued, and laboratory experiments were carried on. Later electromagnetic methods were tried.

The conductivity of numerous specimens of ore was investigated. Galena is the best conductor; marcasite has fair and the zincblende very poor conductivity. Pyrite, occurring occasionally, has better conductivity than the marcasite. Table 1 gives the specific resistance in average figures of the minerals in the district.

TABLE 1.—*Specific Resistance of Upper Mississippi Valley Minerals*

Samples	Resistance, Ohms per Cu. In.
Galena, pure	3
Galena crystals coated with marcasite	80
Galena, marcasite and calcite	700
Marcasite, coarse grained	400
Marcasite in thin sheets	8,000
Zincblende and alternating marcasite layers	23,000
Zincblende, pure	above 100,000
Zincblende, disseminated	above 100,000

The resistance of the Galena dolomite varies considerably and is not much higher than that of the zincblende. All figures relate to fairly dry specimens. The specimens of high resistance showed considerably less resistance in a moist state.

It was evident from the beginning that there was no hope whatever of detecting pure zinc. Only when associated with iron sulfides or with galena did it show a conductivity essentially different from the surrounding rock. But even the zincblende-marcasite ore showed rather low conductivity. However, considering the extremely favorable position and the great size of the flat zinc orebodies with regard to the electromagnetic field used, it was considered appropriate to carry on further experiments until a definite opinion based on field work had been obtained.

In prospecting for the lead, the conductivity tests were encouraging, but the methods were not considered adequate, because of the scattered and irregular occurrence of galena and the rather insignificant size of the deposits.

The inductive method was applied, as current supplied by means of electrodes was not always found entirely satisfactory.

The first field tests were made on a flat-lying zinc-marcasite orebody at a depth of about 200 ft., previously outlined by drilling. A disturb-

ance of the electromagnetic field was obtained, showing a strong secondary field considerably lagging in phase and evidently not induced in the orebody but in the ground itself.

A current is always induced in ground having any conductivity at all, but is generally of insignificant strength; at least in dense, highly crystalline or igneous rocks and in glacial drift. However, the dolomites and limestones of the Mississippi Valley are porous, intersected by numerous cracks and fissures and weathered to great depth, and are capable of absorbing moisture, which contains dissolved sulfides, limestone, etc., and holds a certain amount of salt. The salinity of the ground water makes it a good electric conductor, though the salt content is not very high. The rocks in the Mississippi Valley often show rather good conductivity and then it is possible to induce in the ground strong secondary fields even when using rather low frequency.

The current in the ground was found fairly uniformly distributed with its center of density at a considerable depth not far from the ore horizon. It could then be assumed that a stronger current would concentrate in better conducting parts of the ground at this horizon and accordingly it could be expected that the secondary field would be stronger above orebodies containing marcasite or galena. A test survey based on these principles was carried out on a known orebody and one indication was obtained which closely corresponds with the position of the orebody.

The investigation was extended beyond the limits of the known orebody where indications were found lying approximately in the strike. In order to obtain more information with regard to the origin of this indication, whether caused by marcasite or possibly marcasite and zinc at deeper levels, or by fissures carrying lead, the depth to the conductor from the surface was calculated and found to be about 100 ft. A drill hole put down on this indication encountered considerable lead at 60 ft. This result at first sight proved the calculations of depth to be of little value, showing a difference of as much as 40 ft. from the depth predicted. By later drilling, however, a wide bed of disseminated iron sulfide was encountered on both sides of the first hole at a depth of 90 to 120 ft. This was probably a continuation of the known orebody. The lead in the first hole must be considered more or less incidental.

Conclusions

The results obtained in the Upper Mississippi Valley cannot be considered entirely satisfactory, the main difficulty being the low conductivity of the ore. Flat bodies of ore containing lead may be found by electromagnetic methods; and, provided there is no open and empty space between the ore and the surface, also by means of an equipotential survey. Deposits of marcasite may also be located, but the ordinary

ore, the zincblende, can be found only when associated with sufficient amount of galena or marcasite.

Deposits containing pure marcasite with insignificant amounts of zinc are quite as common as deposits containing both zinc and marcasite in considerable amount. There are also zinc deposits with little or no marcasite. Accordingly, a great number of indications might be due to worthless iron sulfides, while it is possible that no indications would be obtained on the very best zinc deposits.

On the other hand, an extensive and systematic electrical survey will give considerable information as to the general distribution of the mineralization in the district and also indicate whether this mineralization is near the surface or at the depth most favorable for zinc ore, near the bottom of the Galena dolomite.

As the galena occurs mostly in fissures near the surface, help may be furnished by an electric survey in tracing the fissures. These fissures are often easily followed electrically, as they carry a greater content of moisture than the surrounding rock, but the fissures may not be ore-bearing.

As a whole, the result may not seem very fruitful, but there still remains hope of a successful solution of the problem. Even at the present stage of development of the methods of electrical prospecting, quite accurate information may be obtained of geological structure, faults, fissure system, etc., even at a considerable depth. It remains therefore to investigate whether a careful study of topography, geology, structure and occurrence of the ore deposits may reveal any further clue which can be combined with data derived from an electrical investigation. Before these conditions have been thoroughly investigated and duly considered, the possibilities for a general method of electrically finding zinc and lead ore have not been exhausted in the Upper Mississippi Valley.

Survey in the Southwest

In the southwestern portion of the United States, where the climate is arid, conditions for electrical prospecting are somewhat limited.

The rocks are weathered, porous, very susceptible to moisture and the solutions in the ground are often good conductors. The bed rock, therefore, sometimes has very good conductivity. Where the ground is very dry and consequently a poor conductor, water courses often cause strong disturbances in an electrical survey. Sulfide orebodies are oxidized and the oxidation often goes deeper than 1000 ft. Electrical prospecting for ore, therefore, is of more limited application than in areas once glaciated or situated in more temperate regions and as a rule all prevailing conditions must be very carefully considered before electrical prospecting is undertaken.

Even if the orebodies cannot be directly indicated electrically, valuable information often may be obtained from a survey regarding structure, faults, etc., which are essential for finding the ore.

Considerable time has been devoted to the solving of these problems and electrical prospecting has been carried out in several different mining districts in the Southwest, for instance Gilman, Colo.; Hanover and Questa, N. M.; Superior and Jerome, Ariz.; Tintic, Utah.

Often, when the oxidation of the ore was too deep to reach the ore by working at the surface, electrical prospecting underground was tried and applied successfully. Electrical prospecting underground is more difficult than working from the surface and also more limited. In many mines it is hardly possible to obtain any useful result at all; in others good results may be obtained, depending on local conditions.

Except for the limited space available for taking observations, the main difficulties met are the rails and pipelines. Especially the pipelines, which are always connected with water in some way and often form closed circuits themselves or in combination with tracks and orebodies.

In interpreting the result of the investigation, all known conductors must be paid due consideration. The magnitude and character of disturbance from known origin have to be estimated and subtracted before the conclusions are drawn. The same accuracy as in surface work is seldom reached but with the development of special methods for underground electrical prospecting, with sufficient experience, and with a thorough knowledge of the local conditions, satisfactory results can yet be expected.

Survey in Northern California

The sulfide-ore districts in Shasta and Plumas counties in Northern California as a rule offer conditions favorable for electrical prospecting, and some surveys using electromagnetic methods have been carried out in both counties.

In the summer and fall of 1925 some areas in Shasta County were surveyed. The topography is rugged and the hills, though not high, are separated by steep-sided valleys. The area is underlaid by series of sedimentary rocks and volcanic tuffs which are intruded by large irregular masses of porphyry. Well defined lenticular bodies of rather solid sulfide, carrying copper, zinc, and gold and silver values, occur in groups along shear zones in the porphyry or at its contact with other rocks. The oxidation is not very deep, generally not exceeding 30 ft.

The ore has good conductivity and the igneous rock is a poor conductor when dry. A black or gray shale was found to be a good conductor along the strike of the beds, owing to pyrrhotite and graphite content.

Some small orebodies were found. In the course of the survey several strong indications were obtained which could be clearly referred to water courses in shear zones in the porphyry. Lenses or islands of shale enclosed in the porphyry gave strong disturbances but by means of a qualitative survey these were easily identified.

Survey in British Columbia

Conditions for electrical prospecting in British Columbia are generally favorable, although the topography often is very rough. Considerable time has been spent in doing electrical prospecting in various parts of this province.

Britannia District

The Britannia mining area occupies a mineralized zone in a large block of metamorphosed sedimentary and igneous rocks entirely surrounded by the Coast Range granodiorite. The mineralization is confined to a shear zone, striking about east to west and dipping fairly steeply to the south. In the mineralized area, the orebodies occur in series of lense-shaped masses of sulfides separated by disseminated or barren bands of schist of various thickness. The ore is composed of pyrite with chalcopyrite in various proportions. A great difference in the electrical conductivity is found between ore and schist, even when the latter carries a considerable amount of disseminated sulfides. Thus conditions make it possible to trace electrically bodies of high sulfide content in disseminated schist.

In the early spring of 1926, an electrical survey was carried out east of the Britannia Mines where a drift-covered portion of the shear zone was investigated. The area electrically surveyed was located in a deep valley—Furry Creek—between rather steep hill slopes. The equipotential method was used.

The result has been described by Moore and Ebbutt.[2] The following is quoted from their paper: "The equipotential method was used at the Britannia with most interesting results. An orebody was indicated and in the course of development (some 200 ft. below ground) was picked up in its true position." Fig. 8 shows in plan a portion of the area covered by the equipotential survey. "The strong indication was borne out by subsequent developments. A fault was picked up underground. The position of the faulted ore was indicated by the electrical prospecting which furnished the key to the work." Fig. 9 shows in section the position of the vein, relative to the indication on the surface.

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[2] J. I. Moore and F. Ebbutt: Electrical Prospecting at the Britannia Mine. *Trans.* Canadian Inst. for Min. and Met. (1926) **19,** 1017.

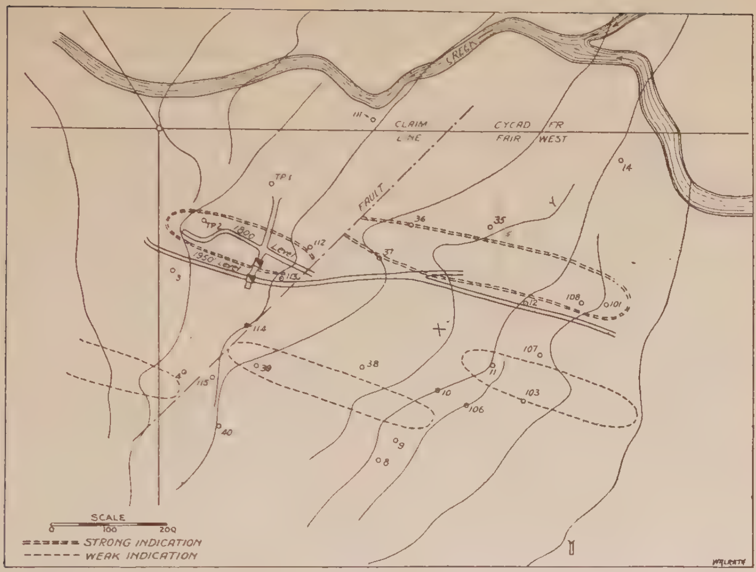


FIG. 8.—PART OF AREA COVERED BY EQUIPOTENTIAL SURVEY AT BRITANNIA MINES.

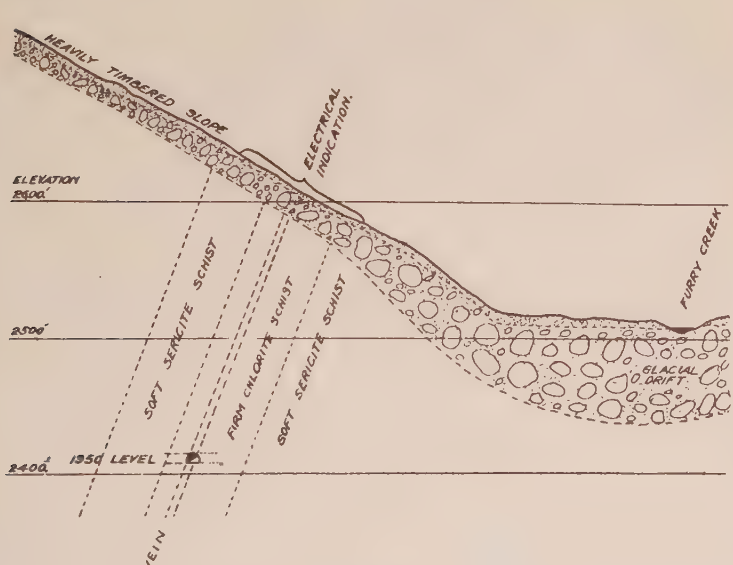


FIG. 9.—POSITION OF VEIN OF FIG. 8, IN RELATION TO INDICATION ON SURFACE (SECTION ON XY).

"It is clearly evident from results obtained at Britannia that the electrical prospecting has a great deal of merit and is destined to serve a most valuable purpose in the future in connection with mining developments."

Kimberley District

During the winter of 1925–26, electrical prospecting was carried out on large areas near Kimberley in the Fort Steele mining division.

The area is underlaid by argillaceous quartzites of the Aldridge formation and often intruded by sills of gabbro. The quartzite beds are often strongly folded. The general strike of the rocks is north to south. The dip is generally to the east. Minor folds modify the simple structure and often anticlines and synclines can be observed.

The ore deposits are of the replacement type and as a rule conform to the dip and strike of the quartzite. The boundaries of the orebodies are generally not well defined. The ore minerals are pyrrhotite and pyrite with zincblende and galena.

There is a great difference in conductivity between normal country rock and the sulfide ore and conditions for electrical prospecting may be considered rather favorable. The fact that the sulfide content of the ore gradually decreases into normal country rock rather than forming well defined walls sometimes makes the outlining of the conductor difficult.

Thin layers of massive sulfides, which are of no commercial value, often occur along faults or bedding planes and cause disturbances. They are, as a rule, possible to identify. Sometimes, however, flat-lying beds may give rather strong disturbances, which only can be identified by means of a minute qualitative survey.

Anyox District

This area is to be considered as rather favorable for electrical prospecting. Large bodies of sulfide occur along contacts between argillite and basic intrusives.

In the summer of 1927 an extensive electrical survey was carried out in the district. Both the equipotential and the electromagnetic methods were used. The topography is irregular with rather steep hills. Except for retarding the work, no considerable difficulties were encountered in the survey due to rough topography.

Often considerable sulfide mineralization is found in the argillite, especially near the contact. The sulfide minerals, mainly pyrite and pyrrhotite, are distributed as narrow veinlets or paper-thin sheets along the beds Where these stringers occur most frequently, or lie close together, they generally cause rather strong electrical disturbances. As

a rule, however, these can be easily identified by the characteristic distribution of the sulfide minerals along the beds. Usually, good conductivity is found along the strike of the beds, but at right angles the beds are poor conductors.

The copper content of the sulfide bodies varies considerably and the electrical work could be used only for determination of the size.

Survey in Cœur d'Alene District

A considerable area has been electrically prospected west of the city of Kellogg in the Cœur d'Alene mining district, Idaho.

Geology

The area is underlaid by quartzite beds of Pre-Cambrian age. The general strike of the formation is northwest to southeast. The dip varies but is mostly to the southwest.

Numerous faults and fissures traverse the formations at about right angles. In the fracture zones, which are mainly composed of shattered quartzite and siderite, the ore occurs as veins or lodes, sometimes rather irregular and ill defined; sometimes large, solid veins, probably old fissures, which later have been filled with ore. The ore minerals are galena and sphalerite, with some pyrite, pyrrhotite and chalcopyrite. The oxidation of the ore is shallow, seldom deeper than 30 feet.

Condition for Electrical Prospecting

Generally the veins carry a sufficient amount of conducting sulfides to render conditions favorable for electrical prospecting.

The area is mostly covered with wash and soil, and outcrops are scarce. The equipotential method was used. The ground is very hilly with steep slopes and heavy vegetation.

The orebodies manifested themselves in a plain and satisfactory way. Dikes of igneous rock, which also traverse the formation at about right angles, caused strong disturbances. The dike rocks seem to weather more easily than the Pre-Cambrian rocks and often form a gougy clay near the surface. The dikes also seem to hold considerable water and often show a conductivity higher than the Pre-Cambrian rocks.

In this district, by means of a qualitative survey, such indications may be easily eliminated. A determination of the specific conductivity of the conductor would reveal whether the indication is caused by clay or water-bearing dikes or by sulfide mineralization.

Survey in Northern Manitoba

During the winter 1926–27, a considerable acreage was electrically investigated north of Lake Athapapuskow, about 80 miles north of The Pas in Manitoba.

Here sulfide lenses of varying size occur in Pre-Cambrian schist. Several large sulfide deposits are known in this region and they are of occurrence quite similar to those discovered by electrical prospecting in Northern Sweden.

Electromagnetic methods were used, the current being supplied by means of grounded electrodes or by induction. The winter was rather severe and for several weeks the temperature was 30° below zero or colder. This is mentioned to point out that an electrical survey, even when using grounded electrodes, is quite possible in a very cold climate.

An excellent check on the accuracy of the survey was obtained when investigating the Baker-Patton orebody, previously known from only two

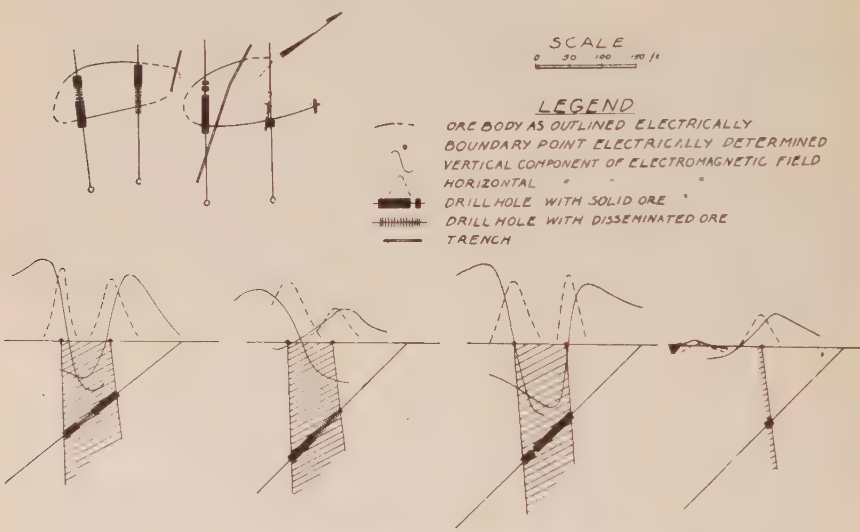


Fig. 10.—Electrical result compared with diamond drilling of Baker-Patton orebody, Northern Manitoba.

trenches. Fig. 10 illustrates the electrical result in comparison with the diamond drilling conducted afterwards. The outline of the bodies near the surface, as electrically obtained from determining the boundaries at various sections, agrees very well with the result in the drill holes at depth. It was thus proved that the electrical survey aided greatly by delimiting the orebodies so that the drilling could be conducted intelligently. It was further proved possible to avoid exploration of narrow stringers of sulfide and irregular and scant disseminations.

Survey in Northern Ontario

Electrical prospecting has been executed in the Sudbury Basin and also tried out in the Cobalt district.

Sudbury District

In the Sudbury Basin the rocks are heavily covered with a thick mantle of glacial drift, often exceeding 100 ft., which renders the observation of geological detail impossible. Large orebodies of zinc, lead and copper sulfides occur near the contact between a tuff and a black slate striking east to west. The mineralization occurs in connection with fault zones cutting through the contact. Where the orebodies are found, the slate is altered and carries a considerable amount of pyrrhotite and pyrite and also graphite. The electrical conditions, therefore, are favorable for locating roughly the mineralized zones. Several square miles of land have been electrically prospected, using electromagnetic methods, and such mineralized zones have easily been located.

From laboratory tests and also from surveys of known orebodies, certain electrical differences were found to exist between the ore and the mineralized slate.

The problem in the field, however, is rather complicated, as sometimes the mineralization of the slate is heavy and uniform.

Cobalt District

The rocks of the area consist of the Cobalt series, mainly conglomerate overlying the Keewatin. In some parts, the conglomerate is covered by the Nipissing diabase. The ore deposits form narrow veins, mostly in the Cobalt series, but a few productive veins have also been found in the Nipissing diabase and the Keewatin. In some parts of the Keewatin, pyrite and magnetite are disseminated and also some graphite. The conglomerate contains a great number of fissures and cracks and many are also seen in the diabase. By following these cracks, some may lead to the discovery of a vein.

Some field tests were made in 1924, using the equipotential method, but nothing conclusive resulted. Later experiments and laboratory tests, however, show that the tracing of a fissure is possible if it occurs within a reasonable distance from the surface, probably not more than 50 ft. There seem to be no means as yet of distinguishing between a fissure carrying commercial ore and one carrying only valueless mineralization.

Survey in Rouyn District, Northern Quebec

This district centers around Rouyn Township, Province of Quebec, Canada, and extends throughout several neighboring townships.

Geology

The area is underlaid by thick beds of Keewatin lavas which vary in composition from acid to basic. Structurally, the beds are tilted and heavily contorted. The general strike seems to be northeast-southwest. Intrusives as dikes, sills and batholiths have intersected the lavas and greatly disturbed them. The area is generally covered with a heavy blanket of glacial drift which comes from long distances and thus gives no clue to the underlying rocks.

Orebodies

The orebodies, as a rule, are comparatively small, irregular lenses of rather high sulfide content, carrying copper, zinc and precious metals. They occur in many different ways, some lying along the contacts of lava flows, some occupying zones of shearing, etc. At Noranda, bodies of massive chalcopyrite and zincblende lie along, or close to and sometimes inside, the old basic dikes. No well supported theory of the origin of the ore has so far been evolved.

Conditions for Electrical Prospecting

Lenses of almost unoxidized sulfides in dense, highly crystalline rocks, thin drift cover composed of rather uniform material and ground water of fairly low conductivity are all conditions which make the district at first sight look ideal for electrical prospecting. However, in working in the district, certain adverse conditions became evident. As to the ore, the chalcopyrite orebodies are excellent conductors, but the zinc orebodies are poor. In other similar districts, zincblende often occurs with other sulfides, so that the mass is fairly conductive, which is not the case in Rouyn. As these zinc orebodies generally occur close to bodies of other sulfides, which are easily indicated by electrical prospecting, the possibility still prevails of eliminating areas that are certainly barren.

The rather frequent occurrence of noncommercial sulfides in masses, dissemination and narrow stringers generally makes observations at close intervals desirable, in order to avoid, as far as possible, the drilling of valueless mineralization. A geological knowledge of the district, after outlining the conducting body electrically, generally enables one to eliminate those without value, and the qualitative survey has been used but seldom. The ability to do this naturally adds to the value of the electrical survey in saving useless drilling.

Operations in Rouyn District

Where the size of the orebodies varies between such wide limits as in Rouyn, it is desirable to use different methods. As between the inductive and galvanic methods of supplying inciting current, for work on

frozen lakes or where it is desired to cover large areas rapidly in the search for large orebodies, the choice would fall on the inductive method. Where small orebodies are likely to be encountered, and close work required, the galvanic method is to be preferred, making observations by the equipotential method or, in the winter, electromagnetically.

The inductive method can be increased in sensitivity by largely increasing the frequency of the current used, but this results in a multitude of indications, as currents are induced in rocks and soil if these are only moderately conductive; in fact, this effect is likely to mask indications arising from ore deeper down so that the object of the survey may be defeated. Furthermore, with any frequency, the current induced in an orebody is dependent on the size of the section exposed to the lines of force of the primary magnetic field, so that, in the case of small orebodies, the amount of secondary current induced is small, and hence the strength of the secondary magnetic field is low and its effect difficult to detect. A large part of our work in the Rouyn field, therefore, has been done with grounded electrodes. By this method, both large and small orebodies may be indicated and, with the moderate frequency used, few disturbing valueless indications are obtained.

Several thousand acres of land and lake areas have been surveyed by our organization during the past year and a half. Little commercial ore was found. Strong electrical indications were frequently obtained, but it was generally possible to identify those caused by bodies of no commercial value. In spite of our advice to this effect, a large number have been drilled or trenched but, so far as we know, in no case has anything of value been found.

Test surveys have been made on some of the known orebodies of the district with results that were in absolute accordance with what was known from underground operations. Unfortunately, these tests cannot be described. They show, however, that similar orebodies, if occurring elsewhere in the district, could easily be found.

Complicated conditions were encountered on and near the Amulet property in Dufresnoy Township north of Rouyn. These were not appreciated at the beginning of the work and there was great difficulty in interpreting the results of the electrical survey. Through the careful geological work by J. C. MacGregor, then geologist of the Amulet Co., the situation is now understood. The flows in this vicinity are badly contorted but generally of rather flat dip. Large and small sulfide bodies occur, with varying sulfide content. Some bodies are rich in copper, some in zinc, and some without commercial value; some are good conductors and some poor. A number are associated with a flat-dipping, heavily mineralized and good-conducting bed of pyrrhotite and pyrite, probably at the contact of two lava flows. A few of the orebodies are found above this bed but the larger ones, or some of them, occur beneath

it, where they are difficult to reach electrically. Thus the ore occurrence is very unusual; and, under such conditions, we have no way, so far, of differentiating between ore and valueless material.

Prospecting in Newfoundland

Prospecting around the Red Indian Lake in the center of Newfoundland started about 1905, when a Mic Mac Indian, named Matty Mitchell, discovered some lead-zinc ore outcropping on the banks of Buchans River,

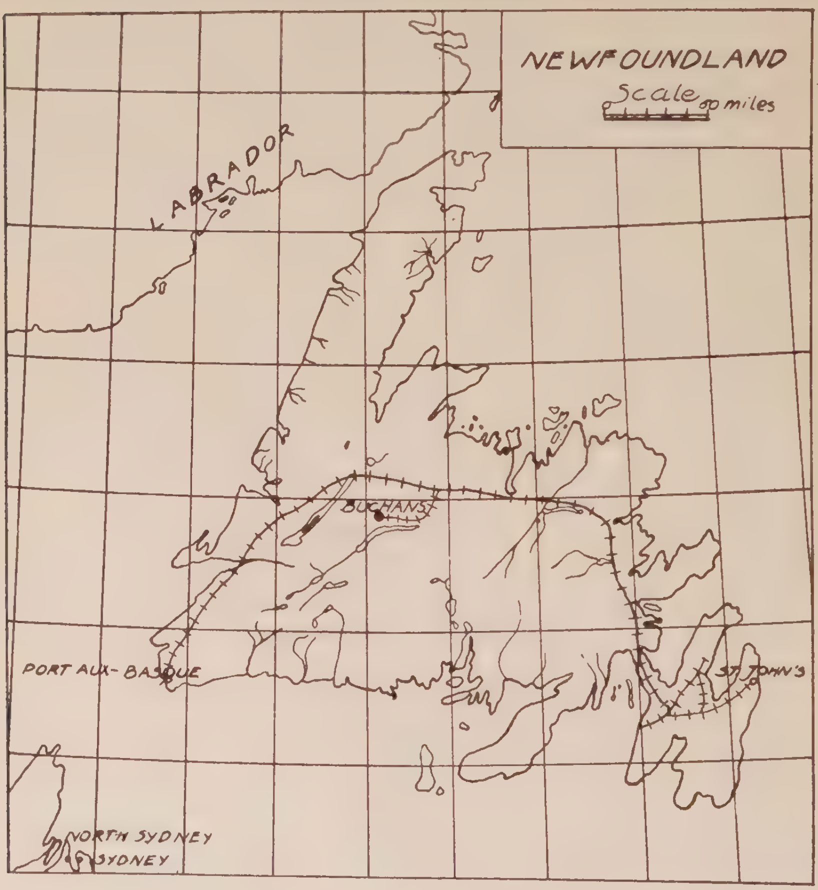


Fig. 11.—Situation of Buchans mine.

about 5 miles north of the lake (see Fig. 11). Exploration work revealed a medium-sized orebody of rich lead-zinc ore. This orebody, called Buchans mine, was developed by the Anglo-Newfoundland Development Co., Limited. A shaft was sunk about 400 ft. and some drifting done. However, the metallurgical treatment was extremely difficult and the mine

remained idle for about 20 years; until H. A. Guess, vice-president of the American Smelting & Refining Co., developed a method (in 1926) for treating the ore and took over the control of the property for that company.

The area surrounding the mine is drift covered and little was known as to the geology of the ore-bearing rocks. A few miles away boulders of rich lead-zinc ore had been found in the glacial moraine and it was suspected that other orebodies existed in the vicinity. Electrical prospecting was tried in the area surrounding the mine, with gratifying result. In the summer of 1926, a number of valuable orebodies were found.

Geology

The bed rock is covered with a rather thick mantle of glacial drift and rock outcrops are rare. The geological conditions are fairly simple, and it has been possible, with the aid of the electrical survey and the few outcrops, to form a general idea of the ore-bearing formation.

The country rock is formed of bedded tuffs and porphyritic lava flows, probably to be classed as Archean. The tuff beds are composed of volcanic debris varying from fine powder of ash and tuff to arkose, grits, greywacke and even coarse conglomerate; thin beds of greenstone were found occasionally interbedded. Near the contact with the porphyrite the tuff beds are slightly altered. The beds composed of fine material here become dense and chertlike.

The contacts with porphyrite are generally not well defined and often form agglomerates of varying aspects. This agglomerate, or pyroclastic breccia, is composed of rounded as well as angular fragments in a basic matrix which, farther from the contact, gradually turns into porphyrite. In some parts the fragments are fresh; in other parts they show heavy alteration.

The tuff beds are sometimes folded and faulted, especially near the orebodies.

The contact between the tuff beds and the porphyrites and sometimes large parts of the agglomerate are mineralized. The orebodies are found as lenses or irregular masses in the mineralization. The ore is composed of complex lead-zinc-copper sulfides in a baryte gangue.

Two types of intrusive rocks are found in the area. A granite porphyry with large grains of quartz occurs in minor batholiths and resembles the acid granite found in this section of the island. It is suggested that this granite is connected with the formation of the orebodies. A younger and basic rock intrudes and cuts across both the granite and part of the mineralized area.

The country has been glaciated during more than one period, which makes determination of the glacial transportation of boulders somewhat uncertain.

The Electrical Survey

The equipotential method was found to be the most applicable, both for reconnaissance survey and for detail work. However, when surveying lake areas during the winter, electromagnetic methods were used. The area around the mine was first investigated in detail and conducting zones of mineralization leading from the known ore were followed. The conditions for electrical prospecting were very favorable, as the orebodies occur in zones of mineralization which are slightly conductive and thus possible to trace electrically. By following the zone east about one-half mile, a large group of indications was discovered, called "Oriental." The same zone was followed to the west about one mile, where another

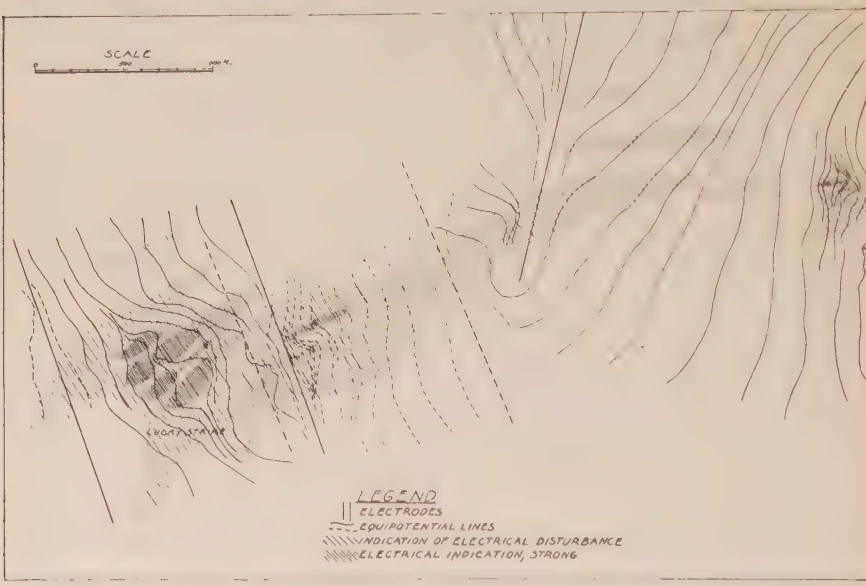


FIG. 12.—AREA SURVEYED ELEC

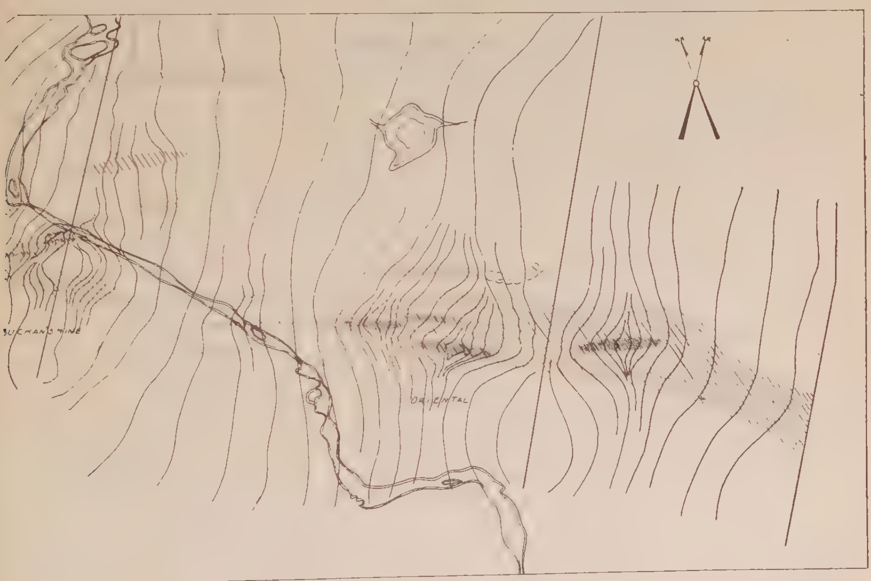
large group of very strong indications was encountered, called "Lucky Strike" (see Fig. 12).

This part of the survey was completed in July, 1926, after only six weeks in the field. Since then several square miles of land and lake area have been electrically surveyed, but this part of the survey is not completed and therefore not included in this account.

At Oriental, some wide and large bodies and several minor ones were indicated electrically. The depth of the overburden here varies from 25 to 60 ft. or more, and the only practical method of checking the indications was by diamond drilling. Two of the large indications have been drilled so far and found to be high-grade lead-zinc ore.

At Lucky Strike a large group of rather wide lenses was indicated on low ground covered with gravel and muskeg. Owing to a shallow overburden, trenching was possible in order to uncover the cause of the indications. By a number of trenches, bodies of high-grade lead-zinc-copper ore were uncovered which well corresponded to predictions derived from the electrical surveys. It is interesting to note that, although very little detail work was done, it was possible to locate and outline portions richer in lead or copper. The space between the lenses thus indicated corresponds either to high-grade zinc ore, nonconducting, or barren rock.

Later extensive diamond drilling and some underground work was done on Lucky Strike and the other orebodies, and according to recent announcement by Mr. Guess, a tonnage of more than three million tons



TRICALLY AROUND BUCHANS MINE.

of ore has already been indicated. The grade of this ore was given as follows: lead, 7 per cent.; zinc, 16 per cent.; copper, 2 per cent.; silver, 3 oz.; gold, 0.03 oz.

Another excellent check on the possibility of locating a rich body of ore inside a mineralized area was obtained during the detail survey immediately west of Buchans mine. A map of this portion is shown in Fig. 13. Here a shaft had been sunk in well mineralized agglomerate. The strongest indication was found to the north of the shaft. An old trench on the surface did not show any increased mineralization at the intersection with this indication. However, the center and west end of the indication were investigated from underground by means of drifts and found

to correspond to a lens of good grade lead-zinc ore about 11 ft. wide. A trench was put down on the east end of the indication but no ore was found. However, after a round of drill holes had been blasted, good ore was encountered even here. Thus it was very well proved that by means of an equipotential survey a high-grade body could be located inside a mineralized area.

After the discovery of Lucky Strike, nearly all exploration work was concentrated there in order to put this deposit in shape for mining. A

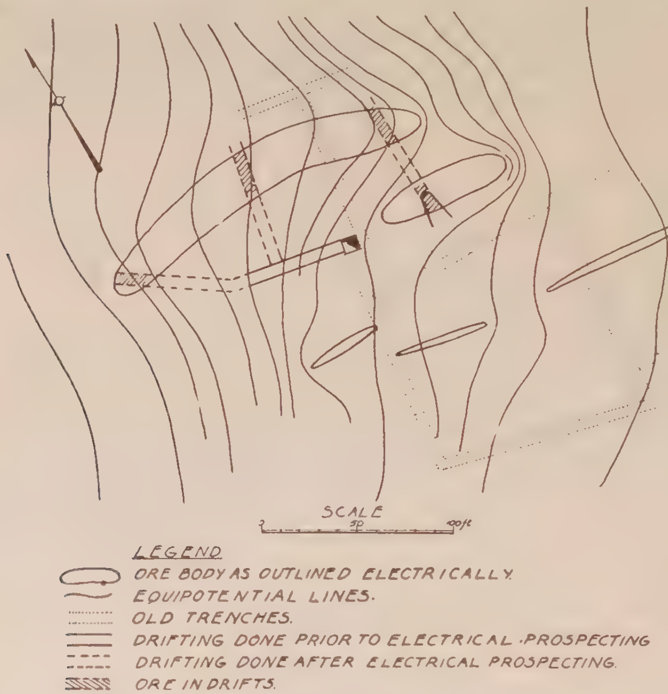


Fig. 13.—Detail survey west of Buchans mine.

mill and a power plant were constructed. A railroad 23 miles long was completed last fall, connecting the mine with the Newfoundland government railroad. A large tonnage of rich ore developed in a short time, so that mill, power plant and railroad could be constructed within slightly more than one year after the discovery, is an outstanding record in the annals of mining history.

Conclusion

In conclusion, it may be said that the value and usefulness of electrical prospecting has been abundantly proved in this district, and it is doubtful whether these very important orebodies would ever have been located without this method.

DISCUSSION

T. Zuschlag, New York, N. Y. (written discussion).—Mr. Lundberg proposes to measure phase differences and interpret them in regard to conductivity variations. In order to do this, it is necessary that the phase differences in question shall be of a measurable magnitude.

Recently, Mr. Jakosky made the statement[3] that "frequencies of less than 5000 per second are not concerned with phase displacement, because these effects usually are of such small magnitude as to be negligible." This statement is not in line with Mr. Lundberg's explanations, as Mr. Lundberg has evidently in mind frequencies of less than 5000 cycles per second.

It is known that phase differences in or between alternating magnetic fields are the natural result of the finite speed of propagation of these fields. In air the speed of propagation of magnetic or electric fields is 300 million m. per sec. In any other medium it is usually lower because the speed of propagation is inversely proportional to the square root of electric conductivity and magnetic permeability. The speed of propagation is further directly proportional to the square root of the frequency. Assuming a field of 50,000 cycles having in certain ground a subsurface speed of one-quarter of its air speed—75,000,000 m.—then a field of 500 cycles, in the same ground, has a subsurface speed of only one-tenth of 75 million; *i. e.*, 7,500,000 m. If the same fields penetrate ground with a conductivity 10,000 times greater than that of the first, the speed of the higher frequency field will be reduced to 750,000 m. per sec. and that of the lower frequency field to 75,000 m. per sec. These speeds, although low compared with the air speed, represent high values. According to Dr. Steinmetz, the speed of propagation of alternating magnetic fields penetrating into solid copper sheets is about 90 m. per sec. for a field of 50,000 cycles and 9 m. per sec. for one of 500 cycles. In other words, any passenger train has a higher speed than a field of 500 cycles penetrating into solid copper.

The quotient speed of propagation divided by frequency represents the wave length. In the case of a field of 50,000 cycles with a subsurface speed of propagation of 75,000,000 m. per sec., we have a subsurface wave length of 1500 m. The air wave length being 6000 m., the ratio of the two wave lengths of this frequency is 1:4. The corresponding values for 500 cycles are 600,000 m. in air, and 15,000 m. for a subsurface speed of 7,500,000 m. per sec., the ratio now being 1:40.

One full wave length means a phase change of 360°. With increasing distance from the source of energization, we have, therefore, a continuous increase of lagging phase, the amount of which for fields of different frequencies is inversely proportional to their subsurface wave length.

Jakosky states that a field of 60,000 cycles has, under certain conditions, a phase displacement of 108°, while a 500-cycle field, under the same conditions, has a phase shift of 0.9° only. Applying Jakosky's phase-shift formula, but using the following corresponding subsurface wave lengths, we find:

For 60,000 cycles and 1,250 m. wave length, a phase shift of 108°
For 5,000 cycles and 4,800 m. wave length, a phase shift of 34.2°
For 500 cycles and 15,000 m. wave length, a phase shift of 9°

These values show that phase displacements in fields of frequencies of less than 5000 cycles can certainly not be regarded as negligible.

Phase differences exert a double influence upon the carrying out of geoelectric investigations. First, they impair the accuracy of the readings, and, second, they

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[3] J. J. Jakosky: Electrical Methods of Geophysical Prospecting. *Engng. & Min. Jnl.* (1928) **125**, 293.

make the final interpretations much more difficult. Theoretically there are two ways to improve this situation. One method proposed by the Radiore Co. is the diminution of phase differences by changing the frequency, while the other consists in measuring the actual values of the phase shift. I prefer the second method, because I believe that the latter, although at first appearing more difficult, will, in the end, result in more efficient work. Due to the relationship between subsurface speed of propagation and phase shift, it promises to furnish more data in regard to the geoelectric structure of the ground.

Electrical Prospecting for Molybdenite at Questa, New Mexico

By Karl Sundberg,* Houston, Tex., and Allan Nordstrom,† New York, N. Y.

(Boston Meeting, August, 1928)

Interesting results were recently obtained in geophysical prospecting at the Questa mine of the Molybdenum Corpn. of America in New Mexico. This paper describes that survey, which was carried out during October and November, 1927, by the Swedish American Prospecting Corpn. It is hoped that the paper will make it possible for the mining man to form an opinion of the applicability of geophysical prospecting under difficult conditions, showing, as it does, the results when prospecting for small small orebodies with comparatively poor conductivity, in a region of rough topography.

Some principles underlying electromagnetic prospecting not known to be published previously will also be given in an elementary way, with special attention to the general structure of the electromagnetic field.

Geological Conditions at Questa

The property is situated in the Taos range of the Rocky Mountains in northern New Mexico. About this region, L. C. Graton says:[1]

> The main range is made up of Pre-Cambrian granites, gneisses and schists but in the vicinity of Red River igneous rocks of later age are present almost to the exclusion of the older series. These younger rocks are divisible into two groups. The older of the two, probably Pre-Tertiary age, is represented by large intrusive masses of monzonite porphyry, usually containing considerable quartz. The more recent group comprises intrusive and probably effusive dark-colored andesites and coarse andesitic breccias, some of which contain fragments of the monzonite; there are also flows of rhyolite and beds of rhyolite breccia. The evidence regarding the relative age of the andesite and the rhyolite is not wholly satisfactory, but there is little doubt that the andesite is the older.
>
> The ore deposits, although occurring more commonly in the monzonite porphyry than in the rhyolite, nevertheless are believed to be genetically related to the rhyolite. They certainly cut the rhyolite or andesite in places, and hence fall in the late Tertiary.

The molybdenite at the Questa mine occurs in veins in the alaskite granite porphyry belonging to the older part of the Pre-Tertiary eruptives. Overlying the granite porphyry is the granodiorite, capping the granite

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* Chief Engineer, Swedish American Prospecting Corpn.

† Swedish American Prospecting Corpn.

[1] W. Lindgren, C. H. Gordon and L. C. Graton: The Ore Deposits of New Mexico. U. S. Geol. Survey *Prof. Paper* 68 (1910).

porphyry intrusion, and over this again lies andesite and andesitic breccia, these being the youngest igneous rocks in the area. All these rocks are exposed in Sulphur Gulch (Fig. 1), in which the adits of the mine are located.

The slopes in this region are very steep and a considerable part of the area is covered with shallow overburden consisting of slide rock and soil (Fig. 2). However, in numerous places the bare rock is exposed, which has given an opportunity to find some outcrops of the veins. There are

FIG. 1.—SULPHUR GULCH. FIG. 2.—TOPOGRAPHY AT QUESTA.

numerous veins, the types most frequently represented being: (1) veins containing molybdenite alone; (2) veins containing other sulfides with little or no molybdenum; (3) veins containing limonite and other oxidation products.

The molybdenum veins at Questa carry the molybdenum in the form of molybdenite, MoS_2, which at the surface is oxidized to a depth of not more than a few feet, the oxidation minerals being molybdite, MoO_3, and rarely ilsemannite, Mo_2O_3. The matrix of the veins is quartz and the distribution of the molybdenite is as follows: On both sides of the vein there are usually slickensides of almost pure molybdenite, the thickness of which varies from 1 or 2 in. down to very thin seams. In the quartz

mass the molybdenite is scattered as well-formed crystals, or there is a network of thin blades of molybdenite. Thus the conductivity of the ore varies, but is generally about 10,000 to 30,000 ohms per centimeter cube.

The veins of the second class are comparatively common, containing instead of molybdenite other sulfides, such as pyrite, galena and stibnite in the quartz mass. These veins occur in the same geological way and their specific electrical resistance is of the same magnitude as that for the molybdenite veins. Thus there is no possibility of determining electrically or geologically whether an indication obtained by an electrical survey is caused by a vein containing molybdenite or by one carrying sulfides without economic value.

The third type of veins is a system of fissures containing oxidized metallic minerals, mostly limonite, and here and there carrying water, which gives them comparatively low resistance (50,000 to 100,000 ohms per cu. cm.).

It must be borne in mind that the possibility of making an electrical survey not only depends on the resistance of the ore prospected for, but also on that of the country rock and the overburden. In this case the barren rock has a conductivity of the magnitude of 10^6 ohms per cu. cm. and the overburden, which almost without exception is slide rock also, has a very high resistance.

In porous rocks the conductivity is not only dependent on the matrix of minerals, but also on the porosity and the character of the water absorbed. In this case there are only two porous rocks, the slide rock and the vein of the third type. At the time of the survey, the first one was very dry and in the second there was here and there some moisture which caused this vein type, though not containing any metallic mineral, to have comparatively low resistance.

All the veins occur very irregularly, having no special direction of dip, and the strike, though mostly west-east, varies considerably, too, causing difficulties in the interpretation of the results.

Exploitation of the Molybdenite

The outcrops of some veins were found 10 years ago and mine work was started on a small scale. In 1923, the present owner, the Molybdenum Corpn. of America, took over the property, and the mines have since then been worked continuously. The total production to date amounts to somewhat more than 2,000,000 lb. of molybdenite, for the extraction of which about 25,000 ft. of development work has been required. These figures give an idea of the difficulties caused by the irregularities of the mineral occurrence.

The ore mined contains 6 or 7 per cent. MoS_2. It is brought down partly on burros to the road in Sulphur Gulch, whence it is hauled in

wagons to the flotation mill at Red River. The mill handles 70 tons per day and gives a concentrate with 75 to 80 per cent. MoS_2, with a recovery of about 90 per cent. and tails containing 0.5 to 0.6 per cent. MoS_2. These concentrates are shipped to the company's smelting plant in Washington.

Electromagnetic Compensator Method Used for the Survey

The Sundberg system was used in making the survey at Questa. As the veins of molybdenite usually are of economic value at a width as small as 5 or 6 in. and the resistance is comparatively high, a very sensitive

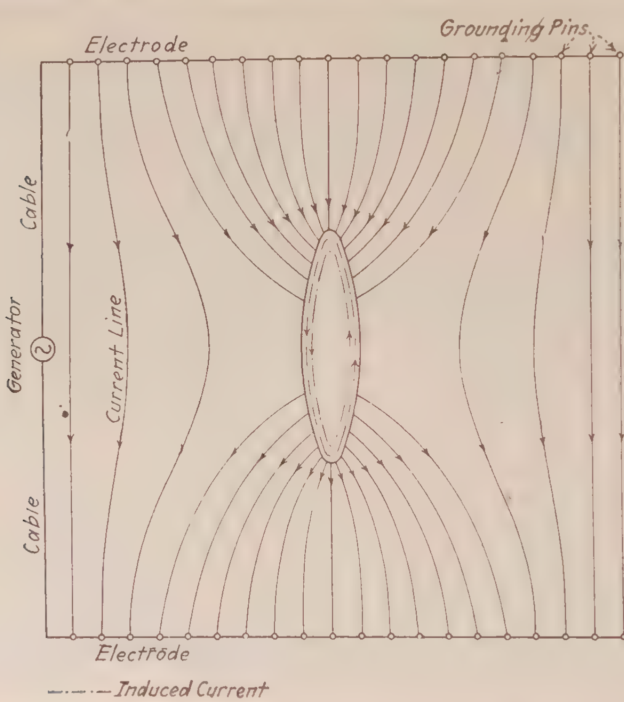

Fig. 3.—Orebody with current flow between electrodes and induced current.

method was required. For this reason the current was introduced into the ground galvanically by means of linear electrodes laid at right angles to the strike of the formation and connected at one end by means of insulated wire to the poles of an alternating current generator (Fig. 3). The electrodes consist of bare copper wire laid out along a straight line and grounded every 10 ft. by means of iron pins.

Any flow of alternating current is surrounded by an alternating electromagnetic field. In the case of barren ground, the current flows uniformly between the electrodes, and the electromagnetic field obtained is called the "normal field." The disturbances in this field due to the presence of

orebodies or other bodies of good conductivity are called the "anomalies" or "secondary fields." The resultant of the normal and secondary field is the actually existing or resultant field. The normal field can be determined either by measurements on barren ground or by theoretical calculations. The normal field thus being previously known, a comparison between the actually determined (resultant) field in a certain area with the normal field will show if any secondary fields exist. The character of the secondary fields will give, from theoretical considerations and practical experience, the most probable position of the conducting body that is responsible for these fields. The corresponding position is marked on a map and called an electrical indication.

Just as we give the strength of different components of the field from a magnet, as, for instance, the earth's magnetic field, in Gausses or gammas (1 Gauss = 100,000 gammas), so we use the same units for the strength of electromagnetic fields. Because the strength of the electromagnetic field at any point under any conditions[2] is proportional to the current used, and for other reasons, it is convenient to use as a unit not the absolute values of the strengths of the fields, but the value per unit of current flowing between the electrodes. As a practical unit we use 0.1 gamma = 1 micro-Gauss (m-G.) per 1 amp. It is of interest to note the great difference in magnitude between the strength of the earth's magnetic field, around 500,000 m-G., and the magnetic fields encountered in electromagnetic prospecting, which generally are of the order of 1 to 100 m-G., sometimes even much less. The possibility of investigating such weak fields depends on the fact that voltages induced by alternating fields can be greatly amplified, which is one reason why alternating currents are used. Another reason is that an alternating electromagnetic field induces eddy currents in any conducting body exposed to the field. Both theory and practice show that these eddy currents generally will increase the strength of the secondary fields, and thus give a more pronounced indication of the orebody.[3] The eddy currents are schematically shown in Fig. 3. Generally eddy currents will be induced in the earth cover, too, and sometimes in the country rocks.

Let us consider a small part—for instance, a small cylinder—according to Fig. 4, of the mass between the electrodes through which the current I flows, and let us first assume that direct current is used and that the current flowing through the cylinder is parallel to its axis and has the direction shown in the figure. This current will produce a magnetic field of which the lines of force will be circles according to the figure. In the case of alternating current, the figure gives direction of current and magnetic field at a certain moment. After one-half cycle (1/1000 sec.,

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[2] Except above magnetic bodies; a case not considered here.

[3] Sometimes we use inductive methods in which the response of the orebody is due entirely to the eddy currents.

if 500-cycle current is used), direction of both current and magnetic field will be reversed. It is usual to represent the time or phase of alternating currents or magnetic fields by angles, one cycle being 360° or 2π. In the case of a sine-current, the strength I of the current at any time t is

$$I = I_0 \sin \cdot 2\pi ft$$

f being the frequency, I_0 the maximum amplitude of current.

Because there is no difference in time between the creation of a current and its magnetic field, there is no time or phase difference between

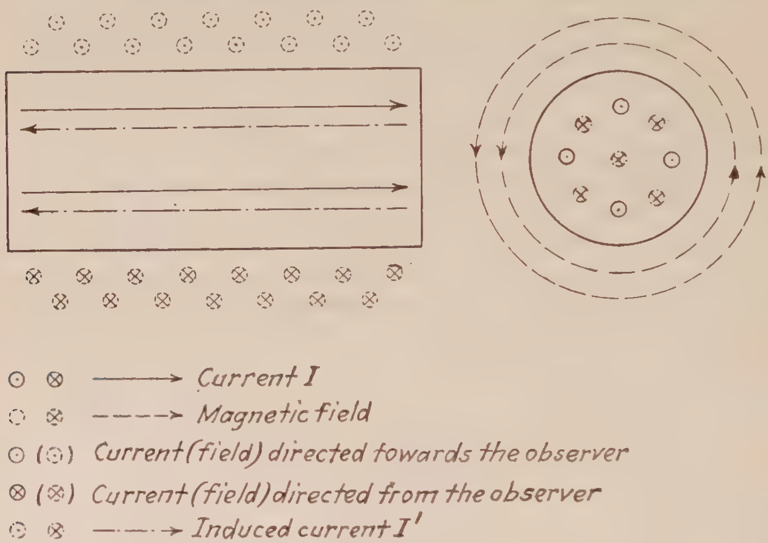

Fig. 4.—Electric current and magnetic field in small cylinder.

the current I and its magnetic field H, the amplitude of which at any moment thus can be represented by

$$H = H_0 \sin \cdot 2\pi ft$$

H_0 denoting maximum amplitude of the magnetic field.

This alternating magnetic field will produce an induced current I' in the conducting cylinder. According to physical laws, the amplitude of the induced current at a certain moment is proportional to the rate of speed with which the magnetic field changes or to $\frac{dH}{dt}$. The amplitude of I' thus can be represented by

$$I_0' = I' \cos 2\pi ft = I'_0 \sin \cdot \left(2\pi ft + \frac{\pi}{2}\right)$$

I_0' being the maximum amplitude of the induced currents. The induced current thus has its maximum when the current I is zero, there being a

phase difference between I' and I of $\frac{\pi}{2}$ or 90°, in time equal to $\frac{1}{4f} = \frac{1}{2000}$ sec., if 500 cycles is used. In the case considered of a small cylinder, the direction of the induced current I' apparently will be the same as the direction of the current I, both being parallel to the axis of the cylinder. In a time diagram, where time differences are represented by angles according to Fig. 5, the currents I and I' are represented by the vectors shown, differing in time 90°. Because I and I' have the same direction in space, we can take the resultant I_R of the two in the time diagram. The result thus is that we have, instead of the current I, which would be present if direct current were used, the current I_R, differing in phase by the angle φ or the time $\frac{\varphi}{2\pi f}$ from I. It is apparent that

$$I_R = \sqrt{I^2 + I'^2}$$

and

$$\tan \varphi = \frac{I'}{I}$$

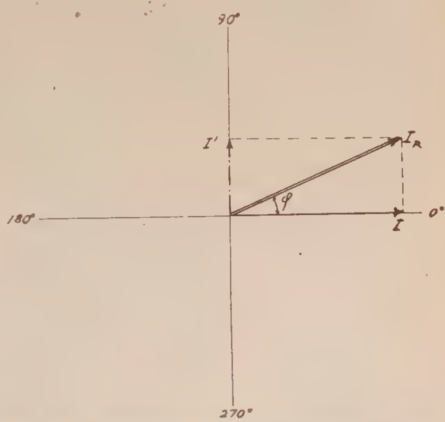

FIG. 5.—REPRESENTATION OF PHASE DIFFERENCES BETWEEN ELECTRIC CURRENTS.

The considerations above refer to the current through the small cylinder only; as a matter of fact, we have to consider the influence of currents in the whole area beneath the electrodes, in the electrodes and in the wires leading to them. Due to the influence of all these, the induced current apparently will have a direction in space which differs from the direction the assumed direct current I would have. For reasons not necessary to give here, the induced currents also generally differ in phase from I, not 90° but some other time angle. Thus, in fact, we have at any point underground a current which is the resultant of a number of components differing both in direction (in space) and phase (in time). Let us consider the phase of the current through the wires to the electrodes, the primary current, as the phase angle 0° in a time diagram according to Fig. 5 and call the corresponding axis of the diagram the real, and the perpendicular axis the imaginary. With reference to this diagram, it is then apparently always possible to divide any current vector of the arbitrary phase φ into two components, one parallel to the real axis and thus in phase with the primary current, and the other parallel to the imaginary axis and thus 90° out of phase from the primary current. Let us call these two components the real, or in-phase, and the imaginary, or out-of-phase, components, respectively.

Turning back to the currents between our electrodes, we thus can represent the current at any point underground by a number of real and imaginary components of different amplitudes and directions (in space). All of the real components, because in phase, can be represented by a resultant real component obtained according to known principles of adding vectors. In the same way all the imaginary components can be represented by a resultant imaginary vector. We thus can represent the current at any point by two components, one real and the other imaginary. If amplitude and directions of these two components are given, the current is determined.

We now turn to the electromagnetic field at any point. Apparently the real components of the currents give a number of electromagnetic field components in phase with the primary current and with varying amplitudes and directions in space. We call these components the real components of the electromagnetic field. They give, by vectorial addition, a resultant real component. In the same way we obtain a resultant imaginary component of the field. We thus can represent the electromagnetic field at any point by two components differing in direction, one the real, or in-phase, component being in phase with the primary current, and the other, the imaginary, or out-of-phase, component being 90° out of phase with respect to the primary current.

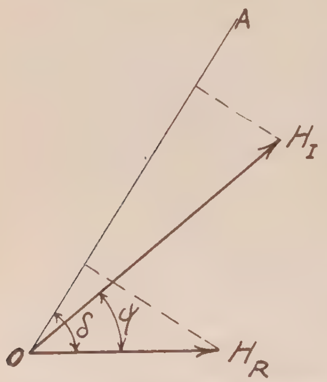

FIG. 6.—ELECTROMAGNETIC FIELD COMPONENTS IN THE POLARIZATION PLANE.

Let us consider the plane of the real and imaginary components of the field (either the electric or the electromagnetic) and assume that the angle (in space) between these two components is ψ according to Fig. 6. Along an arbitrary direction OA which makes the angle δ with H_R we can represent the field by two components, one real,

$$H_R{}^A = H_R \cos \delta$$

and one imaginary,

$$H_I{}^A = H_I \cos (\delta - \psi)$$

The resultant field in the direction OA thus is

$$H^A = \sqrt{H_R{}^2 \cos^2 \delta + H_I{}^2 \cos^2 (\delta - \psi)}$$

and the phase φ with respect to the primary current is determined by

$$\tan \varphi A = \frac{H_I \cos (\delta - \psi)}{H_R \cos \delta}$$

It can easily be proved that the end of the vector H^A describes an ellipse when the angle ψ varies from 0° to 360°, the phase φ^A at the same time varying from 0° to 360°. The direction along which the field

component is in phase with the primary current apparently is perpendicular to the direction of H_I and along a direction perpendicular to H_R the field component is 90° out of phase with respect to the primary current. Along a direction perpendicular to the plane of H_R and H_I no field components exist.

Because of the character of the electric and electromagnetic fields described above, the fields are said to be elliptically polarized and the plane of the field ellipse is the polarization plane. We represent the position of the polarization plane of the electromagnetic field according to Fig. 7, which is a plane view in which the long lines (1-2-3), give the direction or strike of the field ellipse in the horizontal plane and the short lines (2-4) indicate the direction toward which the polarization plane is inclined, the figure at the end of the lines 2-4 giving the inclination or dip in degrees from a vertical plane through 1-2-3. We call this inclination β.

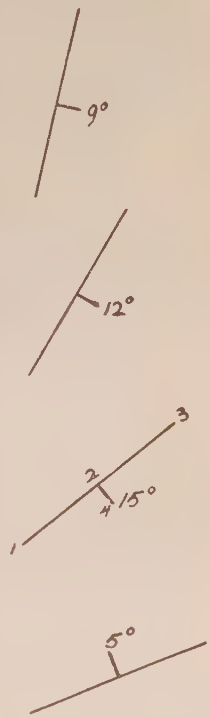

FIG. 7.—REPRESENTING DIRECTION AND DIP OF POLARIZATION PLANE.

The field ellipse, of course, is completely determined by phase and amplitude of two arbitrary components in the polarization plane. In practice we usually investigate the vertical component V and the horizontal component H along the strike of the polarization plane. The latter component we usually refer to as the horizontal component H not forgetting that really a second horizontal component exists perpendicular to H, in phase with the vertical and with the amplitude $V \cdot \tan \beta$. A complete electromagnetic survey thus comprises determination at every point of strike and dip of polarization plane and amplitude of real and imaginary parts in micro-Gauss per ampere of horizontal and vertical components. Because the essential instrument used in such a survey is called compensator, we refer to such a survey as a survey by the compensator method. The compensator used was invented by Karl Sundberg and E. D. Lindblom, of Aktiebolaget Elektrisk Malmletning, Stockholm, Sweden.

The Two-frame Method

Especially for reconnaissance work, but often also for detailed work, we use a more simple method which we call the two-frame method (invented by Sundberg). In a two-frame survey, direction and dip of the polarization plane is determined and the ratio between the field components V_1H_1 at one point and the part of corresponding components V_2H_2 at a second point, which are in phase with V_1H_1. Assuming that

the phase differences between corresponding components at the two points are not very large, which is often found to be the case, this method gives a very good picture of the resultant field. In practice, the results are generally represented in an arbitrary scale in which amplitudes of the field components are given. The amplitudes are obtained by assuming the field to be undisturbed in a certain area where the survey shows the ratios obtained to be the same as the known ratios of the undisturbed normal field. The amplitude of the normal field being known, the amplitude at any point within the surveyed area of the resultant field components is obtained and the components of the secondary field by comparing the resultant field components with corresponding components of the normal field.

The Three-frame Method

Our three-frame method is a modification of the two-frame method and is characterized by obtaining the true ratio between the field components at two points and the difference in phase between the components. This method makes a complete analysis of the field possible in a manner similar to the compensator method.

Interpretation

In practice, it is often unnecessary to calculate the secondary field, because a comparison between the resultant and normal fields gives the position of the indications closely enough for practical purposes. It would carry us too far to discuss details of interpretation of results, and we will give here one fundamental principle only, which is strictly correct for simple in-phase conditions, but which has in practice proved to be very valuable even in rather complicated cases. The use of phase angles or imaginary components in interpreting results is not discussed here.

Let us consider the case of the electromagnetic field from a long horizontal straight current. The magnetic field components from such a current at a point at the horizontal distance X and the vertical distance Y from the current, in an arbitrary scale, are as follows:

$$V = \frac{X}{X^2 + Y^2} \text{ (represents the vertical component)}$$

$$H = \frac{Y}{X^2 + Y^2} \text{ (represents the horizontal component)}$$

In a horizontal plane, Y is constant and equal to the depth to the current beneath this plane, which might be the surface, the current representing a buried orebody. Calculating the components for different distances, X, from the current, we easily find V to have its inflexion point and H its maximum immediately above the current. The same relation is found to be strictly true for a vertical sheet of current independent of the way in which the current is distributed along the sheet.

In the case of steep-dipping orebodies, therefore, the ore is found beneath the points where the horizontal component of the secondary field has its maximum and the vertical its inflexion point.

The Questa Survey

At Questa, the electromagnetic field was investigated by both the compensator and the two-frame methods. One complication in the interpretation of the results is caused by the steep grades, but considering

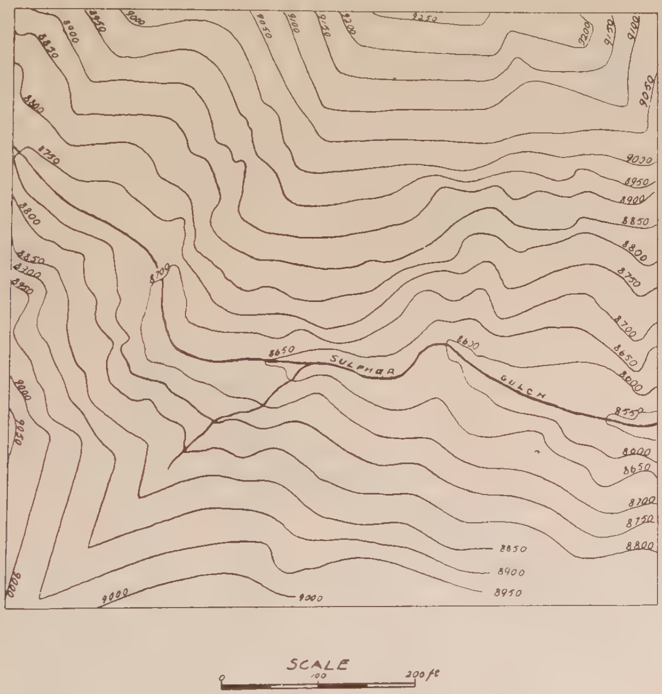

Fig. 8.—Level lines of part of electrically surveyed area at Questa. Figures give elevation above sea level in feet.

that current flow follows the ground and comparatively conforms to the surface when undisturbed, the general grade can be considered to be the "horizontal component," the "vertical component" being perpendicular to this. Fig. 8 shows contour lines over part of the territory surveyed, illustrating the rough topography, which is also shown in Figs. 1 and 2.

Altogether 65 acres were investigated in seven weeks, a rate unusually slow because of the rough topography and close intervals of observation necessary on account of the irregular character and small size of the orebodies. Had the topography been favorable, this property could have been surveyed in two to three weeks. In the case of large orebodies and favorable topography, we have surveyed on reconnaissance as much as 20 to 40 acres a day, as, for example, in our present work in Newfoundland.

In Fig. 9 a typical example is given of the electrical results. There are shown, in section, dip of polarization plane, amplitude of the in-phase (real), parts of the horizontal and vertical field components. The out-of-

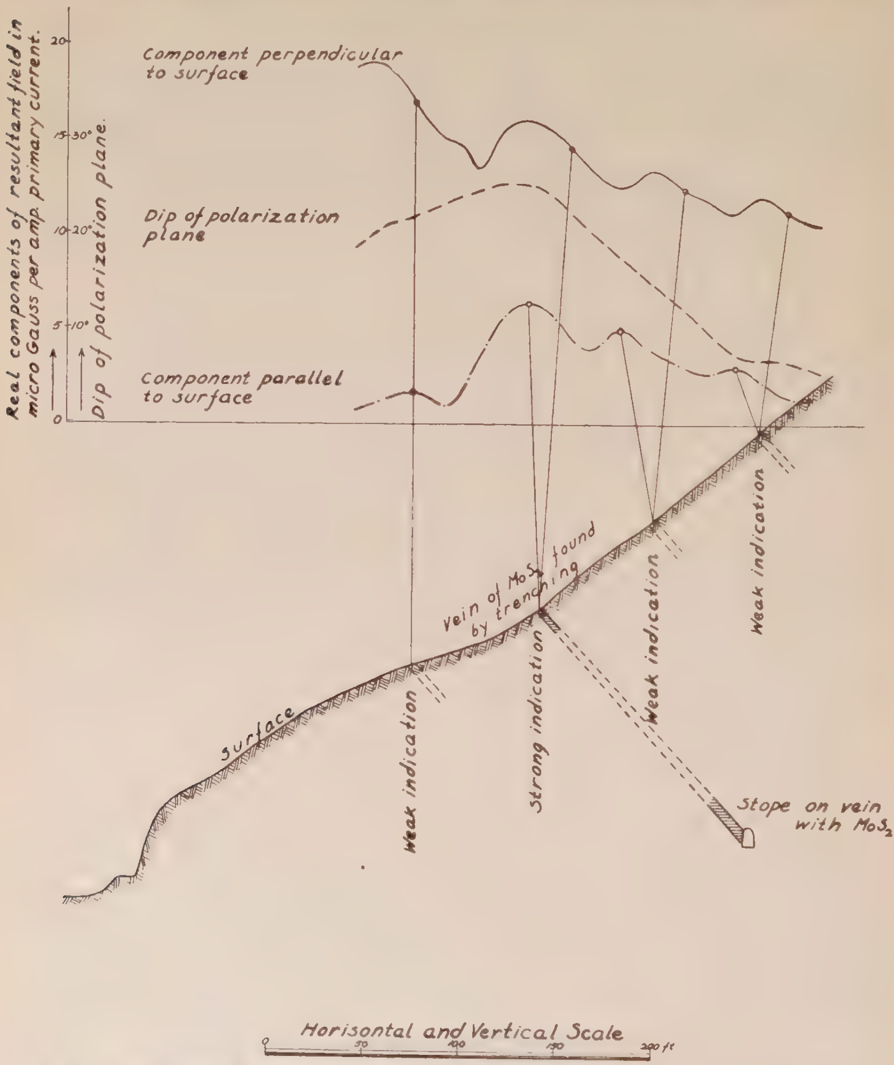

Fig. 9.—Amplitude of electromagnetic field components, dip of polarization plane, electrical indications and result of trenching and drifting.

phase (imaginary) parts of the field components were negligible in this special case. Of the four indications shown, the one marked "strong" has been tested by the drift shown in the figure and by some small surface trenches. The indication, by subsequent development work, proved to correspond to very good ore.

Another indication was tested by trenches at the surface and one drift. Two veins were found, one containing some galena and the other molybdenite.

A third indication was tested from a tunnel and a quartz vein was found, containing considerable molybdenite.

A fourth indication was tested by trenching immediately after the electrical survey was made and a vein was found within an hour after the electrical readings were taken.

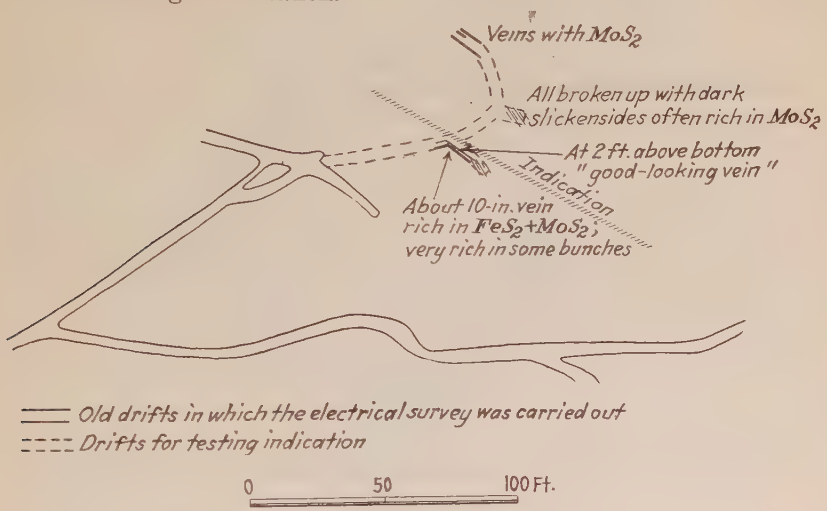

FIG. 10.—RESULT OF UNDERGROUND ELECTRICAL PROSPECTING AT QUESTA.

Some work was done underground in the drifts marked as "old" in Fig. 10. The indication shown in the figures was located and the drifts indicated were afterwards driven to test the indication. A vein 10 in. wide was found, which was rich in molybdenite and pyrite.

Seven indications chosen to be tested by trenching, drifting or raising (depending on local conditions) have given heavy mineralization; four contained molybdenite of good or reasonable economic value; one, some molybdenite but mostly pyrite; and one was a network of small veins, of which one contained galena and had a resistance of 2000 ohms per cu. cm.—the lowest resistance found for any sample from Questa.

As a general conclusion of the results obtained, this survey has shown the possibility of using electromagnetic prospecting for finding small veins with poor conductivity in country of very rugged topography. Officials of the Molybdenum Corpn. of America say that, as a result of the electrical survey, the mine can now be operated 24 hr. a day—the first time in its history—and that a substantial amount of ore is now in reserve.

Operating Principles of Inductive Geophysical Processes

By J. J. Jakosky,* Los Angeles, Calif.

(Boston Meeting, August, 1928)

All electrical geophysical methods depend for their operation upon the effects produced by the flow of an electric current. By studying these effects it is possible to predict the general axis of current flow. The greater flow of current is in the path of greatest effective conductivity; and since the effective conductivity of a mineralized zone is different from that of its surrounding envelope (usually much greater), it is possible to locate such a mineralized zone by the distribution of current. Due consideration must be given, of course, to geological structure, to type of mineralization, and to other factors. A number of methods are used to cause this flow of current, and various methods are available for detecting the presence of such current flow and for studying its effects. In a rather detailed way, this paper will discuss the theoretical and practical phases of the inductive methods of geophysical prospecting.

Electromagnetic Generation of an Electromotive Force

The inductive method is so named because the current flowing in the conductor is obtained by electromagnetic induction, instead of by the use of ground electrodes through which a current is passed as in the applied potential systems, or by direct contact with the orebody. An alternating current flowing in a coil will create an electromagnetic field around the coil. This field will have the same frequency as the current and will radiate or travel outward from the coil in closed magnetic or flux circuits. These flux circuits (in air or a homogeneous medium) will be perpendicular to the plane of the coil and will extend or travel outward with uniform velocity in every direction. Maximum field exists in the plane of the coil. Such a coil used for transmitting or as the "energizer" will exhibit similar figure-eight characteristics to those exhibited by the direction-finding coil which will be described in detail later.

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Apparatus for Creating an Alternating Electromagnetic Field

Two types of energizing systems are shown to illustrate the general principles. Fig. 1 shows a so-called "high-frequency" apparatus, operating at frequencies from 30 kc. (30,000 cycles per second) to 50 kc. The wire or coil through which the high-frequency current flows is inclosed in the waterproof housing or "doughnut" *A*. The lower part of the coil case

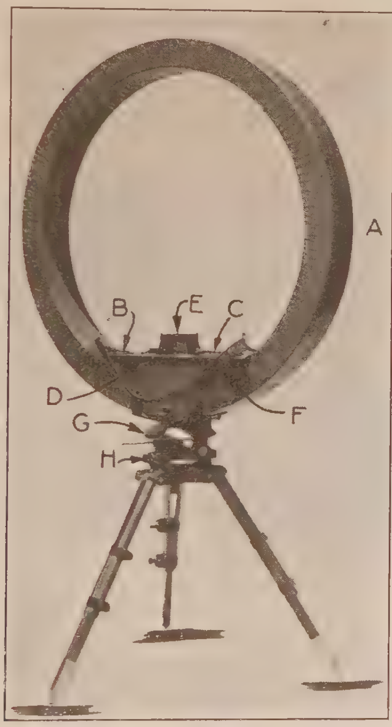

Fig. 1.—High-frequency apparatus.

contains the frequency changing apparatus, which consists essentially of a high-frequency oscillator using two 7½-watt tri-electrode vacuum tubes operating on each half of the 500-cycle supply. Proper plate and filament potentials are obtained from a transformer. The entire equipment, including tube sockets, transformer, blocking and loading condensers, grid resistor, radio-frequency ammeter *B*, leveling bubbles *C*, etc., is mounted on an aluminum chassis or cover-plate *D*, as shown. A glass-covered waterproof "porthole" *E* is provided for observing operation or changing the tubes. A shoulder strap *F* allows the apparatus to be easily carried when the tripod legs are folded. The loop can be rotated about a vertical axis by means of a graduated horizontal turnplate *G* and universal joint *H*.

Fig. 2 shows a low-frequency loop. This consists of a number of turns of rubber-insulated stranded wire bound together into a single cable to facilitate handling. The flag shown is placed at the electrical center of the loop and is used for proper alignment of the direction-finding coils, as will be described later.

Power supply for both the high-frequency and low-frequency systems is obtained from a specially designed 600-cycle 150-watt hand-cranked alternator, a disassembled view of which is shown in Fig. 3. Two handles *A* for cranking are provided, and connected to a 40 to 10 sprocket-chain drive on a 128 to 14 gear drive *B* on the alternator *C*. The alternator turns over at a speed of 4000 r. p. m. and the cranks rotate at a speed of approximately 100 r. p. m. Ball bearings are provided throughout and

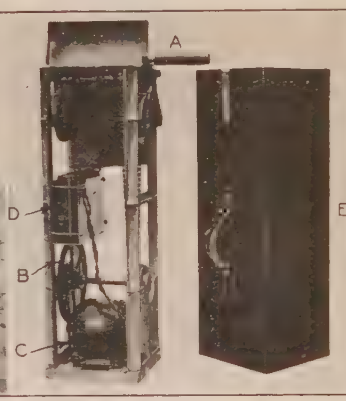

Fig. 2.—Low-frequency loop. Fig. 3.—Alternator and its waterproof case.

the outfit cranks easily for the generation of the necessary power. Approximately 50 watts is required for the high-frequency apparatus and 150 watts for the low-frequency loop. Direct current for the field of the alternator is supplied from three No. 6 dry cells *D*. The entire apparatus is contained in a steel waterproof case *E*.

Theory of Electromagnetic Induction

Whenever an alternating magnetic field cuts a conductor an electromotive force is generated. The magnitude of this e. m. f. is, among other things, a function of the strength of the magnetic field and the rate of change of the field; that is, its frequency. The higher the frequency, the greater is the induced e. m. f.

Of course, the current induced in the conductor will have the same frequency as the original current flowing in the energizing loop. Regardless of the alternating current employed—whether a so-called low fre-

quency from 50 to 3000 cycles per sec., or the higher frequencies up to many thousands of cycles per second—practically the same fundamental phenomenon takes place. These relationships may be expressed by the formula of electromagnetic induction:

$$E = 2\pi f M I \qquad [1]$$

wherein E = the induced e. m. f.

f = the frequency of the magnetic field (or the frequency of the current flowing in the energizing circuit).

M = the mutual inductance between the primary or energizing circuit and the secondary circuit.

I = the current flowing in the primary circuit.

Mutual Inductance

The mutual inductance between two electrical circuits is a constant determining the ease with which energy from the first circuit can be transferred to the second when the two are coupled by the magnetic lines of force due to the current in the first circuit. The greater the mutual

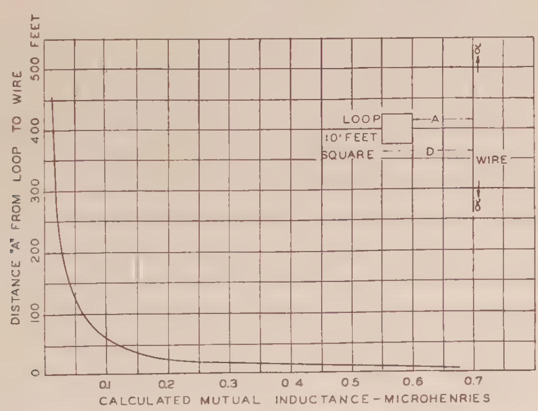

FIG. 4.—RELATIONSHIP BETWEEN MUTUAL INDUCTANCE AND DISTANCE BETWEEN THE LOOP AND WIRE.

inductance, or coupling, the greater the energy that will be transferred between the circuits. Ordinary transformer action is the most familiar example of this phenomenon. As seen from equation 1, the larger the value of M, the greater is the value of the induced electromotive force.

The range in values of M for the average energizing loop and an ore-body (assuming no absorption of the field between the energizing loop and the conducting body) may be shown by considering the case of a rectangular loop and a wire of small diameter and infinite length. The loop and the wire lie in the same plane with one side of the loop parallel to the wire. If h is the length of the loop parallel with the wire, d the width of the loop

and A the distance from the wire to the near side of the loop, a direct calculation gives us for the mutual inductance M the expression:

$$M = 2h \log_e \left(1 + \frac{d}{A}\right) \quad [2]$$

In the case of a square loop $h = d$, and $M = 2d \log_e \left(1 + \frac{d}{A}\right)$.

The magnitude of these values may be illustrated best by a practical example. Consider a vertical energizing loop 10 ft. square placed directly above the conductor so that they both lie in the same vertical plane, assuming no absorption of the magnetic field. The relationship between M and the distance between the loop and the wire is shown by Fig. 4. It will be noted that the mutual inductance decreases rapidly with the distance between loop and wire.

When the medium between the loop and the wire is of such a character as to decrease by absorption (which is the dissipation of energy by eddy currents) the strength of the magnetic field, the value of M decreases in proportion to this decrease in field strength, since absorption is equivalent to decreasing the coupling between circuits.

Magnitude of Induced Voltage

The magnitude of the voltage induced by current flowing in a loop and a wire or other conductor having a small diameter may be calculated from the preceding data. The current flowing in the 10-ft. square energizing loop is assumed to be 5 amp. and, as before, with the loop and conductor lying in the same plane and one side of the loop parallel with the conductor. These values, for various frequencies and mutual inductance between the loop and the conductor, are shown in Fig. 5. The induced e. m. f. decreases rapidly with a decrease in M; that is, an increase in the distance between the energizing coil and the conductor. In geophysical applications, this means that the induced e. m. f. becomes less with an increase in depth of the conductive orebody.

Factors Affecting Current Flow

Inasmuch as electrical geophysical apparatus employing the direction-finding coil system for the location of underground conductive masses operates on *current flow* instead of *potential difference* or induced e. m. f., it may be well to consider briefly the relationships existing between e. m. f. and current flow as encountered in practical geophysical applications of the inductive methods.

The current flowing in any conductor due to an induced e. m. f. may be expressed by the relationship $I = \frac{E}{Z}$ where I = current, in amperes; E = induced e. m. f. in volts, and Z = effective alternating current impedance, in ohms.

The impedance, however, is dependent on many factors and is composed of two quantities called the resistance and the reactive components. These are added vectorially and are 90° out of phase. The impedance may therefore be expressed by the simple relationship of

$$Z = \sqrt{X^2 + R^2} \qquad [3]$$

where X = the inductive or capacitive reactance
and R = the resistance

When the circuit is inductive, X may be expressed by

$$X_L = 2\pi fL$$

where X_L = the inductive reactance
f = the frequency
and L = the inductance

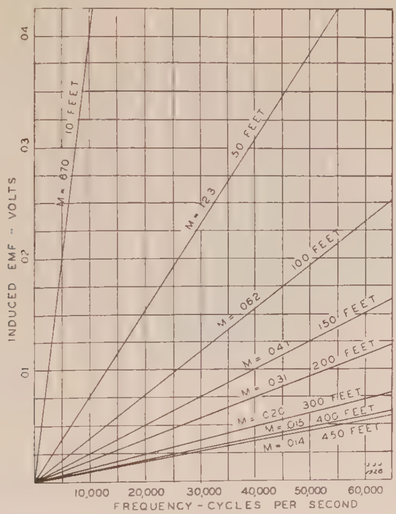

FIG. 5.—MAGNITUDE OF VOLTAGE UNDER VARIOUS CONDITIONS.

All orebodies possess a certain amount of inductance, but it is small and usually may be neglected even in mathematical calculations.

When the circuit is capacitive, X may be expressed by

$$X_C = \frac{1}{2\pi fC}$$

where X_C = capacitive reactance
C = electrostatic capacitance

The capacitive reactance component is 180° out of phase with the inductive component. In a series circuit containing both inductive and capacitive reactance the components are subtractive, and the circuit is said to be predominantly inductive or capacitive depending on which component is of the greater value. When both the inductive and the

capacitive reactances are of equal value the circuit is said to be in *resonance* and the impedance reduces to the resistance.

Under the practical conditions usually encountered in geophysical work, the capacitive reactance may be of very high value, and in all cases where measurements or calculations have been made by the writer, the inductive component is comparatively small and completely masked by the capacitive component. This is especially pronounced in all broken, faulted, and disseminated ores, and in so-called massive ores containing fractures which are filled with nonconducting or highly resistant depositions such as quartz, calcite, altered feldspar or clay, and some of the oxides. These fractures may vary in size from very minute veinlets to fractures, faulted zones or even igneous dikes.

The magnitude of the capacity existing between portions of a fractured ore may be seen from the following typical tests. A view

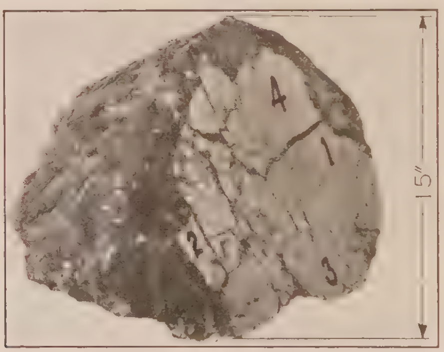

FIG. 6.—SAMPLE OF OUTCROP IN KAMISKOTIA, CANADA.

of the specimen is shown in Fig. 6. This particular sample[1] is a surface outcrop on the Duprat mines property in Kamiskotia, Canada. Samples cut from the specimen were analyzed chemically and submitted to a microscopic examination. The assay results, given in Table 1, show that the marcasite is impure, in that it contains silicate material, lime and alumina. The following information and the data in the table are from work being conducted by R. E. Head of the Department of Metallurgical Research and R. N. Anderson of the Radiore Co. at the Department of Metallurgical Research, University of Utah.

"The microscopic examination shows that this impurity is contained in veins which fill the badly fractured marcasite. The fractures run in all directions and vary in size from very minute veinlets to $\frac{1}{16}$ in. wide. The content of these veins is probably quartz, calcite and altered feldspar or clay. There is also some altered marcasite in the form of iron oxide

[1] Submitted by E. H. Guilford, general manager of The Radiore Co. of Canada, Ltd.

or limonite. Small specks of marcasite are also present in the veins. The marcasite is free from other sulfides and shows little alteration, the iron oxide being only a film next to the impurities contained in the veins. Both the small and large samples were nearly enclosed in a covering of this impure material, the quartz predominating."

TABLE 1.—*Chemical Analysis of Fractured Marcasite**

Material	Sample 56 Small, Per Cent.	Sample 57 Large, Per Cent.
Iron (Fe)	38.7	43.1
Lime (CaO)	0.76	0.76
Sulfur (S)	42.5	44.1
Silica dioxide (SiO_2)	11.2	5.4
Aluminum oxide (Al_2O_3)	12.7	13.4
Insoluble	12.6	6.0

* Analyst: H. Cowles, Department of Metallurgical Research, University of Utah.

The capacities existing between different mineral parts of the specimen (see Fig. 6) were measured at 25 kc. by means of the usual substitution capacity method, with results as follows:

Capacity between	Micro-microfarads
1 and 2	11
1 and 3	7
1 and 4	7
2 and 3	3
2 and 4	7
3 and 4	8

In any circuit predominantly capacitive, the impedance to current flow will decrease with an increase in frequency, as may be seen by the following mathematical relationship obtained from the preceding equations:

$$I = \frac{E}{Z} = \frac{E}{\sqrt{R^2 + X^2}} = \frac{E}{\sqrt{R^2 + \left(\frac{1}{2\pi f C}\right)^2}} \qquad [4]$$

It can thus be seen that as the frequency is increased, or as the capacity is increased, there is a decrease in the impedance and a resultant increase in the current flowing. This decrease in impedance at the higher frequencies more than offsets the slight increase in effective resistance due to redistribution of current caused by skin-effect. These relationships can be shown by actual measurements made on small specimens of the marcasite.

Four test specimens ⅛ by ⅛ by 1 in. were cut from samples 56 and 57 (from the work of Head and Anderson) and studies made of each of these specimens.

Specimen R-1 shows a number of nonconducting veins, none of which cut entirely through it. In testing, a current of 30 ma. was initially used. This was reduced to 20 ma. because of arcing that took place inside the specimen. The variation of impedance with frequency is shown by curve R_1 in Fig. 7.

Specimen R-2 contains fewer veins and consequently shows less impedance (curve R_2).

Specimen R-3 initially contained a thick transverse layer of insulating material near one end, and proved to be a nonconductor at the lower and intermediate frequencies. This end was cut off and constant arcing still took place within the specimen, making it difficult to obtain a balance. The results for this specimen are accurate only to about 100 ohms (curve R_3). The greater the number of fractures, the greater is the change of impedance with the change in frequency.

Specimen R-4 was also a nonconductor due to a thick transverse insulating vein. Removing this, the sample became a fairly uniform conductor (curve R_4). This capacitive effect shows up much greater on disseminated and faulted ores.

In observing these curves it will be noted that all the samples of this particular marcasite had a characteristic "hump" at approximately 50 kc., and that the average impedance decreased with an increase in frequency. This decrease in impedance as the frequency is increased shows that the specimens possessed properties making them predominantly capacitive.

These tests were made on a special balanced differential transformer-type alternating-current bridge designed by the writer and will be described in a later paper. The results obtained on minerals and ores from various parts of the world will be given in a publication on the results of a cooperative study now being conducted by the Department of Metallurgical Research of the University of Utah and the Radiore Company.

Impedance of Disseminated Ores

Disseminated ores also exhibit quite different properties toward direct and alternating current. A disseminated ore may be considered as composed of small electrically conducting particles distributed in a matrix. As a rule this matrix is calcite, quartz, or similar material, and has a low electrical conductivity; that is, a high resistance. If two conducting particles or masses are separated from one another, an electrostatic capacity exists between these two particles, which is dependent on the geometric configuration and arrangement of the particles and the dielectric constant or specific inductive capacity of the separating medium (in the case of a disseminated ore it would be the matrix material). A disseminated ore may therefore from an electrical point of view be considered as a resistance shunted by a number of very small capacities con-

nected in series-multiple. Some disseminated ores have been found to have practically an infinite resistance or impedance to direct or low-frequency alternating current, but to be quite good conductors to higher frequencies of 20 kc. (20,000 cycles per sec.) or more. Disseminated ores occurring in dry or desert regions are particularly noticeable in this respect. This is due mainly to the absence of moisture in the matrix, which makes it practically a perfect insulator.

Impedance of Faulted Zones

The action of the average broken or faulted orebodies when subjected to alternating magnetic fields of different frequencies is very noticeable. The impedance of such an orebody decreases very appreciably with an increase in frequency. Consider a general case where the faulted ore is

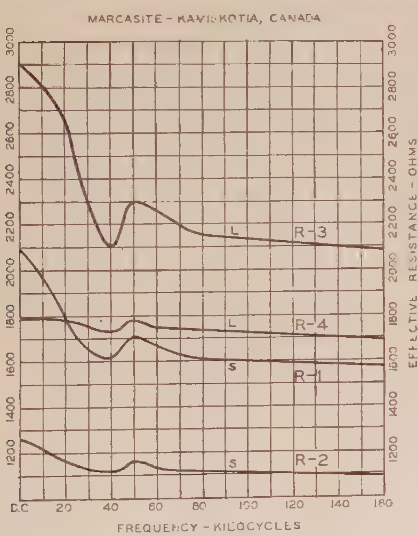

FIG. 7.—RECORD OF VARIATION OF IMPEDANCE WITH FREQUENCY IN FOUR TEST SPECIMENS.

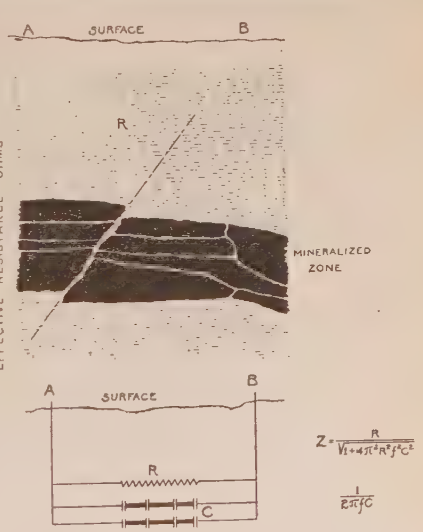

FIG. 8.—RECORD OF IMPEDANCE IN FAULTED ORE.

distributed in the manner shown in Fig. 8. The impedance of this system of conductors and effective capacities may be represented by an equivalent series parallel circuit of pure resistance and capacities, as illustrated in the lower part of the figure. The impedance of such an equivalent circuit is given by $Z = \frac{R}{\sqrt{1 + R^2 4\pi^2 f^2 C^2}}$ which at the higher frequencies approaches $\frac{1}{2\pi fC}$.

Thus an increase in frequency causes a decrease in the effective impedance with a resultant greater current flow. In many cases such orebodies can not be sufficiently energized with low frequency, but are

easily energized with the higher frequencies. This may be illustrated by considering the specimen shown in Fig. 6. The estimated effective area between particles 1 and 4 is 10 cm. The capacity between these particles is 7 micro-microfarads. Assume that this same geometrical configuration is increased until the effective area between 1 and 4 is 25 sq. ft. The capacity would then become 16,240 micro-microfarads. Calculation will show that the impedance of such a system will be 196 ohms at 50,000 cycles per sec.; 9800 ohms at 1000 cycles per sec.; and if the body were dry and the matrix nonconducting, the direct-current resistance would approach infinity.

If such a body were in a wet region, its direct-current resistance would be considerably less than infinity but might not differ sufficiently from the surrounding country rock to make easy its detection and a study of its electrical characteristics.

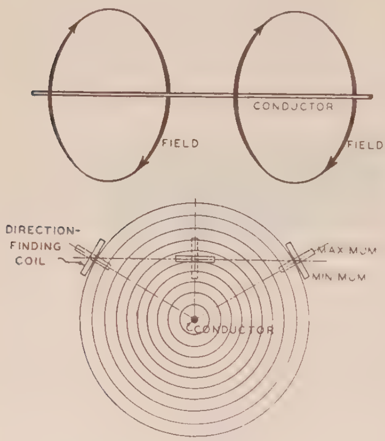

FIG. 9.—ELECTRICAL FIELD SURROUNDING A SIMPLE CONDUCTOR IN A HOMOGENEOUS MEDIUM. END VIEW OF CONDUCTOR AT BOTTOM.

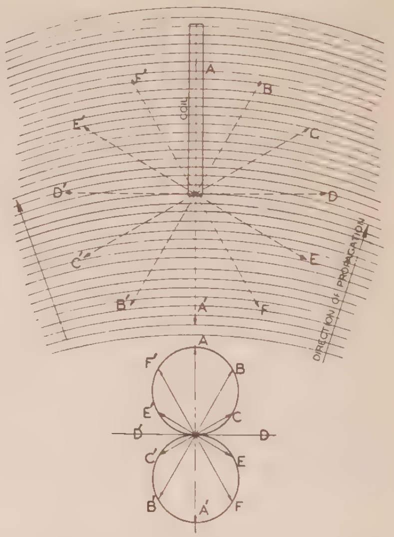

FIG. 10.—PLAN VIEW OF A DIRECTION-FINDING COIL IN A UNIFORM FIELD. "FIGURE-EIGHT CURVE."

DETECTION OF CURRENT FLOW IN A CONDUCTOR

An alternating current flowing in a conductor sets up an alternating electromagnetic field having the same frequency as the current. In the case of a simple conductor, such as a small-diameter wire of great length suspended alone in air, the field will surround the wire and travel outward from it in the form of concentric circles or envelopes. These conditions are illustrated in Fig. 9. The lower portion of this figure illustrates an end view of the conductor showing the magnetic field surrounding the conductor and traveling outward from it uniformly in all directions. The

electrical geophysical methods employing direction-finding coils depend for their operation on this alternating magnetic field surrounding the conductor.

The most satisfactory form of detecting equipment with alternating current, which allows almost direct field calculation of the location of the conductor, is that employing the direction-finding coil. By the use of such a coil the difficulties often encountered in conveniently obtaining good ground connections in the methods where equipotential curves, etc. are plotted, are overcome, especially when the ground surface is swampy or covered with heavy brush or moss, snow, and ice. Such a coil, with its associated apparatus, is easily portable and allows quick field manipulation. The complete apparatus consists of the direction-finding coil, mounting head, and tripod, and an amplifier (for the audiofrequency range); or a detecting and amplifier set, the output of which is usually connected to a pair of headphones.

Direction-finding Coils

Factors Governing Induced E. M. F.—The total voltage induced in a coil placed in an alternating magnetic field is proportional to the strength of the alternating field, its rate of change or frequency, the number of turns of wire in the coil, and the effective area of the coil. The effective area of the coil is used in this case to give an indication of the number of lines of force that pass through the coil when it is placed in a field of uniform intensity. The amount of flux linked with the coil is proportional to its area and the cosine of the angle between its direction and the direction of propagation of the lines of force.

Properties

The "Figure-eight" Curve.—To arrive at the polar curve of a direction-finding coil, the simplest way is to regard the generated e. m. f. as a function of the flux linkage. Fig. 10 shows a plan view of such a coil in a uniform field. The axis of rotation of the coil is vertical. When the coil is in the position AA', perpendicular to the direction of the field, the maximum number of lines will link or pass through the coil. In this position the maximum voltage is induced in the coil. As the coil is rotated the number of lines linking the coil decrease, until at position DD' the coil is parallel to the direction of the field and no lines link it. In this position zero voltage is induced in the coil. The effective area of the coil is therefore varied from the actual area when perpendicular to the direction of the field to zero when parallel with the field. If 20 lines link the coil in position AA', we can calculate the number of lines linked at other positions. At B there will be 17.5; at C, 10; and at D, zero lines. If the rotation of the coil is continued we have -10 at E, -17.5 at F, and -20 again at A'. The negative sign is used to denote a

reversal of the generated e. m. f. and inspection will show that the flux is passing through the coil in an opposite direction after position D or D' is passed. If the successive positions of the coil are drawn as vectors, and the length of the vector is taken to equal the number of lines of force linked with the coil in that particular position (the induced e. m. f. being a function of the flux linkage), we will obtain the vectors shown in the lower part of the figure. Upon joining the ends of the vectors, two circles are obtained; one circle corresponds to the positive values of linkage as found above and the other to the negative values.

This diagram is known as the cosine or figure-eight diagram, and is characteristic of a simple coil in a uniform, alternating magnetic field when the dimensions of the coil are small compared to the wave length.

"Sharpness" of Maxima and Minima.—In practical direction-finding, the accuracy with which a bearing can be taken depends to a large extent on the sensitivity of the receiving apparatus to small changes in the direction of the plane of the coil. As is well known, the figure-eight diagram shown herewith is the basis of all radio and electromagnetic direction-finding systems. Readings might be made either by noting the position of the coil when signals are loudest or by observing the direction corresponding to the minimum signals. When the maximum strength is observed, the coil is in position OA or OA', and, as may be noted from the polar diagram, a small change in angle will make a relatively small change in signal strength. For instance, shifting the coil from OA to OB (an angle of 30°) gives a relative change of $\frac{20}{17.5}$. In the position OD or OD' a small change in angle will produce a very great change in relative signal strength. If the coil were rotated through the same angle of 30° from OD to OE, the theoretical relative change would be $\frac{10}{0}$. (Note that $\frac{10}{0}$, numerically equal to infinity, does not mean infinite signal strength.)

The curve, therefore, is very steep around the minimum position, which means that the apparatus is far more sensitive as regards precision if adjusted for the zero signal instead of for the maximum signal. To this greater sensitivity of the directional properties of the coil is added the greater sensitivity of the average human ear in determining the existence or nonexistence of a signal, rather than the maximum intensity of such a signal. If the readings are being made in a hard wind, or where disturbing noises are present, the operator often will have difficulty in determining his minimum positions. In actual geophysical work, the minimum angles may be read accurately under average conditions to 0.5° for proper phase conditions. (The question of phase relationship will be treated in a later paragraph.) For this reason the minimum point or direction is read in practice, but the direction usually recorded is that for maximum signal strength, which in a simple case would be 90° from the position of

minimum. If, for instance, the minimum signal were obtained at an angle of 65° with the vertical, the point of maximum signal strength would be 90° from that position, or at −25°.

Apparatus

The most recent direction-finding apparatus developed for Radiore geophysical work is illustrated in Fig. 11. The mounting head *A* on which the direction-finding coil is pivoted is, provided with a sighting

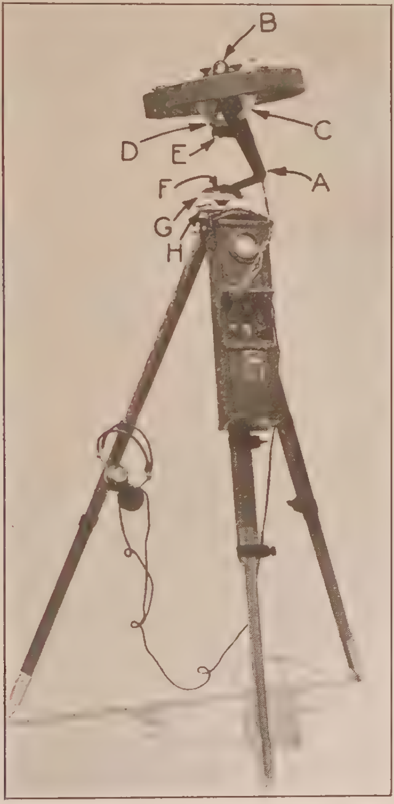

Fig. 11.—Radiore direction-finding apparatus.

arrangement *B* (similar to gun peep-sights) whereby the axis of rotation of the coil may be aligned quickly with the center of the energizing coil. A graduated vertical arc *C* is attached to the pivoted plate holding the direction-finding coil. The vertical-angle index mark and level-bubble arc are attached to movable arm *D*, adjusted by a thumbscrew *E*.

The head rotates on vertical axis *F* and azimuth angles may be read on scale *G*. The entire assembly is mounted on a ball-and-socket plate *H*. The direction-finding coil is electrically connected to the vacuum-

tube set-box containing a detector (so arranged that it functions as an audiofrequency amplifying stage when using low-frequency coils) and a two-stage peaked transformer-coupled audiofrequency amplifier. The entire apparatus, consisting of heterodyne control dial, compensating controls and batteries, is contained in a waterproof aluminum box 4½ by 4½ by 17 in. A double-range voltmeter is placed underneath a waterproof "porthole," by means of which filament and plate voltages may be read. The control knobs are placed on a recessed panel, for protection against mechanical injury. The conventional type of headphones are provided so that the operator can read the point of minimum signal strength when determining direction of the resultant wave. The same mounting head and set-box are used for both low-frequency and high-frequency work, it being only necessary to fasten the proper direction-finding coil on the rotatable plate with two thumb screws.

Corrections

At the higher frequencies the voltage induced in the direction-finding coil is supplemented by another e. m. f. called the antenna effect. This antenna effect produces a signal, the intensity of which is often independent of the position of the coil. The magnitude of the antenna-effect e. m. f. is dependent largely on the shape and size of the direction-finding coil, the distance between the set-box and the coil, the height of apparatus above ground, the frequency employed and the "capacity effect" between the headphones and the hand or body of the operator and the direction-finding coil. The antenna-effect e. m. f. may aid or oppose the inductive e. m. f. of the coil, which together with a phase difference would cause errors in the indicated direction if it were not removed by a proper compensating arrangement in the set-box.

The exact interpretation of data from a geophysical system employing a loop energizing coil and a direction-finding receiving apparatus would be considerably simplified (especially when employing the so-called high frequencies) if the field surrounding the energizing coil were purely induction. This type of field is responsible for transformer action. Instead, however, the field about a coil carrying an oscillating current is quite complex, especially at distances near the coil (less than a wave length) such as are employed in geophysical work. In the vicinity of the coil there are three fields or components, which must be considered not only in the design of the initial equipment, but also for proper interpretation of the geophysical field data when weak or deep conductors are to be accurately located.

Surrounding the coil there are three fields, the electrostatic, the induction field, and the radiation field.

The *electrostatic field* caused by the alternating charge on the radiating system decreases inversely as the cube of the distance. This field leads

the current by 90°. The *induction field* is the one generally met with in electrical engineering practice and varies inversely as the square of the distance from the coil. The *radiation field* varies inversely as the distance from the coil and is 90° out of phase with the induction component. The radiation field makes radio transmission possible. When the induction field is at its maximum value, the radiation is at minimum and vice versa. Near the coil the induction field is much more intense than the radiation field so that the phase difference between them is no obstacle in finding a minimum setting for the direction-finding coil. For an open or capacitive-type antenna, the fields are of equal intensity at a distance of a wave length divided by 2π.

A closed coil or loop will produce a radiation field similar to an open antenna. This is due to the fact that although the field from each vertical leg of the loop is practically equal in intensity and opposite in direction, they are not exactly opposite in phase except at points lying equidistant from the two vertical legs of the loop. By proper design of the energizing loop and the direction-finding equipment, it is possible to minimize the confused electric conditions existing in the neighborhood of an antenna or loop, and to secure proper operation of the direction-finding equipment even when operating very close to the energizing coil.

Practical Relationships between the Energizing and the Direction-finding Coils

From what has been said, it can be seen that when the direction-finding coil is so placed that its axis of rotation is in the same plane and passes through the center of the energizing loop, minimum or zero signal will be obtained when the direction-finding coil is at right angles to the plane of the energizing coil. Maximum signal will be obtained when the direction-finding coil is in the same plane as the energizing coil. These relationships are fundamental and hold true in the elementary case, regardless of the position of the plane of the two coils. This is illustrated in Fig. 12. It is to be noticed that the axis of rotation for the direction-finding coil is horizontal *only* when the direction-finding coil is at the same elevation as the energizing coil. Initial setting up of the Radiore inductive equipment therefore consists of two steps: (1) proper alignment of the energizing coil so that its plane is always vertical and passes through the axis of rotation of the receiving coil, and (2) alignment of the direction-finding coil so that its axis of rotation passes through the center of the energizing coil. To allow this second step to be done accurately and quickly, alignment sights and a ball-socket joint are provided on the head of the direction-finding coil, as described previously.

When the equipment has been set up as described, the operator of the direction-finding coil knows that the position of the coil for minimum sig-

nal will normally be horizontal. Should he obtain any angle or dip other than zero (measuring from the vertical) and a strike not pointing toward the energizing coil, he knows that some disturbing influence is present. This disturbing influence may be another field caused by induced current flowing in an underground conductive mass, or a distortion of the initial field from the energizing coil, so that it is not being propagated as it would be if it were located in a uniform homogeneous medium. These effects will be discussed later.

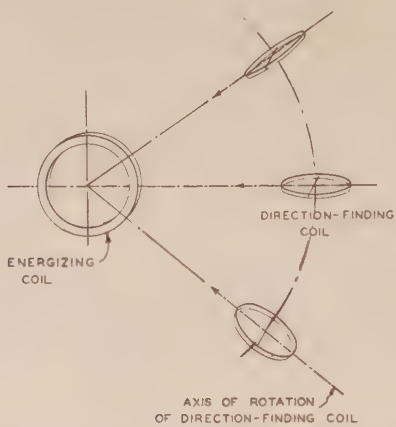

FIG. 12.—RECORD OF RELATIONSHIPS BETWEEN DIRECTION-FINDING COIL AND ENERGIZING COIL.

OPERATING CONDITIONS

When the energizing coil is placed so that its field (called the primary field) "cuts" a conductor, an e. m. f. is induced and a current flows in that conductor. As mentioned, this induced current sets up a field (called the secondary field) of its own. A direction-finding coil placed in the vicinity of the conductor will "pick up" both of these fields.

If two electromagnetic fields of the same frequency and in-phase cut the direction-finding coil, the position of the coil for maximum or minimum signal strength will be determined by a single resultant of the two fields. If, for instance, one field is horizontal and the other wave is tilted so that it makes an angle of 60° with the horizontal, the direction-finding coil, under proper conditions, "points" somewhere between these two vectors, the exact direction depending on the relative strengths of the two fields. If the fields were of equal strength, the resultant vector would lie equidistant between them.

The elementary conditions prevailing in actual operation can best be illustrated by Fig. 13. Here is the plan view of a conductor of considerable length and small diameter placed so as to be in the field of the energizing coil. The direction-finding coil now has two fields linking it.

At position C (note lower right-hand part of the figure) the component fields would exert the following effect: the energizing coil, being vertical, will tend to cause the direction-finding coil to give the loudest signal when it, too, is vertical as represented by the vertical vector. The field surrounding the conductor will tend to produce the loudest signal in the direction-finding coil at the angle shown by the vector pointing toward the conductor. The resultant effects of the primary and secondary fields are added vectorially, and the coil actually gives the loudest signal when in

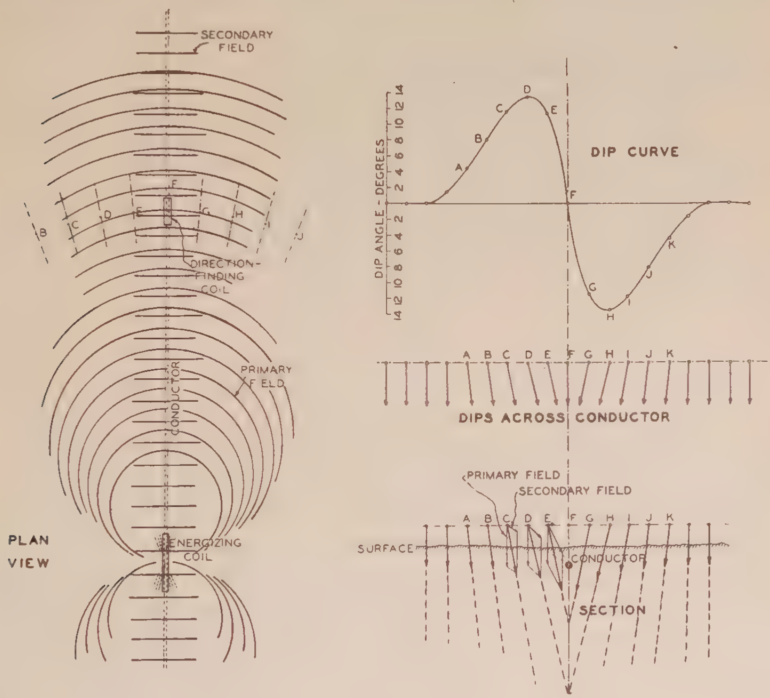

FIG. 13.—PLAN VIEW OF LONG CONDUCTOR OF SMALL DIAMETER SITUATED IN THE FIELD OF THE ENERGIZING COIL.

the position shown by the resultant vector. Moving the direction-finding coil to the position F, which is directly above the conductor, obtains a vertical angle. At this point both the primary and the secondary fields will induce the loudest signal in the coil when it is vertical. As the coil is moved beyond the vertical position, the direction of the dip angle changes, as shown by the vectors G, H, etc. Thus it can be seen that as a traverse is taken across a conductor through which an induced current is flowing, a series of dips will be obtained on the direction-finding coil. For purposes of illustration, we will assume that a series of readings is being taken on a circular traverse across the surface of the ground above the conductor, as shown in the plan view. Beyond point A, the second-

ary-field vectors due to the distance from the conductor are quite weak and not sufficiently strong to give a noticeable deflection to the angle of the direction-finding coil. The resultant direction for all practical purposes is vertical, or a zero dip. As the direction-finding coil is moved along the traverse toward the conductor the dip angle becomes increasingly larger until a maximum dip is reached, after which it decreases until a zero dip angle is obtained when vertically over the conductor. As the readings are continued beyond a point over the conductor the same condition results, except that the dips are in the opposite direction. This is illustrated by the arrows in the diagram marked "Dips across conductor." The dip curve shown plots the angles to scale.

In this illustration a circular traverse is shown, in order that the primary field may be constant and the change in resultant angle will be due only to variation in intensity of the secondary field and its change in angle. In practice, however, the traverses are taken along straight lines perpendicular to the conductor. The relative distance between energizing and direction-finding coils allows this to be done without any error within practical limits.

Effects of Phase Shift between Primary and Secondary Fields

The effects of phase relationship between primary and secondary fields should be considered. If these fields arrive at the receiver so that they reach their maximum or minimum values simultaneously, they are said to be in-phase. Under such conditions a definite resultant can be obtained for any given ratio of primary to secondary fields and sharp definite minimum readings will be obtained with the direction-finding loop. This can be illustrated by means of Fig. 14. The primary field is represented in magnitude and direction by the vectors P along the horizontal plane. The secondary field is likewise represented by vectors S, and, in this illustration, making an angle of say 45° with the horizontal plane. If the two fields are exactly in-phase their resultant will be the line R, and if a direction-finding coil is revolved about any axis of rotation lying in the horizontal plane and perpendicular to the vertical plane no signal would be heard in the headphones when the coil is perpendicular to line R.

A shift in the phase relation between primary and secondary fields is due largely to the following factors: (1) average depth of the conductive body as compared to the distance between energizing and receiving equipment; (2) distortion of wave front and difference in velocity of propagation through the air (through which the primary field travels in reaching the receiver) and through the earth (through which the energizing and the secondary field travels); (3) transformer action whereby the current is induced in the conducting body; (4) distribution of current in the conductor and the three fields existing in the vicinity of the energizing coil.

When working at the higher frequencies, the phase relationship between the primary and secondary fields becomes of increasing importance. At higher frequencies the phase shift may be of sufficient magnitude to introduce a serious error in the indicated direction unless compensated for. Errors of 10° or more due to phase relations may be encountered in field work.[2]

Usually, however, the two fields are not in-phase and then their resultant will no longer be constant in direction and magnitude but will be of the type known as a radius-vector, the locus of which will be an ellipse

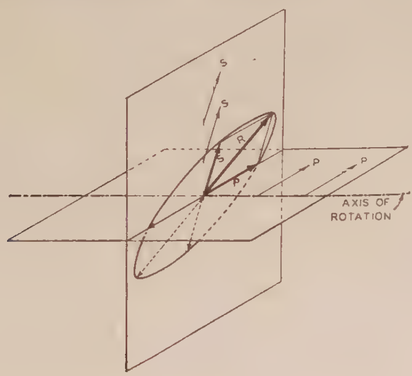

Fig. 14.—Effects of phase relationship between primary and secondary coils.

lying somewhere within another ellipse passing through the extremities of the P, S, and R vectors. This resultant is a continually rotating vector and completes one revolution for each cycle of the alternating current. Under such a condition we find that an absolute zero signal can be obtained only when the direction-finding coil is parallel to this plane. In practice, the axis of rotation of the direction-finding coil is somewhat as shown in Fig. 14 and a zero signal can not be obtained. Such an "out-of-phase" condition is readily recognized in practice by the operator. Out-of-phase conditions may be corrected for by numerous methods, including shifting the energizer frequency until the proper relationship is obtained at a point where readings are being made. Because of the comparatively long wave lengths used by the Radiore Co., this method is by far preferable, and the energizing apparatus is so designed that proper phase relationships may be obtained merely by moving a multipoint switch.

[2] J. J. Jakosky: Fundamental Factors Underlying Electrical Methods of Geophysical Prospecting. *Engng. & Min. Jnl.* (1928) **125**, 238, 293.

On account of the operating characteristics of the Radiore direction-finding equipment, it is not necessary for the primary and secondary fields to be exactly in phase. It is necessary, however, that they be close enough in phase for their resultant radius-vector to be an ellipse having a major axis somewhat greater than the minor axis. The greater the difference between these two axes, the sharper the reading, although an experienced operator will have no difficulty in obtaining readings of an accuracy sufficient for field operations when the ratio of major to minor axis is three or more. This ratio varies within wide limits, depending on the strengths of the primary and secondary fields, depth of the orebody, etc. This means that the frequency of the energizer need not be shifted every time the two fields are not *exactly* in phase, but only when their resultant radius-vector does not have sufficient ellipticity for sharp readings.

If the primary field alone is present, a zero signal will be obtained when the direction-finding coil is parallel to the primary vector. Because an infinite number of planes may be passed through a single line, when the primary field alone is present the axis of rotation of the direction-finding coil may be placed in such a position as to give zero signal regardless of the position of the coil. This is one method of definitely determining the presence of a secondary field and usually can be employed at the lower frequencies when the secondary field is too weak to cause readable dip angles.

Relationships between Depth and Length of Orebody

When working over a conductor of considerable length and uniform depth the ratio of primary to secondary field varies with the distance between the direction-finding and the energizing apparatus. This is due mainly to the less attenuation of the induced current when traveling over the conductor, as compared to the attenuation of the primary field. As a result the secondary field vector may be relatively large at considerable distances from the energizer, while when very close to the energizer it may be completely masked by the strong primary field. Since a certain minimum relationship exists between field strengths of the primary and secondary fields and their relative propagation angles, at which a readable dip angle may be obtained, it can be seen that the deeper a conductive orebody lies, the longer must be its effective length for optimum operating conditions. Increasing the power of the energizing system in order to impart more energy to the current induced in the conductor will not materially change conditions, inasmuch as the primary field is usually increased in proportion. The greater the power used for energizing, the less sensitive may be the direction-finding apparatus. The trend in design of Radiore equipment is towards lower powered

energizing apparatus and more sensitive direction-finding apparatus. This results in greater over-all portability of the equipment.

The depth at which an orebody may be detected and its characteristics determined by geophysical methods depends largely on its shape and mass. A conductive body in the shape of a sphere is one of the most difficult types to work with by inductive methods. Such a body, however, would give optimum results with the torsion balance or other gravitational methods. A long sheet or vein conductor having the same mass as the sphere would be almost ideal for the inductive process, while its detection might not be possible with the gravitational methods, especially if it existed at any depth.

Length of Orebodies

The *electrical length* of a conductive orebody varies with the type of mineralization and the frequency of the energizing current. If the body is a massive mineralized area containing no faulted zones or fractures filled with an electrically nonconducting material, the body will be electrically conductive to current of high or low frequency and to direct current. If, however, the body is fractured, broken, or faulted, and replacements or depositions of calcite, lime, quartz, etc. have taken place in such a manner as to insulate various portions of the body, its electrical behavior will be quite different with currents of different frequencies. When using high frequencies the induced current will readily pass through these breaks because of the electrostatic capacity existing between the various parts of the body, as has been discussed earlier in this paper. Low-frequency currents will suffer much greater impedance and considerable difficulty will be obtained in energizing such a broken body. It will thus be seen that the effective length of an orebody may vary with the frequency. The change in energizing frequency from low (1000 cycles) to high (50,000 cycles or more) will furnish considerable information as to the structure of the body and has been successfully applied in a number of cases.

It must not be assumed, however, that breaks or faults are not detected by using the high-frequency equipment. Such breaks are clearly indicated by reversal and shape of the *index curve* due to the change in ratio of primary and secondary fields, and the shift in phase relationship at the break or faulted zone. The phase relationships existing between various parts of a broken conducting body depend on their relative length, the mutual inductance between these component bodies and the energizing system, the effective resistance of the separating medium and the operating frequency. At best, it is a complicated relationship. Detailed studies have been made of numerous faulted or broken zones and absolute check obtained between the electrical readings and the geologic conditions.

Under such conditions the high-frequency equipment possesses another advantage in that one set-up of the energizer will usually be sufficient to read the entire body, while with the low-frequency source of power it is usually necessary to move the energizing apparatus past each broken or faulted zone. This is of especial disadvantage when the conducting bodies are of short length as compared to depth, as difficulty is usually met with in detecting the presence of a secondary field in the region of the energizing coil where a strong primary field exists. This explains why some types of broken and faulted ores existing at considerable depth may be detected and mapped when using high frequencies, while the lower frequencies gave no indication of a conductor.

Strike Readings

The azimuth angle or direction of the direction-finding coil when in a vertical plane is called the "strike" and represents the resultant direction

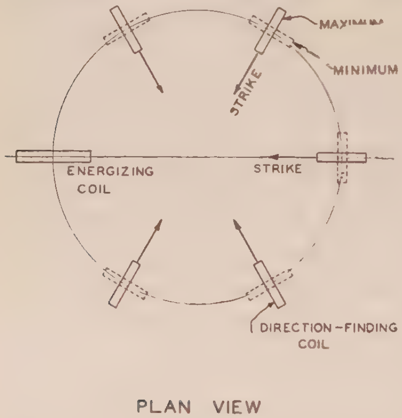

Fig. 15.—Relationship of relative positions of energizing coil and direction-finding coil.

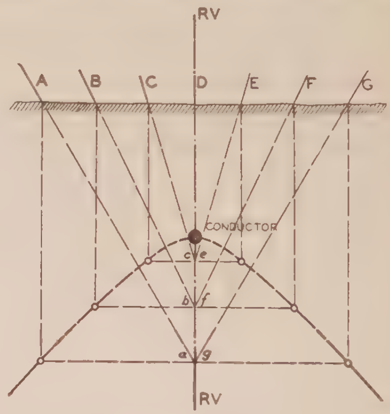

Fig. 16.—Method of plotting the depth of a conductor.

of the fields cutting the direction-finding coil with reference to a horizontal plane or plan projection. The relationships between the relative position of energizing coil and the direction-finding coil may be seen in Fig. 15. It will be noticed that the strike of the direction-finding coil is toward the energizing coil only when it lies in the plane of the energizing coil. When the direction-finding coil is not in the plane of the energizing coil, the directions for strike are as indicated. This is one of the reasons why the energizer in Radiore practice is always "pointed" toward the direction-finding apparatus. When in this position, the receiving operator knows that any strike other than toward the energizing equipment is caused by some distorting influence which may be due to distortion of the primary field wave-front, as will be described in more detail later, or a secondary field created by an induced current flowing in a conductor underground.

When the energizer is placed in the vicinity of a conductive zone, the secondary field induced in such a zone will have a certain direction with reference to the primary field and usually results in a strike different from what it would be normally were that field not present. Strike and dip angles are two of the important electrical phenomena to be noted in a reconnaissance survey for the indication and resultant location of a conductive mass.

Determining Depth of Conductor by Curve

A previous paper[3] described an empirical method of plotting to obtain the depth of a conductor. Briefly, this consists essentially of the procedure illustrated in Fig. 16. Through each observation point on a given traverse are drawn lines making an angle with the vertical equal to the dip angle. Through the points of intersection of these lines with the vertical RV, at which zero dips were obtained at the surface, are drawn lines parallel to the surface. Then perpendiculars are dropped from each observation point to its corresponding horizontal line. A smooth curve drawn through the intersections of these horizontals and verticals is the so-called correction curve. The apex of this curve is the position of the conductor.

Plotting the correction curve as described virtually amounts to assuming that the secondary field vector is horizontal, that the primary vector is vertical, and that a definite relationship exists between the magnitude of these two vectors. In this position the secondary vector is between the upward angle produced by distortion of wave front (to be described later) and the downward angle caused by drawing a smooth curve which neglects the "tip" that normally belongs on the complete curve of the general type: $y = Ke^x + K'e^{ax}$. The relation between the magnitudes of the primary-field and secondary-field vectors is taken into account in actual practice by proper distance of operation between the energizing and direction-finding apparatus. This distance is dependent on the energizing power used, approximate depth of orebody, average conductivity of overburden, and type of mineralization. This type of curve need not be considered as empirical, however, in location of the plan view of the conductor. The reversal or change in direction of the dips at a point immediately over the electrical axis of the conductor is indicated by the point of inflection of the curve, and is usually further checked by plotting the dip curves, etc. The curves therefore furnish an excellent graphic method for determining the plan view of the conductor, and allow the field operator to determine something of the nature of his conductor by the general shape of the curve, as well as to check any errors in field readings. By ascertaining

[3] J. J. Jakosky: *Op. cit.*

the depth of the electrical axis of the conductor at different traverses, it is possible to draw a profile view of the entire conductor.

Study of Electrical Readings

Various methods may be used in studying the data obtained from an electrical survey. Graphical methods are more rapid and by their use considerable information can be gained of the nature and geometrical configuration of the conductive body. Curves characteristic of certain

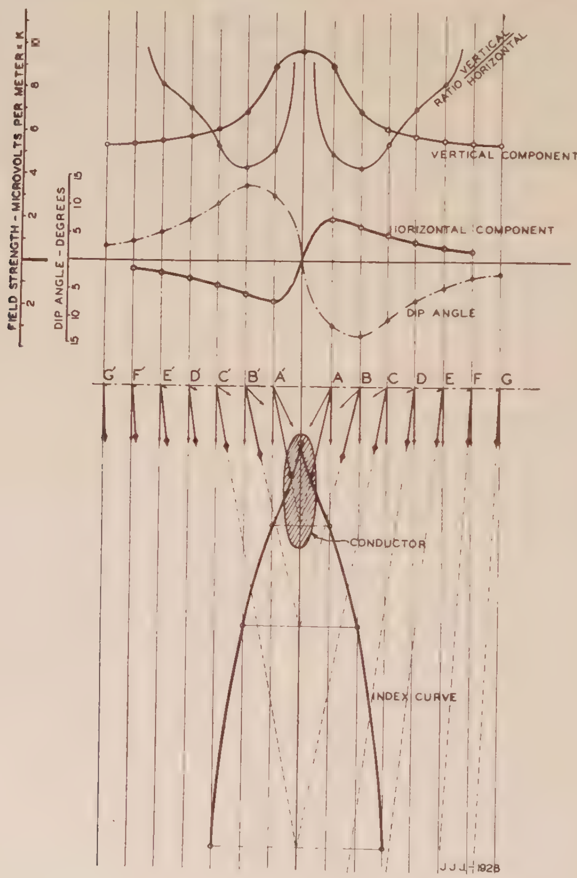

Fig. 17.—Record of electrical readings on elliptical conductor on edge.

types of orebodies have been analyzed mathematically and completely verified by field measurements and subsequent mining operations. The application of such curves will be illustrated in a general way for three cases by using an elliptical conductive body, assuming even current distribution, no adjacent conductors, or wave distortion, and with the

center of gravity of the body at the same distance below the surface in each of the three cases.

In Fig. 17 is shown the conductor "on edge," and traverse readings made at stations A, B, C, A', B', C', etc. The index curve for this type of conductor is as shown. Above this are plotted the dip angles, the vertical and horizontal components of the vectors and the ratio of vertical to horizontal component. Note the general shape and symmetry of the curves.

In Fig. 18 is shown the same elliptical orebody with its major axis horizontal and its geometrical center at the same distance below the surface as in Fig. 17. A decided difference can be noted in the shape of the index curve, as well as in the values of the other components.

In Fig. 19 is shown a conductive body of the same shape, making an angle of approximately 30° with the vertical. The unsymmetrical curves immediately show this condition, and also indicate the general direction of the dip of the orebody. Such a conductor will present an unsymmetrical electrical arrangement when readings are made at the surface of the ground and the strike and dip curves will be unsymmetrical in appearance. By proper interpretation of dip and strike curves it is possible to determine the angle of dip of a mineralized zone. Especially is this valuable in regions where overburden exists and the structural geology of the underground structure is not definitely known. In such instances it is quite impossible to plan the proper diamond-drilling program without knowing the dip or at least the direction of the dip of a conductor. Inspection of the dip curve shown in the figure will indicate that the diamond drilling should be on the side of the conductor having the smallest dip angles. Actual field practice has proved that these theoretical curves check very well with the results of field practice, and that after drawing the index curves it is possible to predict the general shape and depth of the orebody well within reasonable mining tolerances.

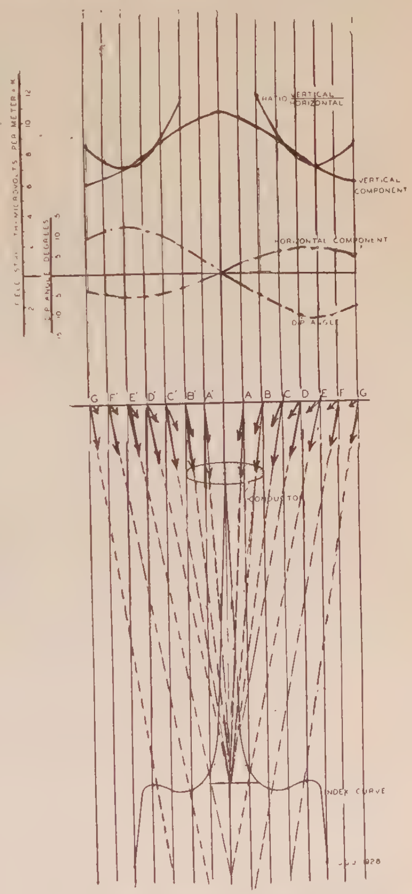

Fig. 18.—Readings on elliptical orebody with major axis horizontal.

Field Procedure during Detailed Survey

After the electrically conducting or mineralized areas of a property have been determined in the reconnaissance survey, a detailed survey is usually conducted to ascertain the depth of the electrical axis and the characteristics of the conducting zones. The energizing apparatus is set

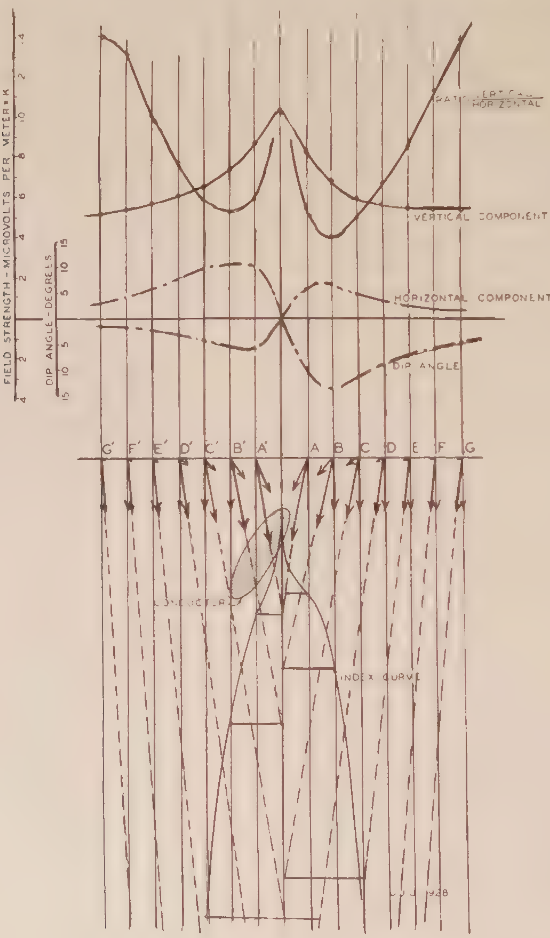

Fig. 19.—Readings on an elliptical orebody at an angle of approximately 30° from vertical.

up over the conductor and the plane of the coil is placed in the general direction or strike of the conductor. Readings are then made with the direction-finding coil at various stations called *traverses* along the conducting zone or "indication," by the procedure described in connection

with Figs. 13 and 16. Eight or ten readings are taken at each traverse The traverses are from 40 to 200 ft. apart, depending on local conditions and the information desired. During each set-up of the direction-finding and energizing apparatus the operators must have the energizing coil vertical, the plane of the energizing coil passing through the center of the direction-finding coil, and the axis of rotation of the direction-finding coil passing through the axis of the energizing coil. Resultant strike and dip angles, distances between energizer and direction-finding coil, traverse and station numbers, topography, etc., are recorded. For certain work the power and operating frequencies are recorded in addition.

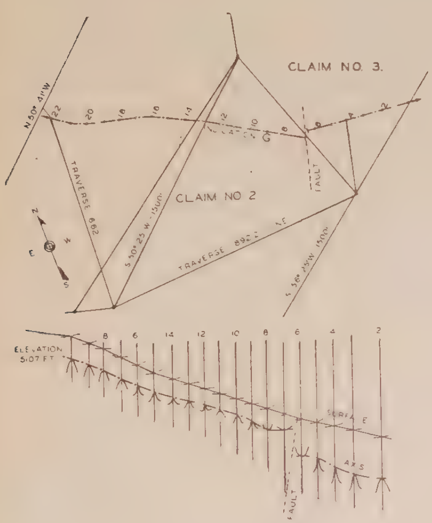

Fig. 20.—Portion of a map of a typical electrical survey.

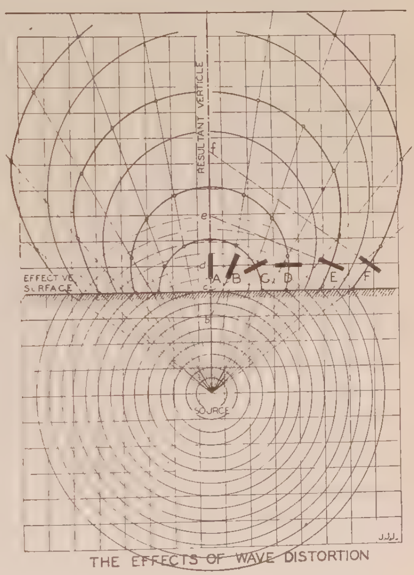

Fig. 21.—Wave-front when the velocity ratio is 4 to 1.

Completed Survey Map

A portion of a map of a typical electrical survey is illustrated in Fig. 20, showing data from a survey conducted in Inyo County, California. The "indication" in the plan view is tied in by the usual surveying methods to known property corners, bench marks, and other features, to allow later location of the indication should the electrical survey stakes be removed. In drawing the profile view of the conductor the depth of the conductor is obtained by plotting correction curves for traverses taken at intervals along the *indication*. By drawing a curve through the indicated depths at each of these traverses, the electrical axis of the conductor is located. The correction curves for each traverse are shown in

the figure. These curves are drawn by imagining the traverses as being rotated 90° to allow the curves to be plotted in the plane of the paper. Note the reversal of the index curves at the fault, and the displacement of the conductor.

Distortion of Secondary Field

Fig. 9 shows the magnetic field surrounding a conductor and traveling outward from it uniformly in all directions. Under such conditions the direction-finding coil can be used to locate by triangulation the position of the conductor where two or more different readings have been made. In actual practice, however, the wave-front traveling outward from the conductive orebody is not a true circle, and the conductor cannot be located by the simple process previously described. The wave-front is distorted because of the existence of three factors: shape of the conductor, irregular current distribution in the conductor, and difference in velocity of propagation for the wave in penetrating different media.

As is well known, the velocity of propagation of an electromagnetic wave varies with the character of the media through which the wave passes. Measurements made in various locations give values of the velocity of propagation in air to that through the earth from about 1.5 to 4.5. In other words, the wave travels from 1.5 to 4.5 times as fast through the air as through the earth.

In Fig. 21 is plotted the wave-front for conditions where a 4 to 1 velocity ratio exists. It will be noted that the curve is not a circle after the wave emerges above the effective surface of the earth, plotted by assuming an ideal condition where the earth is homogeneous. With the secondary field alone present and the wave-fronts as indicated, the position of the direction-finding coil for maximum signal strength will vary with the distance from the vertical position above the conductor. When immediately above the orebody, the direction of maximum signal strength is downward, and toward the orebody. Secondary-field readings taken at points on either side of the vertical position above the conductor will not give a direction towards the orebody. A reading made at point B would give an intersection at point b. A reading at F will give an intersection at f, which is above the surface of the ground. It must be remembered, however, that the intersections obtained in practice when the conventional vertical energizing-coil system is used are due to the resultant effects of the primary and secondary fields. The presence of the primary field usually causes the resultant intersections to be below the surface of the ground.

Distortion of wave-front tends to give an indicated depth less than that of the actual conductor. This factor has been considered in the working out of the empirical correction curve already described.

Distortion of Primary Field

The primary field from the transmitter is also subject to distortion, which, as before, is due to the difference in velocity of propagation of the different media through and over which the wave travels before reaching the direction-finding coil. The distortion of the wave-front also varies among other factors with the height of the energizing coil above the surface of the earth.[4]

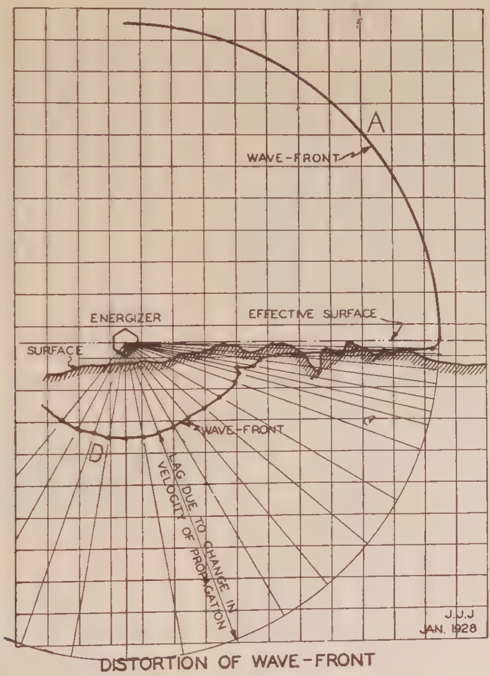

Fig. 22.—Calculated position of wave-front of primary field.

In Fig. 22 is shown the calculated position of the wave-front. Note how it is distorted near the surface of the earth, the amount depending on topographic and other conditions. The two points A and D represent the instantaneous position of the wave-front of some single wave. When traveling through air the wave has reached point A in the same time it has reached point D when traveling through the earth. Note that distortion occurs regardless of the frequency employed in the energizing system.

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[4] F. Hack: Die Ausbreitung ebener elektromagnetischer Wellenlängs eines geschichteten Leiters. *Annal. der phys.* (1908) **27,** 43.

G. W. Pierce: Principles of Wireless Telegraphy, 122-127. New York, 1910. McGraw-Hill Book Co.

J. Zenneck: Wireless Telegraphy, 246–262. New York, 1915, McGraw-Hill Book Co.

The distortion increases with the frequency but within the range of operating frequencies, and the range in earth resistivities of the overburden encountered in practical geophysical applications, the increased distortion due to the employment of the higher frequencies has been found to be well within experimental error. The question of frequency will be discussed later.

Phantom Dips

Owing to the distortion of the primary field or improper alignment of energizing and receiving equipment, it often happens that a small (usually less than 20°) "dip" or improper strike direction is obtained. These are called phantom dips or strikes and are readily recognized by the experienced operator. Such dips and strikes are often obtained when the

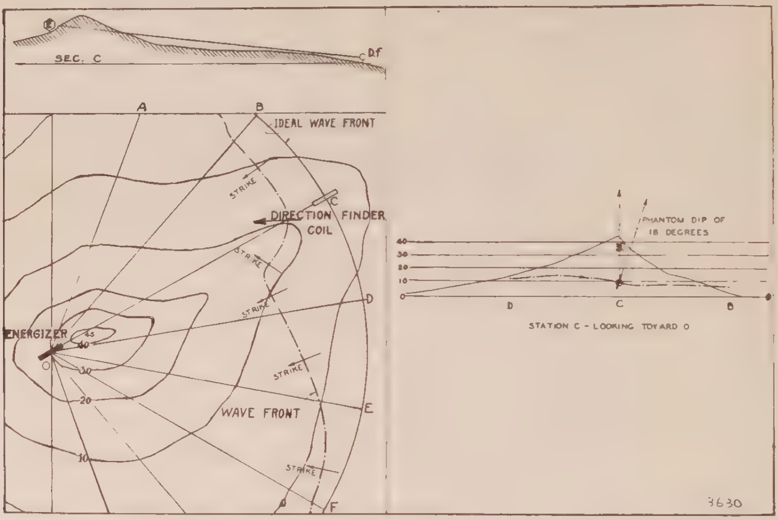

Fig. 23.—Distortion of wave-front caused by placing energizer "back" and to one side of a hill.

energizer and direction-finding coils are located on a ridge, in a narrow valley or canyon, or at the edge of a deep cut or precipice. Usually the greater the distance between the energizer and the direction-finding coils, the greater is the wave-front distortion. This accounts for the Radiore practice of keeping a relatively short distance between energizing and direction-finding apparatus. Such a system, of course, requires many more moves of the energizing equipment per day, but this was considered in making the equipment portable and easy to set up.

Fig. 23 is an illustration of distortion of a wave-front due only to the fact that the energizer was placed "back" and a little to one side of a hill. The heavy dot-and-dash line shows the position of the actual wave-front.

No secondary field is present. Note that the strike of the direction coil does not indicate the true direction of the energizer. The right-hand view (which looks from point C toward O) shows that the wave-front is distorted vertically as well as horizontally. At that point a vertical distortion or phantom dip of 18° is obtained, and a strike error of 27°.

Effects of Frequency

The Radiore high-frequency method differs from the conventional inductive methods in that it uses a comparatively high frequency; *i. e.*, 30 to 50 kc. These frequencies were adopted after a series of tests to determine the best frequencies for the complex electrical and physical conditions met with in field practice. The various electrical geophysical methods depend for their success on the existence of a difference in electrical conductivity between the orebody and the overburden. Granting that there is a difference in conductivity, the problem is to cause a current to flow through the orebody. Other things being equal, a higher frequency means a higher induced e. m. f., not considering the decrease in magnetic intensity due to absorption of the magnetic field by the overburden. If the conductivity of the overburden is appreciable, the question is whether the increase in the e. m. f. at a high frequency is more than compensated for by the decrease due to absorption. It must be remembered that absorption takes place at all frequencies both high and low. There is a balance between an increase in e. m. f. due to an increase in frequency as compared to a decrease in field strength with the increase in frequency.

Without going into a detailed mathematical discussion, it is possible to illustrate the effects of high frequency by means of a numerical example. Applying a theoretical absorption equation:[5]

$$H = H_o e^{-2\pi\sqrt{\frac{f}{\sigma}}Z} \qquad [5]$$

where H is the magnetic intensity at any depth Z; H_o is the intensity at the surface, f is the frequency and σ is the specific resistance of the overburden, we find that there is an increase in the induced e. m. f. at the higher frequencies. For purposes of illustration, consider an overburden having the comparatively low specific resistance of 10,000 ohms per cu. cm. and 100 ft. (3048 cm.) thick. Assuming first a frequency of

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[5] Also see J. H. Morecroft: Principles of Radio Communication, 146 and 844. New York, 1927; John Wiley and Sons.

J. H. Jeans: Mathematical Theory of Electricity and Magnetism, Chap. 15. Cambridge University Press, 1925.

C. P. Steinmetz: Theory and Calculation of Transient Electric Phenomena and Oscillations, Chap. 6. New York, 1920. McGraw-Hill Book Co.

10,000 cycles, the intensity of the field at that depth is 52 per cent. of that at the surface. Increasing the frequency fivefold (to 50,000 cycles), the intensity is 26 per cent. of that at the surface. Thus increasing the frequency five times only doubles the absorption. If we introduce these factors into our equation $e = 2\pi MfI$, we find that e has increased 2.5 times. Now if we consider that the high frequency will also be effective in decreasing the impedance of the orebody if it is disseminated, broken or faulted, it is seen that the induced e. m. f. in the conductor at high frequency is much greater than at a lower frequency. In average regions the overburden is of a sufficiently high resistance to make absorption a minor factor, so that high frequency is very effective in creating a strong secondary field. Skin effect, causing a redistribution of current with a resultant slight increase in the effective impedance of the conductor, is not of sufficient importance to materially affect the foregoing calculations for average conditions.

Interpretation of Field Data

One of the most important steps in geophysical prospecting is the proper interpretation of the field notes. The fundamentals of geophysical prospecting are well known and all relationships for ideal conditions may be theoretically expressed by mathematical formulas. The proper interpretation of data, however, contains many empirical steps or operations and can be done only in the light of experience. It may seem inconsistent that the interpretation should be so complicated when the fundamentals are definitely known, but this is caused by the fact that conditions underground are complex. Take, for instance, only one of the many factors to be considered—the occurrence of ore. If conductive orebodies occurred as straight cylinders of uniform diameter, and in a homogeneous medium, the problems of depth, etc., would be relatively simple. Actually, however, a mineralized zone may be large in one place (ore shoot) and then diminish to a very small cross-section (stringer, pinch-out) at another place. Numerous connecting zones (fingers, shoots) branch out from the main body, each of which will conduct a certain amount of current with its resultant secondary field. Ore bodies also occur at irregular angles or dips, and may be in thin sheets or veins. Since any geophysical instrument operates on the combined resultant of all of these component forces, and the nearest component has the greatest effect on the instrument, the correct interpretation of data may become very complex from the one viewpoint of ore occurrence alone. To this may be added the topographic disturbances and distortions to the primary and secondary fields, etc. Proper interpretation of geophysical data requires that due consideration be given the electrical, geological and tectonic, and mining data on each particular property.

Previous publications[6] have described the results obtained. Only a few types of survey data will be given in this paper to illustrate some interesting points regarding the application of the inductive process to geophysical exploration.

Exploration Profitably Preceded by Geophysical Examination

Although it is well recognized that geophysical methods alone are not a direct means of locating commercially valuable orebodies, such methods are of real economic value in outlining the development program of a property and securing information of value to the geologist regarding structural features and the location of the conductive mineralization. One valuable feature of the electrical methods usually is their ability to eliminate barren areas and to furnish information which the geologist

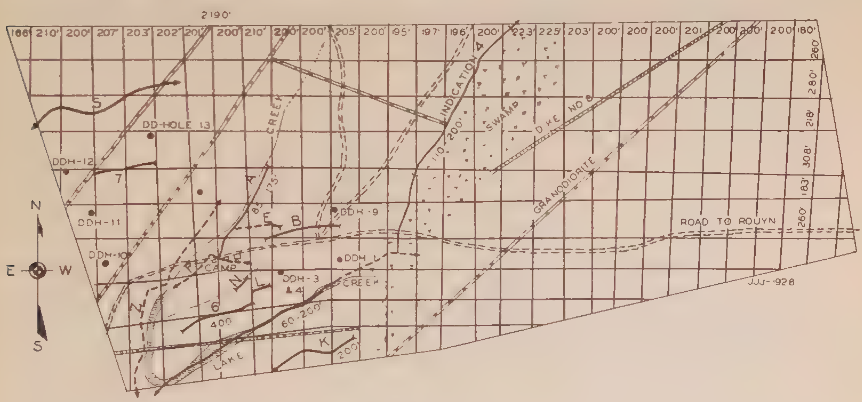

Fig. 24.—Record of a diamond-drill survey in the Rouyn district.

and mining engineer may use to determine the most probable mineralized areas likely to contain commercially valuable ore deposits. This may be illustrated by the exploration conducted by a Canadian company in the Rouyn district.[7] This company had previously explored the region by a systematic system of diamond drilling, as shown in Fig. 24. The drill cores gave some promise of mineralized zones to substantiate what little surface geology could be studied through the overburden typical of that district. The same property was later surveyed and the indications shown on the map were obtained. It is safe to predict that had the electrical survey been conducted as one of the initial steps in the development of the property, the diamond drilling could have been used in proving up the indications and determining their value. Subsequent drilling on the indications has shown good mineralized zones.

[6] E. H. Guilford: The Radiore Method. Canadian Mining and Metallurgical *Bull.* (1928) 644.

J. J. Jakosky: *Op. cit.*

Continuity of Mineralized Zones

An interesting example[7] of the continuity of a mineralized zone may be seen in Fig. 25. This indication was located during a survey of a property in Des Meloizes Township, Quebec, Canada, and was found to be of proper electrical character to warrant further exploration work. The indication has now been checked over a distance of 2000 ft. between drill holes No. 1 and No. 8, and every drill hole, with one exception, has

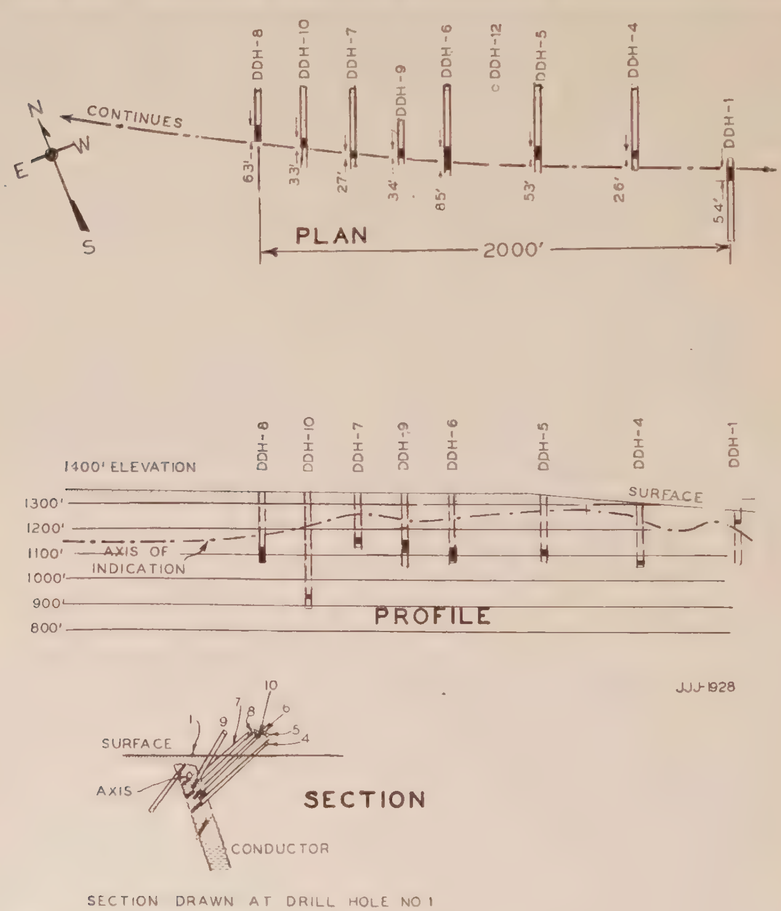

Fig. 25.—Continuity of mineralized zone indicated electrically.

cut a wide zone of sulfide mineralization. The conductor is a dipping sheet vein and possesses considerable depth.

The axis of the conductor was determined by use of the correction curve. Note that this axis is close to the top of the conductor. As explained, this is because the portions of the conductor lying close to the

[7] Data from E. H. Guilford.

surface have a much greater effect than the lower portions. As a general rule the effect of a differential conductive area will vary inversely as some power greater than the square of the distance to the direction-finding coil. An end view of the conductor is shown; also the drill holes. The mineralized portions of the drill holes are indicated by solid black.

Discovery of Zone Not Indicated by Geological Survey

An illustration[8] of a case in which surface geology failed to give any hint of a possible mineralized zone is illustrated by an indication located on a property in Inyo County, California (see Fig. 26). This is a lead-silver district which has been mined intermittently since 1865. Experience from previous ore occurrence showed that the ore values always occurred in the limestone adjacent to the limestone-monzonite contact. A major conductor (indication *D*) was located during the electrical

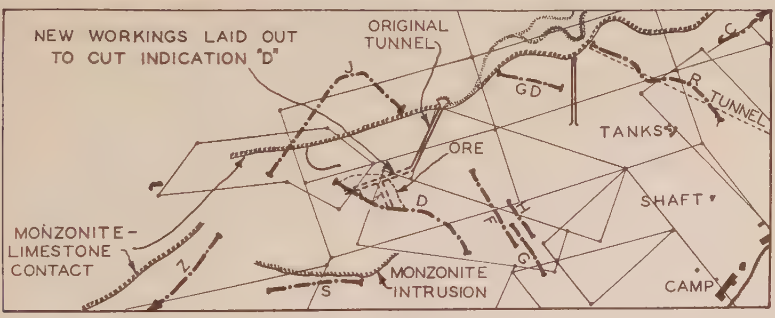

FIG. 26.—ELECTRICAL DISCOVERY WHERE GEOLOGY GAVE NO INDICATION.

survey, but because of its distance from the contact as shown on the surface considerable doubt existed as to its being of special value. Because of its proper electrical characteristics, however, a crosscut was driven from the tunnel to indication *D*, where it was found that a tongue of monzonite (probably an offshoot from the main monzonite intrusion to the north) existed and the contact of this tongue with the surrounding lime had created conditions favorable for ore deposition. The high-grade oxidized ore associated with the sulfides was of excellent value, carrying approximately 21 oz. silver and 33 per cent. lead. The conducting sulfide zone consists of galena, sphalerite, chalcopyrite and pyrite and is over 150 ft. thick.

Corrections for Topography and Dipping Conductor

An example of the corrections that must be made for topography when working over dipping-vein conductors is shown in Fig. 27, a preliminary map taken from a Mexican survey. The locations of the

[8] Data supplied by H. E. Olund, geologist of The Radiore Co. of the United States.

indications projected to the surface are shown by the light dotted lines, while the calculated vein locations, after correcting for dip of the vein and rake of the conductor axis in the vein, are shown by the heavy dotted lines. This property has been opened up by underground workings under the indications L_1 and L_3. No ore was encountered in the drift through one of these faulted blocks and the electrical survey has shown the displacement of the vein in this block. Attention may be called to the apparent existence of a conductor on the continuation of the indication parallel with each fault. This probably is due to conductive material in the faulted zone, caused by the "drag" and wet gouge. The dip of the faulted plane, together with the dip of the vein of 54° to the west, accounts for the displacement between the electrical indications and the surface fault projections. The conductive zones in this property are from 400 to 700 ft. deep. A diamond-drill hole was driven from the

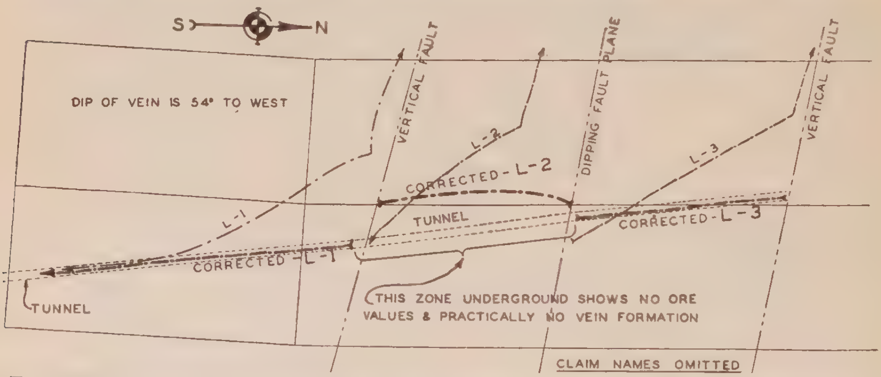

FIG. 27.—EXAMPLE OF CORRECTIONS FOR TOPOGRAPHY WHEN WORKING OVER DIPPING-VEIN CONDUCTORS.

tunnel to intercept the indication L_2 and to establish identity with L_1 and L_3, but after penetrating about 30 ft. encountered such a badly broken zone that further drilling was impossible. A crosscut will now be driven.

TYPE OF INFORMATION TO BE DERIVED FROM A SURVEY

Application of the inductive method described in this paper has been instrumental in the location of many indications, or mineralized areas, in various parts of the United States, Canada and Mexico. Diamond drilling, trenching, and mining subsequently have been employed on many of these indications and ores or mineralized areas of various values and types have been located. Geophysical prospecting is passing the stage where the mining engineer or exploration company is quite interested to learn that a conductor was *actually located* by any geophysical method. This first period of amazement and scientific education has passed and the questions now to be answered are of a practical nature and

pertain to the best application of various processes to the particular problem at hand, the proper interpretation of the results, and what direct bearing a geophysical survey will have on the problem of developing a pay mine and the blocking-out of valuable ore reserves.

As mentioned before, all electrical geophysical methods operate on the same two fundamental principles; namely, the flow of an electric current through a conductive zone and the detection of that flow of current. These principles may be employed successfully in the self-potential process, and various modifications of the applied potential or ground contact and the inductive processes. The differences between the various processes then are mainly methods of field procedure, type of equipment employed, and the effect of local geological and electrical conditions. Certain kinds of information may be more readily obtainable with one type of equipment or method than with another, and each method usually will be found to have advantages in some particular application or when operating under certain local conditions. Taken as a whole, however, the data obtained from a complete survey by any of the recognized methods will be of real value and help when properly interpreted and applied in the development of a property.

During the past two years the writer has had an opportunity to study different reports and maps of surveys by the different processes. In some cases, the same territory was surveyed by different companies using quite different field equipment and procedure. In some instances certain information was obtainable from one type of survey which could not be noted from another. As a whole, however, the results are quite in agreement, the chief differences being due to the policies of the company conducting the survey. One company feels that it is quite important to obtain the approximate depth of the conductor in addition to the plan location. For this purpose the correction curve was developed and the proper field procedure evolved to obtain the desired degree of precision. Other companies feel that the width of the conductor, together with its plan view, is of vital importance. As a result, their field procedure is planned to obtain this information. Another company stresses the value of more extensive geological information. It can not be definitely settled which method or procedure gives the most valuable information to the client, since local conditions vary greatly. In one instance, the mining company was only interested in knowing the plan location of their mineralized zones. From then on the development work could be conducted more efficiently (considering time and relative costs) by tunneling from an old working.

In other cases it was considered better practice to make a complete study of the property and obtain as complete information as possible regarding the plan location of the mineralized zones, depth and approximate width, together with the relative conductivity. It is not a question

of how much information may be obtained from a geophysical survey, but only a question of conducting the survey and examination to a stage that gives the optimum return on the expenditure considering the entire development program. The experience of the Radiore Co. to date is that ordinarily the most information for the least expenditure can be obtained by locating the plan view of the conductors (the mineralized zones) and determining their approximate depth and width. Such work can be done at the rate of about 8 to 25 acres per day (with a crew of four or five men), depending on topographic conditions and amount and type of mineralization present. The information derived from such work may then be studied in connection with the geology and known ore occurrence of the district and the exploration program more efficiently carried out. In some cases, however, more complete studies have been required, lasting over a period of months.

The amount of geophysical work that should be done is somewhat comparable to the sampling of a mineralized area. Numerous systems or procedures are used for sampling and as a rule the greater the number of representative samples taken the more probable will be the agreement between the assay results of the samples and the final ore recovered. There is an economic limit, however, to sampling.

Consideration must be given to the training and personnel of the company for which the work is being conducted. A majority of the operating companies have excellent engineering and geological staffs who have a thorough knowledge of the property and are therefore capable of interpreting the results of the survey and applying it to their specific development or structural problems. In such cases the geophysicists may act only as general consultants in the interpretation of data. Sometimes, however, it is necessary to conduct a geological examination of the property and to take an active part in planning the exploration and development program. In every case the greatest benefits are derived by proper cooperation between the geophysicists, the mining engineer, and the geologists for the interpretation of all available data; the work of one is complementary to that of the other.

ACKNOWLEDGMENTS

The writer desires to acknowledge the material assistance and cooperation received from his associates of the Radiore Co., particularly from A. B. Menefee, H. E. Olund, B. M. Snyder, E. H. Guilford, M. Brenner, H. O. Walker, and R. N. Anderson. Especial acknowledgment is also made to Dr. D. A. Lyon and H. E. Head, of the Department of Metallurgical Research of the University of Utah and the Inter-Mountain Station of the U. S. Bureau of Mines, for assistance and help in the cooperative studies now being conducted on the electrical properties of various ores, etc.

DISCUSSION

K. Sundberg, Houston, Tex. (written discussion).—Mr. Jakosky's interesting paper presents more in detail the viewpoints published in his earlier paper.[9] I commented on that paper,[10] discussing the frequency and the method of interpretation in the Radiore method. I concluded (1) that serious difficulties might be encountered because of the comparatively high frequency used; (2) that the fact that the frequency is varied during the survey makes the correlation of results difficult: (3) that "imaginary indications" corresponding to no conductors but due to an incomplete procedure of interpretation apparently can be obtained by the Radiore process.

In Mr. Jakosky's answer to that discussion of mine,[11] he tries to prove that my criticism is unimportant, and partly incorrect, especially with reference to "imaginary indications," the presence of which Mr. Jakosky denies. I prepared an answer, which I thought I had sent to Mr. Jakosky and to the *Engineering and Mining Journal*, but recently I found that this answer never reached its destination. Therefore I submit it herewith.

I did not enter at all into a discussion of Mr. Jakosky's viewpoint regarding the effect of breaks in orebodies, which he considers equivalent to the dielectric in condensers. According to my opinion, it is very doubtful whether it would be possible to show the existence of the corresponding capacitive effect. In any case, it seems somewhat irrational to try to locate a conducting body, such as an orebody, by a capacitive, that is, nonconductive effect.

In discussing the masking effect of the overburden, Mr. Jakosky suggests that I have neglected an important point, namely, that the field caused by the current in the orebody will have an axis but the field from currents in the overburden will not. Unfortunately, it often happens in practice that the field from currents in the overburden does have an "electrical axis," because the conductivity of the overburden is not constant but changes from point to point. Frequently lenses of better conducting overburden are found which give indications of the same character as those related to the orebodies, if too high frequency is used.

Though entering into a discussion of my elementary theoretical points, Mr. Jakosky states that no process containing as many unknown and variable quantities as does electrical prospecting is amenable to exact mathematical treatment and that, whatever the apparent results indicated by such mathematics as we may be able to apply, the actual results have proved that the Radiore process usually gives the indication within ordinary mining tolerances. Most geophysicists at the present time agree that the most difficult and most important part of a geophysical survey is the interpretation of the results, the classification of indications into those of possible importance and those of none. Anyone familiar with geophysical prospecting knows that mathematics is becoming more and more a necessity, both in the interpretation of geophysical results and in developing the technique of geophysical methods. I also think most geophysicists agree with me that any serious geophysical method should function perfectly under ideal "mathematical" conditions.

Mr. Jakosky apparently is a little bothered regarding the "imaginary axis," or indications which do not correspond to conductors but are due to an incomplete interpretation and he tries to prove mathematically that the "imaginary axis" does not exist. I have pointed out (see Fig. 28) that indications corresponding to the "imaginary axis" *must* be obtained in all cases where the primary field has a vertical compo-

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[9] J. J. Jakosky: *Op. cit.*

[10] K. Sundberg; *Engng. & Min. Jnl.* (1928) **125**, 579.

[11] J. J. Jakosky: *Idem.* (1928) **125**, 820.

nent, which is the case at points above or beneath the level of the transmitter and, in any but a perfectly level country, it must be difficult, if not impossible, to keep the direction-finding coil always exactly level with the energizer. We all agree that the general character of the vertical component of the secondary field is according to Fig. 28, and it is obvious therefore that for one conductor two points generally exist where the sum of the primary and secondary fields is zero; *i. e.*, the resultant field horizontal, which means two indications according to Fig. 28, one of which is "imaginary," because only one conductor exists. If the primary vertical component should be larger than the maximum secondary vertical, of course no point exists where the field is horizontal, and the presence of the conductor is not revealed by the Radiore interpretation, if I have correctly understood the procedure of Radiore. This statement holds for any position of the energizer relative to the orebody.

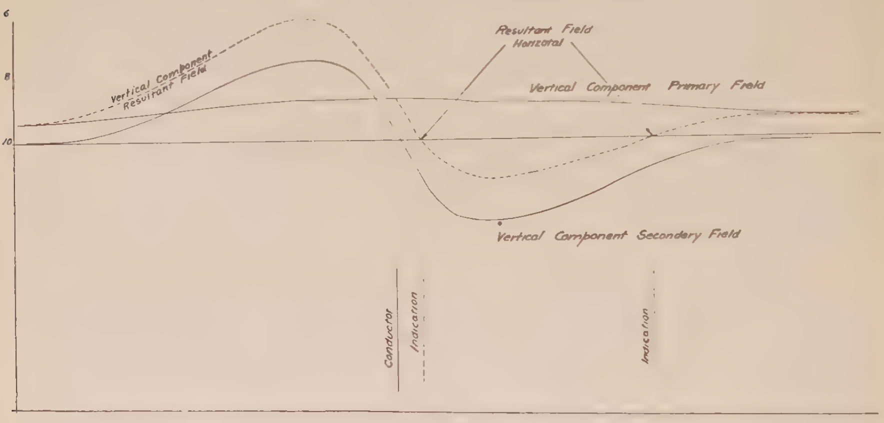

FIG. 28.

Mr. Jakosky believes that my corresponding mathematical equation (III) is incorrect and gives another relation (his equations B and C), which is identical with mine except that the factor b in my equation is $\frac{b}{2}$ in Mr. Jakosky's. This difference is due to the fact that Mr. Jakosky uses the maximum *vertical* component of the secondary field as unit for the vertical component of the primary field, whereas I use the maximum total secondary field as unit. This difference, of course, is of no importance and Mr. Jakosky in solving his equation B obtains values which, for a certain b value, b, are identical with the value I obtain for my b value, $\frac{b}{2}$, or, in other words, our corresponding results are identical. Mr. Jakosky's conclusion from his calculation, that the resulting errors in locating the conductor are not nearly as serious as I predicted, therefore is erroneous.

But Mr. Jakosky's "proof" of the nonexistence of the "imaginary axis" is not sound. He investigates the "boundary condition for a zero primary field" by substituting $b = 0$ in his equations B and C and gets $X_0 = 0$, $X_0 = \infty$, just as I did. He further says: "When the direction-finding coil is vertical, it follows from both theory and practice *that the conductor must be directly below the coil; obviously the conductor cannot be both "here" and at an infinite distance at the same time.* From these considerations it follows that equation C is an extraneous root and therefore may be discarded." But even if this proves that equation C is an extraneous root for $b = 0$ (which it does not), does this prove that equation C is to be discarded for all values of b? I think not, but if I had such a hard nut to crack, I think I should argue some-

what as follows: "It is clear, therefore, that one conductor can give two indications, one at a certain short distance from the conductor, the other at a greater distance. Because only one conductor is present, only one indication can be correct; the second one does not apply to the actual case and must, therefore, be discarded. So, for practical reasons, we discard the indication which is at the greater distance from the conductor." I do not believe, however, that such a "proof" will satisfy any critical mind.

Regarding the frequency, I cannot consider Mr. Jakosky's argument convincing. I consider high frequency warranted for ore prospecting under special conditions, but not in the most common cases. In the discussion in the *Engineering and Mining Journal*, I have given my reasons for believing it to be often a disadvantage.

Some Applications of Potential Methods to Structural Studies

By E. G. Leonardon* and Sherwin F. Kelly,* New York, N. Y.

(New York Meeting, February, 1928)

The first to appreciate and foresee the value of applying electrical measurements to structural studies was Prof. Conrad Schlumberger, Professor of Physics at the School of Mines in Paris. One of his earliest practical undertakings in this line was carried out, in the field, at Fierville-la-Campagne, Normandy, in 1912. His book, published in 1920,[1] still stands as an authoritative, fundamental treatise on the use of the electric current in geological studies, in spite of the rapid evolution of geophysics during the last seven years. A remarkable feature of this book is that, even then, it stressed the role of electricity in structural investigations and made public the first successful results obtained therewith.

Stratigraphic studies have been pursued continuously by Conrad Schlumberger and his associates since 1912, the only interruption being the World War. The results attained demonstrate the wide possibilities of electrical exploration in structural work. This field of activity is, at the least, as varied and valuable as the search for ore. It is curious that until very recently it has so slightly attracted the attention of geophysicists and geologists on this continent.

The purpose of this paper is to present briefly some examples of the practical application of the potential methods to geological studies. These examples are chosen from a number of problems solved in the field by our firm, in the course of its consulting practice.

Certain fundamental points are briefly stated before the field operations and interpretations are described.

Conductivities of the Rocks

Compared with metallic substances, rocks are poor conductors. However, it would be a grave error to affirm that they are completely insulating and that no electrical current can flow through them. It is surprising to encounter, even now, in certain discussions about the applicability of the potential methods, deductions based on this assumption that rocks are *absolutely* nonconductive.

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* Schlumberger Electrical Prospecting Methods.

[1] C. Schlumberger: Étude sur la Prospection Électrique du Sous-Sol. Paris, 1920. Gauthiers Villars.

Rocks do not transmit a current as metals do, without any *transport of ions.* On the contrary, their conductivity is due to the moisture they contain, which confers upon them an *electrolytic conductivity.* This conductivity, therefore, varies according to moisture content and the quantity of ionized salts in solution.

An approximate idea of the limits between which the conductivities of various geological formations can be expected to vary is given by the following:

	Resistivities in Ohms per Meter Cubed
Formations containing dissolved sodium chloride	0.2– 10
Clays	10– 30
Calcareous shales	20– 40
Shaly limestone	40– 200
Igneous and metamorphic formations	200–1000 and up
Rock salt	10^{15}

These figures show the enormous differences that can exist in nature between the conductivities of several members of a geological complex. The study of this electrical property, by an appropriate method, must then present a fecund field for applied geophysics. It will permit the solution of many geological problems where rocks of different electrical characteristics are in contact.

The Potential Method and the Map of Potentials

One way to study the conductivities of the subsoil is to pass an electric current through the ground. This acts as a limitless conductor: the current enters the earth at one point, A, and leaves it at another point, B. Points A and B are called the "earth contacts." Other forms of "earths" can be imagined, such as, for instance, replacing the points A and B by linear contacts. Even more complicated combinations could be considered, if a particular problem should indicate the advantage of their employment.

The current, when flowing from one earth contact to the other, does not follow a straight line but uses all the existing volume offered for its passage, and its distribution can be mathematically calculated if the medium is homogeneous.

As previously pointed out, those unfamiliar with this question of terrestrial conductivity may fear that the current will not flow in certain cases. However, experience in grounding return circuits in various electrical installations (such as telegraphs, traction systems, etc.) proves that no trouble is encountered in this respect, and that the current always passes. As a matter of fact, far from the contacts, the resistance to the passage of electricity is extremely low, on account of the large cross-section offered, and the resistance of the earth circuit is practically con-

centrated around the contacts. The resistance of the "grounds" varies widely according to the soils in which they are placed, and also with the care with which they are made. In practice, in very dry siliceous sands, a total resistance of 30,000 ohms has been sometimes encountered, whereas in very conductive regions such resistance can be reduced to some tens of ohms. The intensity of the current, therefore, will vary within wide limits for a given electromotive force. Consequently, the electrical determinations may sometimes be concerned with minute quantities and will necessitate the use of highly sensitive instruments.

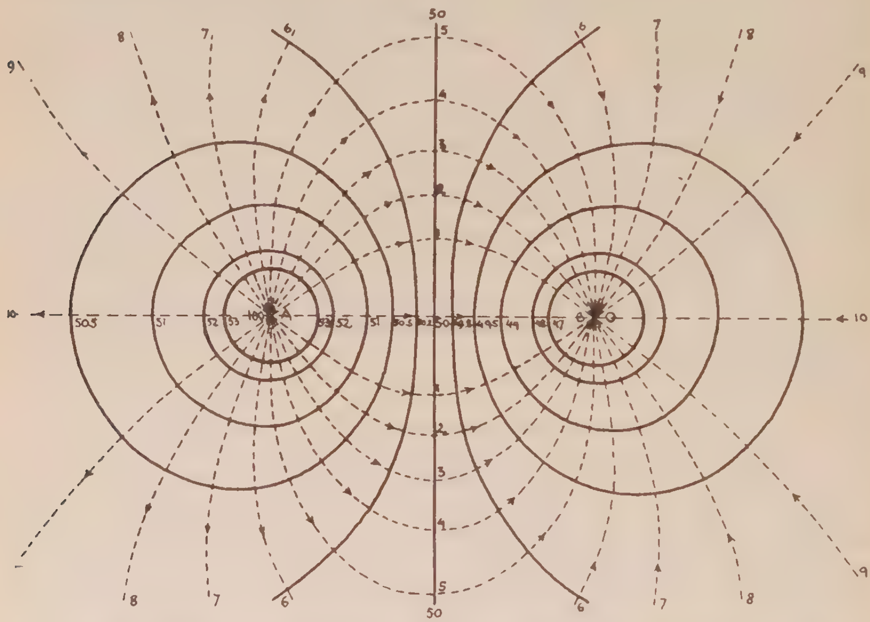

Fig. 1.—Equipotential curves observable at surface of homogeneous ground (solid lines).

No attempt is made here to expound the mathematical theory underlying the potential methods, since this is already known from a number of previous articles; but, in order to make this discussion clear, a few fundamental principles will be recalled briefly to mind.

Fundamental Principles

In the case of homogeneous ground, the application of Ohm's law for a limitless conductor permits the calculation, *a priori*, of the equipotential surfaces, and of the equipotential curves observable at the surface of the ground. These latter have the form shown by solid lines in Fig. 1.

The potential on each curve is known and consequently the variation of this quantity may be studied along any given line. This gives a "potential profile." For example, along the line AB, the theoretical profile in the case of a homogeneous and isotropic soil has the shape indicated in Fig. 2.

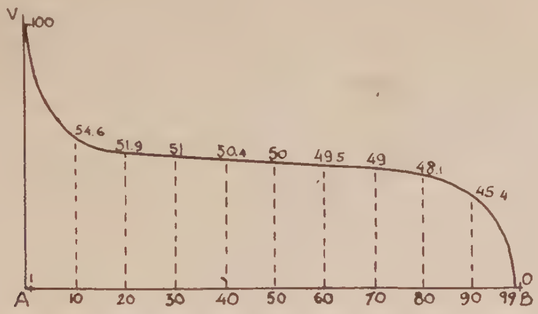

FIG. 2.—POTENTIAL PROFILE BETWEEN THE EARTH CONTACTS IN HOMOGENEOUS AND ISOTROPIC SOIL.

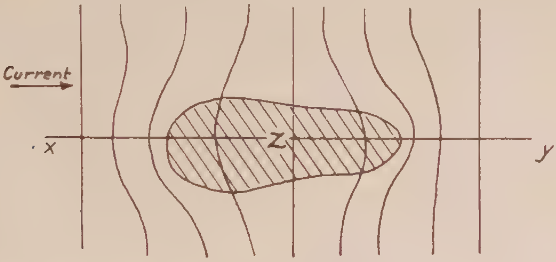

FIG. 3.—DISTORTIONS IN EQUIPOTENTIAL CURVES CAUSED BY A CONDUCTIVE MASS IN A RESISTANT MEDIUM.

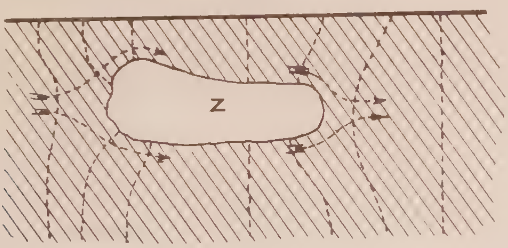

FIG. 4.—DISTORTIONS IN EQUIPOTENTIAL CURVES CAUSED BY A RESISTANT MASS IN A CONDUCTIVE MEDIUM.

When the ground is not homogeneous, distortions appear in the equipotential surfaces, curves and potential profiles. For instance, if a conductive mass Z (Fig. 3) is enclosed in a more resistant medium, it repulses the equipotential curves. The opposite effect is produced if Z is more resistant than the surrounding material (Fig. 4).

These elementary examples show that heterogeneity has a distorting effect on the potential field. The role of electrical exploration is to put in evidence these distortions, and then to interpret them correctly.

Evolution of the Technique

The first thought that comes to mind is to map the equipotential curves in the field and compare the chart thus obtained with the ideal figure for homogeneous media. This elementary procedure is advantageous in many cases and is frequently utilized; but it is slow and laborious, even in open countries, and often impracticable in mountainous or forested areas. It should, therefore, be supplemented by the use of higher mathematics, which apply in electrical exploration as well as in other geophysical methods (gravimetry, seismology, magnetometry).

Ten years ago only maps of potentials, with profiles and equipotential curves, were used to correlate results of field work. These are often difficult to interpret. Today, the resistivities of the rocks are calculated and the results given in the form of a map of resistivities, which comprises equiresistivity curves and profiles of resistivities. The variations of this parameter are thus easy to follow, both vertically and horizontally, and greatly facilitate the geological interpretations.

The principal concern of the mining man and geologist, however, particularly in a science as new as geophysical exploration, is to know the value of the results obtained in practice. This is more important than mathematical theory when it comes to appreciating the real value of the science and its workers. We give, therefore, the exposition of several typical problems showing the range of possibilities of potential methods in structural exploration.

Study of Tilted Strata under Overburden

This study is interesting from a chronological as well as from a technical point of view. It was performed in 1912 at Fierville-la-Campagne in Normandy, France, and precedes, to the best of our knowledge, by more than a decade, any similar attempt by other operators. It is described in Professor Schlumberger's book.

The problem was the following of an iron carbonate bed, intercalated between sandstone and shale strata, steeply tilted, and covered by 200 ft. of horizontal Jurassic clays and limestones. It was attacked as follows:

If a current is caused to flow in a homogeneous soil between two earth connections A and B, the equipotential curves in the neighborhood of A and B are approximately circles. If the rocks, instead of being homogeneous, are made up of tilted strata, the current will flow more easily in a direction parallel to the beds, where aqueous conductive

fissures are present, than perpendicular thereto. This is the case of an anisotropic medium. There will be a longitudinal and a transversal conductivity, which will differ from each other. The equipotential surfaces will be flattened ellipsoids of revolution with their axes of revolution perpendicular to the plane of the beds. The equipotential curves will be ellipses, the major axes of which will be coincident with the trend of the strata.

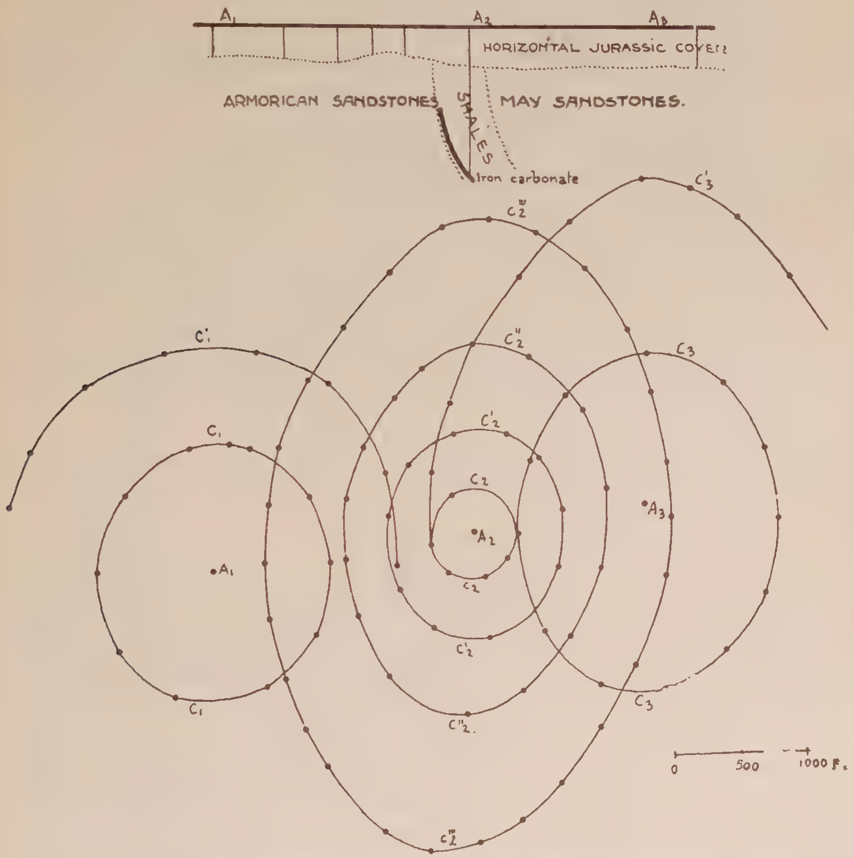

FIG. 5.—ELLIPTICAL EQUIPOTENTIAL CURVES OBTAINED BY ELECTRICAL PROSPECTING OF TILTED STRATA UNDER OVERBURDEN.

Fig. 5 shows the results obtained. The tilted Silurian formations are covered by beds of Jurassic limestone 200 to 300 ft. thick. A number of equipotential curves were traced about the contacts A_1, A_2, A_3. The strike of the beds is accurately determined by the longer axes of the curves. In addition to following the trend of the beds, the contact between the shales and Armorican sandstones was located by means of profiles. These profiles are not shown on Fig. 5.

The drill results were in agreement with the electrical indications. It is interesting to observe that there was serious difficulty in locating the contact by drilling alone, and that, at one place, seven holes were required to accomplish a single determination.

It should also be noted that the curves traced were of large dimensions, the biggest ellipse around A_2 being over two miles long. The block of ground affected by the electrical phenomenon is therefore to be measured in cubic miles, and the determination of strike is the average for a large body of strata. It is not affected by local irregularities, as may be the case with small outcrops, and especially with core drills. The fact that electrical prospecting is capable of giving average values over a large territory is to be particularly stressed, since it provides an advantageous means of completing precise, but strictly local, data furnished by drilling.

General Study of Concealed Anticlinal and Synclinal Folds

The method explained above is efficacious in the study of numerous problems of structure and stratigraphy, such as the study of concealed anticlines and synclines, contacts of tilted formations hidden beneath overburden, etc.

In 1920, a study of the May syncline (Normandy), was made for a subsidiary of the Creusot Iron Works.

Geology

A Silurian syncline was known from surface observations along the river Orne, and from workings southeast of Maltot, near May, on the other side of this river. (Fig. 6. The village of May lies off the map to the right.) The axis of the syncline seemed to lie roughly along the line Maltot-Fontaine Etoupefour.

The syncline is slightly overturned, with its two limbs dipping northeast (Fig. 7). In the area under consideration, it is covered by 130 to 260 ft. of horizontal Jurassic limestones and clays. In places some alluvium covers the Jurassic. The general succession of folded beds is as follows:

	Feet
Gothlandian black shales	0–400
May sandstones	300–900
Calymene shales	150–600
Iron carbonate bed	10– 25
Armorican sandstones	600

Northeast of the syncline in the valley of the Orne, a fault shows that the Silurian is cut off by vertical Pre-Cambrian phyllites (Faille des

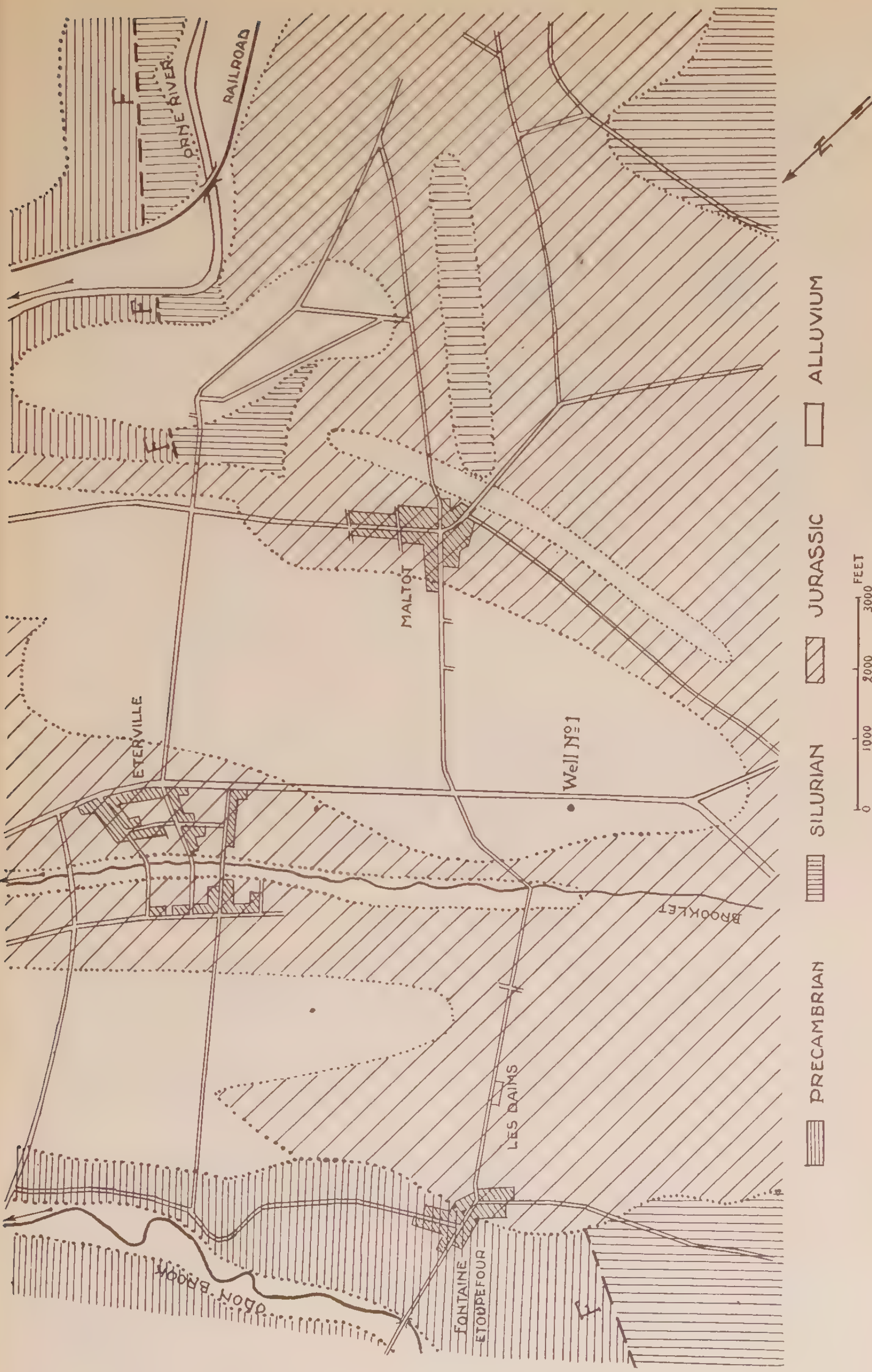

FIG. 6.—GEOLOGICAL MAP OF REGION OF MALTOT (NORMANDY).

Etavaux). A similar occurrence is evident west of Fontaine Etoupefour (Fig. 6).

In all this area, the only drill hole completed, before the electrical exploration took place, was well No. 1 on the road running southwest from Eterville. This drill encountered the iron formation at a depth of approximately 300 ft.

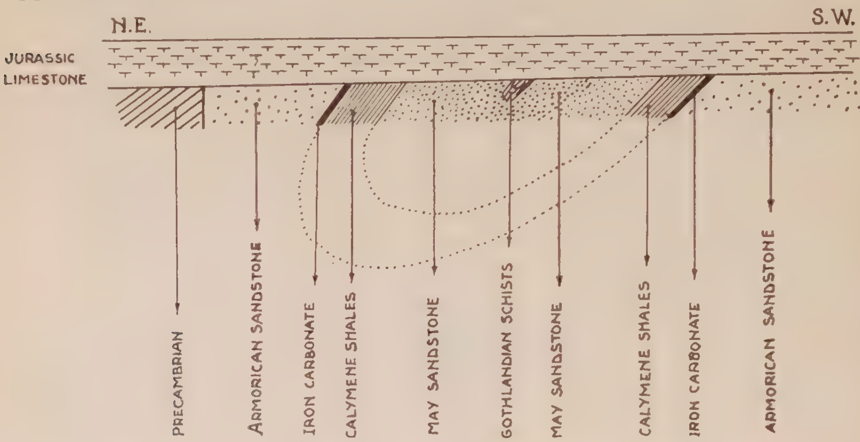

FIG. 7.—VERTICAL SECTION OF THE MAY SYNCLINE, NORMANDY.

Problem

The problem presented for electrical solution was to give a picture of the Silurian beneath its cover. Here it should be observed that geologists had no precise opinion on the connection between the fault near Fontaine Etoupefour and the one on the Orne. In particular, it was believed that the latter (Faille des Etavaux) continued towards Eterville.

Electrical Results

The directions of the various geological horizons were easily followed by means of electrical profiles drawn perpendicular to the axis of the syncline, and giving the relative resistivities of the beds traversed (Fig. 8). Profiles LL' and MM' are figured as examples, and the corresponding strata are noted upon them. The syncline was thus studied over a length of more than two miles.

The Pre-Cambrian formation north of the Etavaux fault, and in the neighborhood of Fontaine Etoupefour, has an average strike different from that of the Silurian. This permitted a study of the trend of the beds by means of equipotential ellipses. In Fig. 8 the direction of stratification is shown by the double-headed arrows at A, B . . . H. These data indicated the demarcation of the two series and proved that the two apparently separate faults were in reality but one. The trace of this fault under the cover is indicated by a dotted line. The conse-

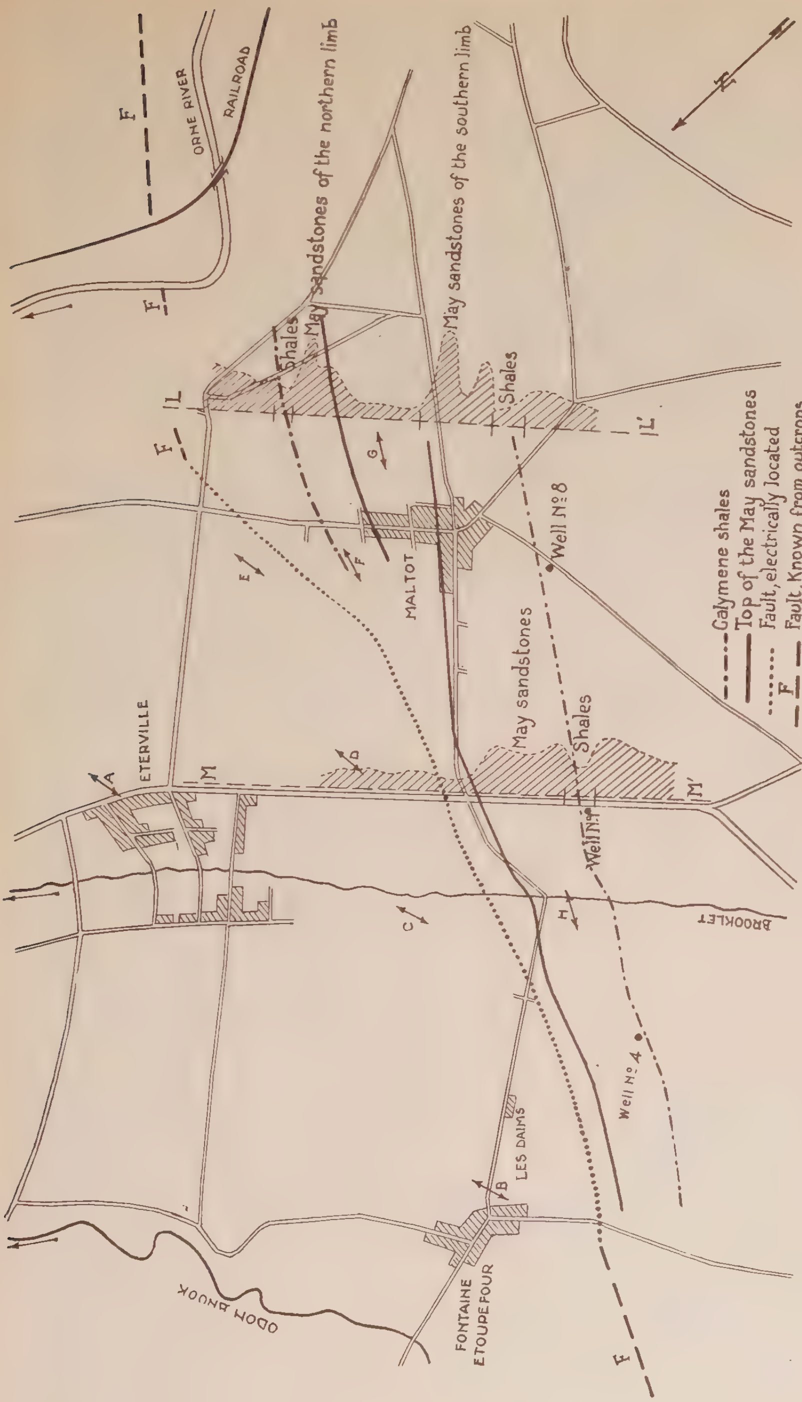

FIG. 8.—ELECTRICAL RESULTS OBTAINED IN STUDY OF MAY SYNCLINE.

quent abrupt cutting off of the northern limb of the syncline was unexpected, but the remaining piece of this limb was judged to be not worth exploitation, and the cost of an extensive drilling campaign in that area was thereby saved.

For the southern limb, two drill holes (Nos. 4 and 8) proved, by cutting the iron formation, the correctness of the electrical results.

Location of a Fault

When an electric current passes from a formation characterized by a certain conductivity into another one whose conductivity is different, a distortion appears in the form and disposition of the equipotential curves. The result is similar to what occurs when a ray of light passes from one medium into another of different index of refraction—a refraction is observed.

This phenomenon is equally well shown on the "profiles of resistivity," that is, on a graph giving the resistivities of the ground at different points along a line crossing the contact in question. Fig. 9 shows the results obtained in Alsace along the Rhine fault, near Lobsann. This fault, which brings Oligocene into contact with Triassic sandstone (Vosgian sandstone), disappears under Quaternary terraces. Its position was accurately indicated by means of profiles of resistivities.

It is also possible, in favorable cases, to determine the throw of the fault, and sometimes its dip.

Study of Horizontal Formations

The method consists in determining the resistivities of the succession of beds encountered in a vertical section at a given place. This is accomplished by means of potential measurements made at the surface of the ground. Such a study constitutes, in fact, a vertical "electric drill hole."

The applications of this technique are numerous: for instance, to determining thickness of overburden, following a horizon marker, or measuring the dip of slightly inclined beds.

This method requires that each formation be fairly homogeneous along its bedding, but that perpendicular thereto the strata should be clearly different one from another. Fig. 10 refers to two examples of such work. On each there is represented:

1. The profile of resistivities, showing the variations of this parameter with depth.
2. The conclusion drawn therefrom with respect to the thickness of the different strata.
3. Below, at the same scale, the results later obtained by diamond drilling.

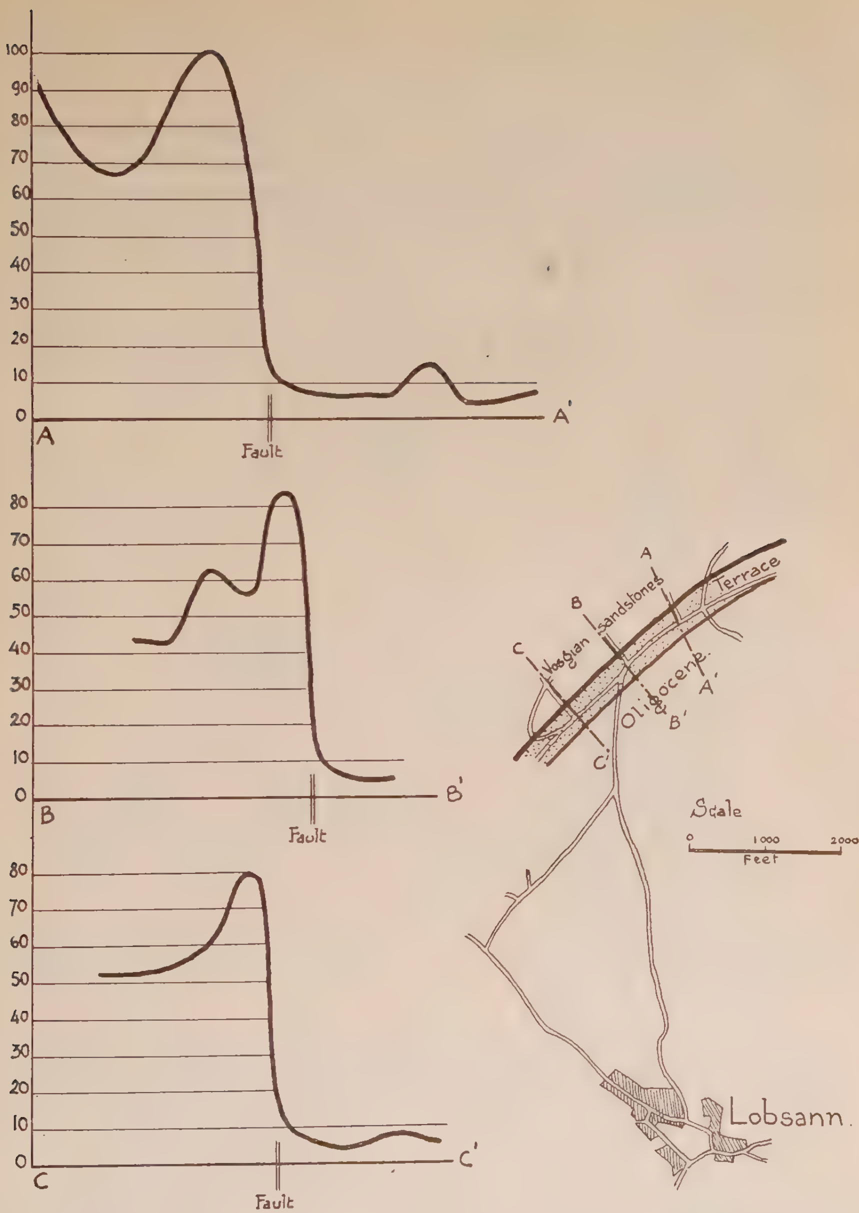

FIG. 9.—PROFILES OF RESISTIVITY OBTAINED ALONG THE RHINE FAULT IN ALSACE

The first problem relates to a search for lignite in the Department of Landes (France) and illustrates work at shallow depth. It was one of a series of ten electrical tests made for the Bureau of Mines, all of which were verified by later drilling.

The second study was made upon the Hettenschlag salt dome in Alsace, discovered by electrical methods. The prediction was concerned with the thickness of the alluvium and the depth to the salt. The accuracy of these results is particularly interesting, since away from the dome the corresponding figures are about ten times those found over the dome.

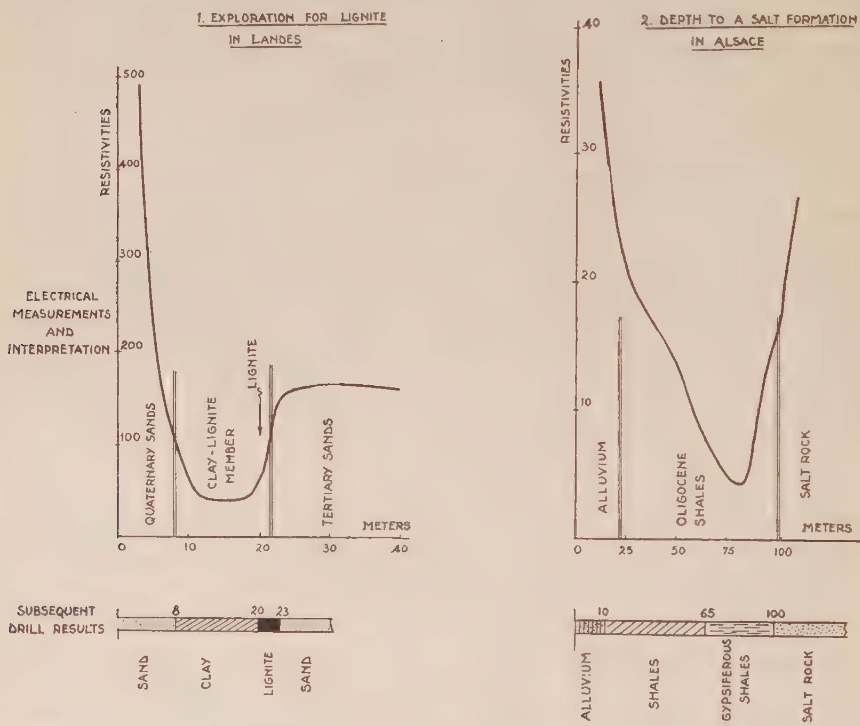

FIG. 10.—STUDIES OF TWO HORIZONTAL FORMATIONS. EACH SHOWS PROFILE OF RESISTIVITY, CONCLUSION REGARDING THICKNESS OF STRATA AND RESULTS OBTAINED BY DIAMOND DRILLING.

EXPLORATION FOR OIL—LOCATION OF SALT DOMES

The problems connected with exploration for oil are numerous. Many of them can be solved by electric methods (such as anticlines, faults, salt cores, etc.). As a matter of fact, wherever differences of conductivities exist between geological formations, the problem of the discrimination of these formations can be solved. There is no definite limit to the depth to which research can be carried. It is merely a

question of scale, and if the scale of measurements at the surface is doubled, the depth of investigation is also doubled. However, if the similarity of effect is to be maintained, the dimensions of the heterogeneity have also to be doubled. It is evident, then, that at great depths only large structures can be investigated.

Fig. 11 summarizes the results of electrical prospecting on the Boldescii anticline in Rumania. The map gives the equiresistivity curves

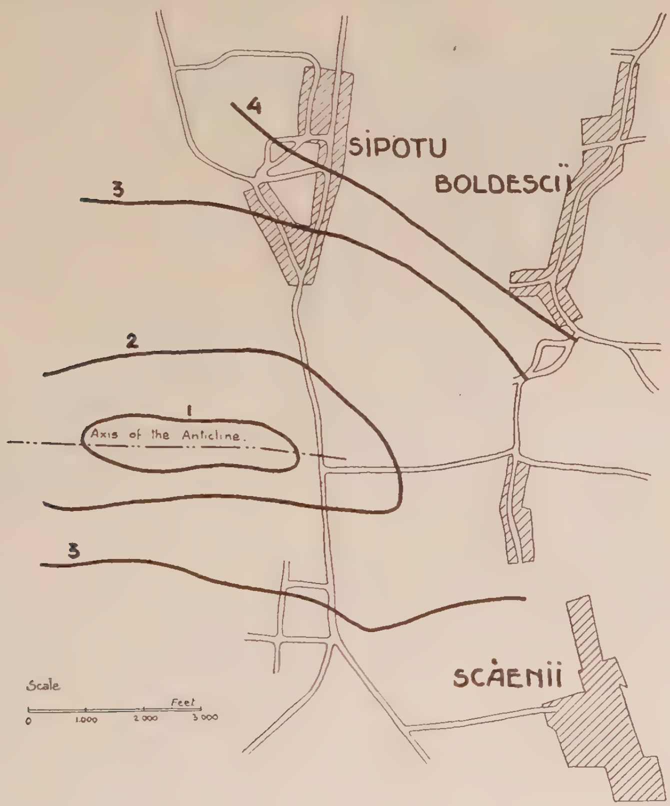

Fig. 11.—Results of electrical prospecting on Boldescii anticline.

at a certain depth. The curves correspond fairly well to the contour lines of the structure. This area was surveyed in 1923 prior to any oil discovery in that region.

Fig. 12 represents the Meyenheim salt dome, discovered in 1926 during the course of an electrical study of the Oligocene potash basin of the Upper Rhine (Alsace). The equiresistivity curves outline approximately the form of the salt intrusion, as may be seen by comparing them with the depth at which various drills encountered the salt. These figures, in feet, are given on the map beside the corresponding drill holes.

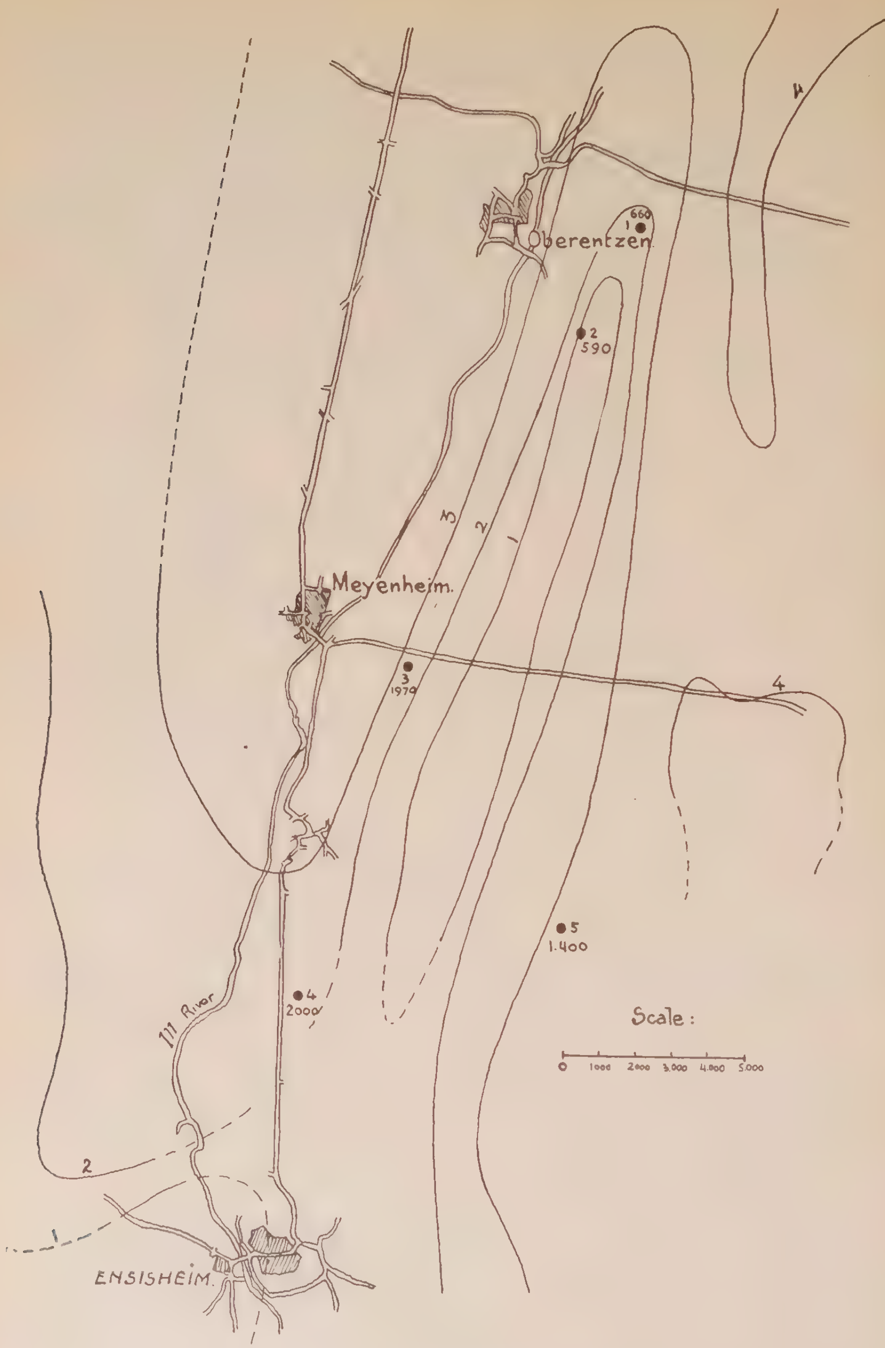

FIG. 12.—EQUIRESISTIVITY CURVES OF MEYENHEIM SALT DOME, SHOWING APPROXIMATELY THE FORM OF THE SALT INTRUSION.

Hole No. 1 encountered salt rock at a depth of 660 ft. and the drill was still in the formation when it was stopped at 4500 ft. Several potash beds were traversed.

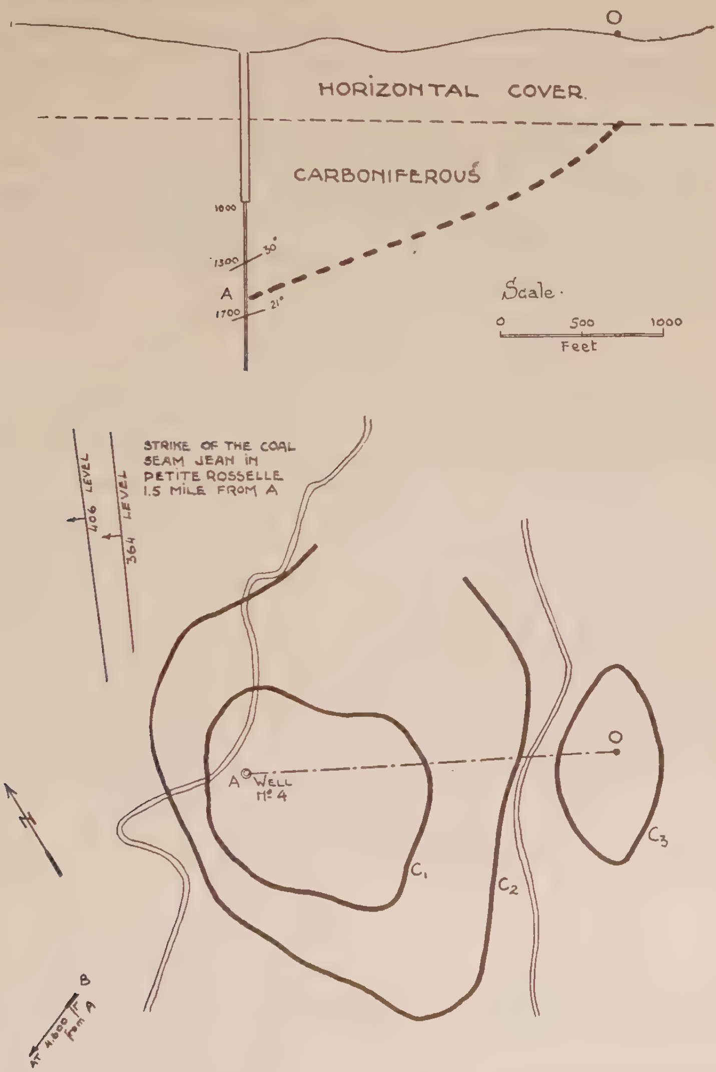

FIG. 13.—EQUIPOTENTIAL CURVES USED IN CONNECTION WITH DRILL HOLES, TO DETERMINE THE DIP OF INCLINED BEDS.

In 1927, another salt dome was discovered in this region at Hettenschlag by the same method of exploration, and also subsequently checked by drilling. A part of this work was described by Friedel.[2]

[2] M. G. Friedel: Sur l'existence d'un dôme de sel dans le bassin potassique oligocène du Haut-Rhin. *Comptes Rendus* de l'Académie des Sciences (1927) 1028.

Map of Subsurface Resistivities

The resistivity is a parameter characteristic of a given terrain. A map of electrical resistivities is therefore a veritable geological map in which the different strata instead of being defined by their lithological or paleontological characteristics, are differentiated by their electrical resistivity.

Frequently geological studies are seriously hampered, if not made impossible, by overburden which may itself be very insignificant, but which completely masks all outcrops (alluvium, glacial drift, vegetation, etc.). In these cases, electrical methods can render valuable services by following, beneath the overburden, a given bed which is known, for example, at only a single point.

Studies Utilizing Drill Holes

The geological information given by drills can often be augmented electrically. For instance, it is possible to determine the average direction, and the approximate value, of the dip of inclined beds. To illustrate this an example is given of work done in 1921 at Velsen (Sarre) for the French Government (Fig. 13). The current was sent into the ground at point A, 1600 ft. in depth. Three equipotential curves C_1, C_2, C_3 were traced. C_1 and C_2 enclose the drill hole A because a large part of the current utilized the casing to return to the surface. However, as has already been indicated, the current has a tendency to circulate along the bedding. This explains curve C_3 centered at O, the point of emergence of the current which followed the stratification. The determination of this "umbilicus" O therefore gives the direction of dip, OA. These results possess the advantage of being affected by a large block of terrain, and thus of giving an average value which is not influenced by local irregularities.

The average dip is also obtained, as is indicated in the cross-section, assuming there is no fault between A and O.

Conclusions

The varied problems discussed show that it would be erroneous to assume that the potential methods are limited to the search for metallic ores. The field of applicability in structural work is at least of equal importance. The results already obtained prove that the method has long since passed from the experimental stage to the domain of practical use.

The physical property studied, electrical conductivity, is entirely independent of the physical properties utilized in other branches of geophysics (gravity, elasticity, magnetism). Electrical prospecting constitutes, then, a fourth line of attack on many geological problems. The differentiation of formations by means of a particular method does

not always meet with the same success in all districts. In this connection, electricity will sometimes supplement, or advantageously replace, other methods in case they encounter difficulties too great to overcome.

The greater part of the work is based on measurements of potential differences. For this it is necessary to obtain very accurate readings, and the personnel must consequently be trained through long practice. However, this alone is insufficient for success, since the correct interpretation is of even greater moment, and only long experience permits the solution of many problems. It must also be noted that the method possesses a great flexibility which permits the attack on a problem to be made from several angles. This, of course, is an advantage, but necessitates, correspondingly, geophysicists well acquainted with the possibilities inherent in the different techniques. For example, it takes more time to train a capable operator in electrical prospecting than in magnetometer or gravimetric work.

Considering the actual applicability of electrical stratigraphic work, it should be a current means of investigation for the well-informed geologist and mining engineer. A large amount of the money expended every year in mining exploration is devoted to hazardous ventures. The ever-growing cooperation between the mining operator and the geophysicist will permit a better utilization of a part of this money. This paper will have achieved its end if it has succeeded in rendering the possibilities of electrical exploration more comprehensible and familiar.

DISCUSSION

J. T. Boyd, Aire Libre, Puebla, Mexico.—I wish to ask about the cost of these methods. At Teziutlan we have a copper orebody which is conductive. We made some geophysical experiments with the dip needle, but with entirely negative results, on account of small percentages of hematite in the soil, which interfered with useful observations. Later work by the equipotential method cost us as high as $600 or $800 (Mexican money) per acre. We have had sufficiently interesting results to make us willing to do a little more work, but we must have more efficient work. We must have some estimate of the acreage that can be covered in a day and some guarantee as to the limit for the price of doing such work.

(Written discussion).—Since the time of the meeting it has been explained to me why the cost of our equipotential work was so high. As this explanation may be interesting to others who also do not understand the highly technical explanations and papers presented by the various experts, I repeat it here.

The usual equipotential field method consists in establishing a long electrical base line by connecting the poles of a small electrical generator to the earth at points some distance removed from each other, and then listening to the sound of the generator through a radio-tube detector and amplifier.

It is plain that if two metal electrodes are introduced into the soil anywhere along this base line, and if they are connected through the input of the detector tube, a current will flow through the detector tube that will be detected and made audible, provided the two points are at a different potential. This audible sound can be amplified through the amplifier and listened to through headphones. If the two

points, however, are at the same potential, no current will flow and there will be no audible sound produced.

The method, then, consists in placing one of the two electrodes in the soil, placing the other some distance away from the first, and moving it around until a point is found at which no sound will be produced, indicating the point of equal potential. When such a point is found the first electrode is moved ahead of the second until a second point with the same potential has been found, and so on.

A line drawn through the equal potential points when all are correctly plotted on a map furnishes a basis for study. The convergence or divergence of a number of such lines begun at different points will indicate the presence of a conductive orebody or its absence.

Unfortunately there is a difficulty that cannot be foreseen which at times makes the cost of the method prohibitive. This is "out of phase" difficulty.

At some locations metal field electrodes can be placed at certain points, say 50 ft. apart, and there will be a sharp distinction between the points where the sound is audible and the point where it is inaudible, which makes possible rapid and accurate progress with this method. In other locations, for reasons not understood, if the electrodes are placed so far apart a different electrical line of force is encountered "out of phase" with the line on which the work is being done. This makes it impossible to find any point at which the sound is inaudible, and therefore prevents location of the point of equipotential.

Under such conditions the distance between the electrodes has to be shortened until a clear point of inaudibility can be found. At times the distance has to be shortened until the two electrodes can be placed only a foot or two distant from each other. When this is the case the method becomes very slow, and on account of the great amount of time spent, also very costly, until the cost is prohibitive.

We have just finished doing some more work by one of the inductive methods and have been very favorably impressed by the businesslike way and speed with which the work was carried out. With the inductive method we covered some 1200 acres at an approximate cost of $5 (U. S. currency) per acre.

E. G. Leonardon (written discussion).—In answer to the question by Mr. Boyd, I should like first to call attention to the fact that his written discussion is concerned with only one method, utilizing alternating current, and hence does not apply to direct current procedures.

As an example of the cost of electrical exploration, I can cite one of the surveys described in our paper; namely, that on a part of the May syncline. All the expenses incurred by the company for which we did the work, including our fee, came to less than $2000 for two months' work. About 7 sq. miles were covered. It must be admitted, of course, that the survey was performed in France in 1920 at a time when the cost of living was low. Therefore the figure given ($2000) can hardly be considered to apply over here. In the United States, at the present time, the same exploration could probably be carried out for $8000 to $9000, fee and all expenses included.

Electrical Prospecting Applied to Foundation Problems

By Irving B. Crosby,* Boston, Mass., and E. G. Leonardon,† New York, N. Y.

(Boston Meeting, August, 1928)

Electrical prospecting by potential methods has been applied to mining problems for some years and determinations of the depth to bed rock have been made, but so far as is known it has not been used previously in the solution of civil engineering problems, such as investigating the foundations of dams.

During the past winter and spring the senior author has been engaged as consulting geologist in studying the site of a large dam on the upper Connecticut River and has had the satisfaction of introducing electrical prospecting to foundation problems. The valley across which the dam is to be built is underlaid by a deeply buried preglacial gorge which is filled with glacial deposits. An extensive drilling campaign was in progress, but the work was unusually slow and expensive on account of the number of boulders in the overburden. The necessity of speeding up the work was pressing and it was decided that much of the needed information could be obtained by geophysical methods. After investigating the various methods and conferring with the company engineers and with the consulting engineer, Albert Crane, a contract was made with the Schlumberger Electrical Prospecting Methods, and an observer with apparatus was sent to the dam site the first of April, 1928.

The problem was to determine the depth to rock at various points in order to prepare a contour map of the bed-rock surface and outline the buried gorge in the vicinity of the dam site. Over one hundred determinations of depth have been made and the bed-rock topography is now known in considerable detail. In addition, some information has been obtained about the nature of the overburden. The drilling has been continued; in fact, two additional drills have been put to work, and the electrical determinations have been checked at several points. At one of these the depth to rock given by the electrical work was 142 ft. and the drill found rock at 147 ft., an error of less than 4 per cent. The electrical work has also been checked by the geological study, and it has been found that when the electrical determinations are in agreement with the geological expectations they are probably reasonably accurate. When the electrical interpretation of the subsurface topography is incon-

* Consulting Geologist.

† Schlumberger Electrical Prospecting Methods.

sistent with the geological interpretation, close cooperation between the geologist and the electrical observer is desirable, and when that is obtained much slow and expensive drilling can be obviated without loss of accuracy.

The rock at the dam site is largely chlorite schist, a metamorphosed rhyolite. It is a hard, dense rock and is probably of Pre-Cambrian age. There is also some phyllite, which is of the same age. The glacial drift that fills the buried gorge consists of till, gravel, sand and clay. One of the test pits encountered a bed of coarse gravel, cobbles, and boulders, which was loose and had many open places. It is such material that has made the trouble in drilling.

Before starting the program of electrical exploration the method was demonstrated on eight of the existing borings with satisfactory results. The electrical observer was not informed of the depth of rock in the holes, and he made the determinations in ignorance of the actual conditions. When he had given his results they were checked with the drill records, and it was after this demonstration that the extensive campaign was decided upon.

The results of the trial determinations at the eight drill holes are described here with the use of diagrams. Later, when the explorations have been completed, it will be possible to describe the situation more thoroughly and publish additional information.

Vertical Exploration by Potential Methods

The principles upon which electrical prospecting by potential methods is based are already too well known to require much elaboration here; hence they will be summarized briefly.

Two points in the ground, known as the "earth contacts," are connected to a source of electrical energy.[1] The current enters the earth by one and leaves by the other contact, utilizing for its passage the entire volume of the practically limitless conductor thus offered.

This flow of current creates an electrical field at the surface of the ground, and if the current is direct and uniform, every part of the ground will be charged with a certain potential. This can be calculated when the terrain is isotropic and homogeneous, and in this case the theoretical map of potentials can be drawn up easily. If the ground is heterogeneous, either at the surface or at depth, distortions appear in the potential map; distortions which must be determined and interpreted.

Two classic examples of these distortions have been given in all the elementary explanations of equipotential methods. They are recalled here because they illustrate a general principle. A conductive mass

[1] Other forms of earth contacts, besides those formed of points, can be utilized. Some experimenters have used two parallel, linear earth contacts, an arrangement that lends itself well to mathematical treatment.

inclosed in a resistant medium attracts the current paths and repulses the equipotential curves (Fig. 3, page 183), and a resistant mass has the opposite effect (Fig. 4, page 183). The effects on the equipotential curves of attraction or repulsion, as well as the variations in the distribution of potentials at the surface, can be determined exactly in certain simple cases.

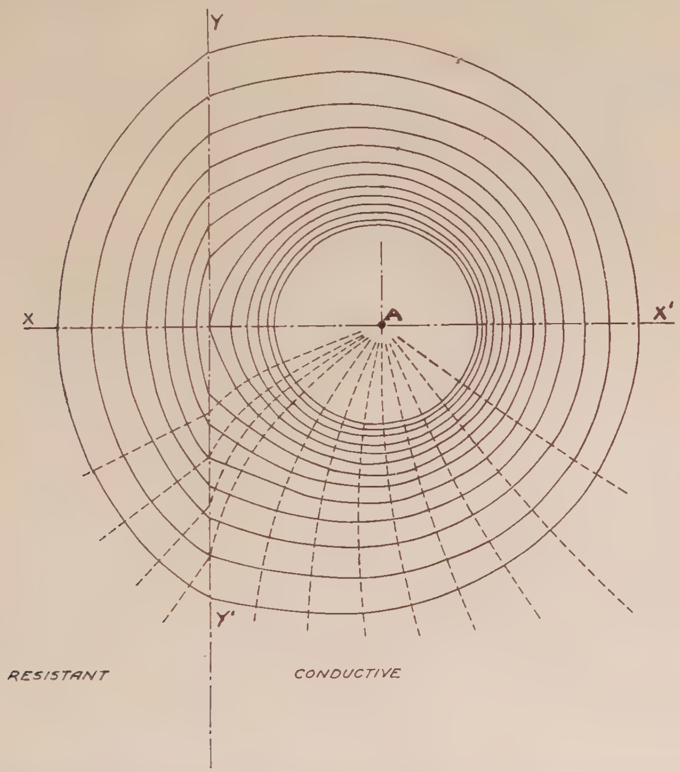

FIG. 1.—CURRENT PATHS (DOTTED LINES) AND EQUIPOTENTIAL CURVES NEAR THE PLANE CONTACT BETWEEN TWO LIMITLESS MEDIA. CURRENT ENTERS CONDUCTIVE MEDIUM AT *A*.

For instance, it is possible, in this manner, to study mathematically the electrical field resulting from the flow of current through two limitless media of different conductivity, which are separated from each other by a vertical contact. Fig. 1 shows the results obtained when the current enters the more conductive medium at point *A*. The other contact, *B*, is supposed to be infinitely distant, and the ratio of conductivity of one medium to the other is assumed to be three. If the current enters the resistant medium, the situation is as shown in Fig. 2. From these two figures it becomes apparent that not only the equipotential curves undergo distortion, but that the value of the potential at a given distance

from A, as well as the rate of drop of potential, are entirely different from the same figures in homogeneous soil.

These last two examples lead directly to the concept of vertical exploration by electrical methods. In nature we frequently have the case of a series of essentially horizontal beds of different conductivities. The problem is to determine the position of the various strata by means

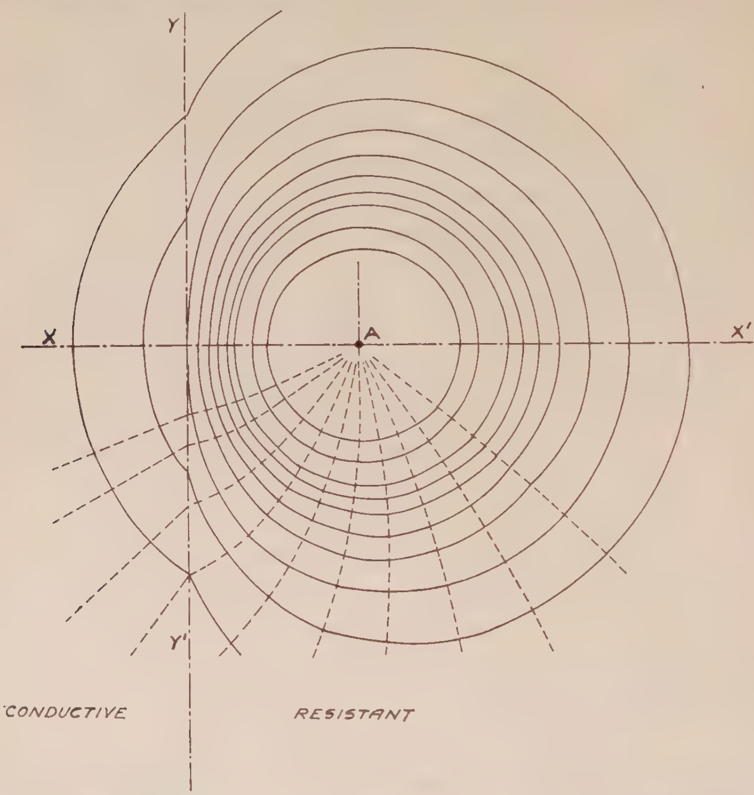

FIG. 2.—CURRENT PATHS AND EQUIPOTENTIAL CURVES (SOLID LINES) NEAR THE PLANE CONTACT BETWEEN TWO LIMITLESS MEDIA. CURRENT ENTERS RESISTANT MEDIUM AT A.

of an electrical survey. At first glance this might seem impossible, since the strata are homogeneous horizontally and the equipotential curves, therefore, are circles centered about the earth contact.[2] Nevertheless, it can be demonstrated easily that the problem is solvable.

In the case of a layer of indefinite extent (No. 2 in Fig. 3) beneath a uniformly thick bed of different conductivity (No. 1 in Fig. 3) it is evident that the equipotential surfaces about the contact A will not be hemi-

[2] As has already been assumed in this discussion, we continue to suppose, for simplicity, that the second earth contact lies at an infinite distance.

spheres, as would be the case with a homogeneous medium, but will have quite different and more complicated shapes. Refractions similar to those in Figs. 1 and 2 take place at the boundary of the two layers. The relation of the drop of potential at the surface to the distance from the contact will also be different from that in homogeneous material, and this fact is the key by which the thickness of the first layer can be determined. The use of calculus, into which we will not enter here, demonstrates that the problem is one capable of an exact solution.

This simple explanation shows the principles upon which the technique of vertical exploration is based. However, in the case of a practical field problem involving a larger number of strata of varying characteristics, the complications allow an approximate solution only. It is, nevertheless, possible to solve the problem whenever the beds are quite

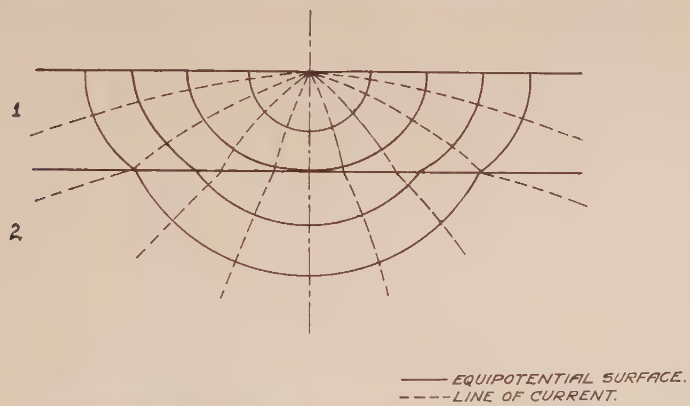

FIG. 3.—SCHEMATIC DIAGRAM OF EQUIPOTENTIAL CURVES AND CURRENT PATHS IN THE CASE OF A HORIZONTAL BED OVERLYING A LIMITLESS MEDIUM OF HIGHER CONDUCTIVITY.

homogeneous horizontally and well differentiated vertically. It should be noted here that the disturbances observed in the field of potentials depend on the distances from the surface, and upon the thicknesses of the different beds, as well as on their relative conductivities. It must not be expected, therefore, that a very thin layer can be located at great depth.

As has been explained, vertical exploration beneath a point consists of a study of the surface distribution of potentials about that point. Among the different operating procedures that can be adopted, one of the simplest and easiest has already been outlined; it consists of establishing a symmetrical distribution of potentials around the earth contact at which the measurements are made. This can be done by having the second earth contact sufficiently distant to prevent its exerting any appreciable influence in the small area where the readings are made. The radiating profiles that can be traced from the first point should then be identical. The fact that they are not always so is due to the lack of

perfect horizontal homogeneity of the beds, and to the topography. By tracing a number of profiles the local irregularities can be eliminated and the average distribution of potentials obtained.

The employment of maps of potentials, and the comparison of results registered on them with the normal distribution in homogeneous ground, is not convenient for the interpretation of electrical observations. A much clearer idea of subsurface conditions can be obtained by translating

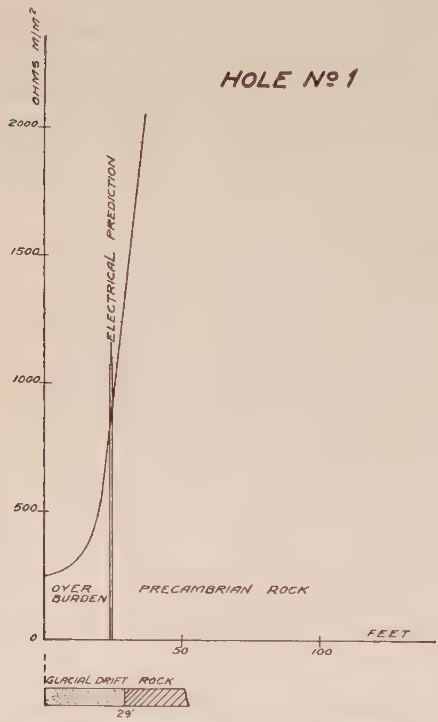

FIG. 4.—DETERMINATION OF DEPTH TO BED ROCK, WITH CONDUCTIVE AND FAIRLY HOMOGENEOUS OVERBURDEN.

FIG. 5.—THIS DIAGRAM SHOWS A POOR CORRELATION, PROBABLY DUE TO IRREGULAR SUBSURFACE TOPOGRAPHY.

the results into average resistances in function of the depth. It is possible, moreover, by using a few supplementary hypotheses, to determine the resistivities of the rocks at different depths. Experience has shown that the employment of these quantitative data is very advantageous and renders the interpretation quite easy.

TREATMENT OF THE PROBLEM AT THE DAM SITE

The geological conditions at the site in question were described briefly in the first part of this article. From what was known concerning the rocks and the overburden it was believed that this method of electrical

exploration would give valuable information. The schists at this place are similar to Pre-Cambrian schists which have been studied electrically in numerous places in Canada and New England. They are highly resistant, of the order of 3000 to 4000 ohms per meter cubed, as is to be expected for such dense, metamorphosed rocks. On the other hand, the unconsolidated formations, except when composed of dry sand, are better conductors, usually having a resistivity of one-eighth to one-tenth that of the schists.

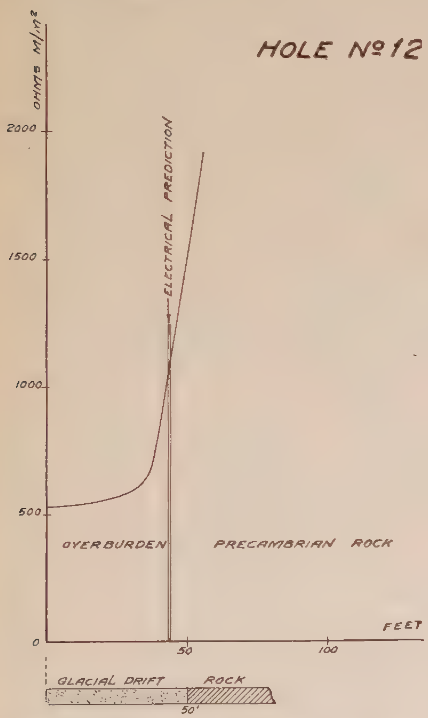

FIG. 6.—DIAGRAM IN THE CASE OF FAIRLY HOMOGENEOUS OVERBURDEN. ACCURACY FAIRLY SATISFACTORY.

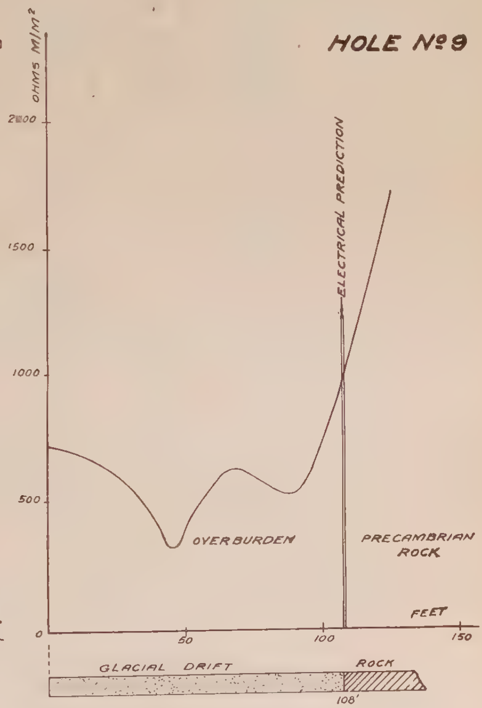

FIG. 7.—FORM OF RESULTS WHEN THE OVERBURDEN IS COMPOSED OF DIFFERENT LAYERS.

The various graphs (Figs. 4–11) show the resistivities calculated as a function of depth for holes 1, 7, 9, 10, 11, 12, 13 and 14. The resistivities in ohms per meter cubed are plotted as ordinates against the depth in feet to which they correspond.

These profiles can be arranged in several groups. Those at holes 1, 7 and 12 are of similar appearance. The overburden consists of essentially homogeneous material with a resistance of approximately 500 ohms or less per meter cubed (500 ohms at hole 12; 180 ohms at hole 7; 250 ohms at hole 1). Beneath this overburden is the schist, with a much higher resistivity, of 3000 to 4000 ohms, and more, in fresh, unaltered rock.

The electrical prediction is easy to make and places the rock on the rising part of the curve. It should be noted that in these three cases the depth to rock is not great and consequently the irregularities due to the topography, either of the surface or of the bed rock, may have a comparatively large effect. This is the case with hole 7, where there is a marked discrepancy between the actual depth to rock found by drilling and the depth determined electrically. Probably the drill struck a high point,

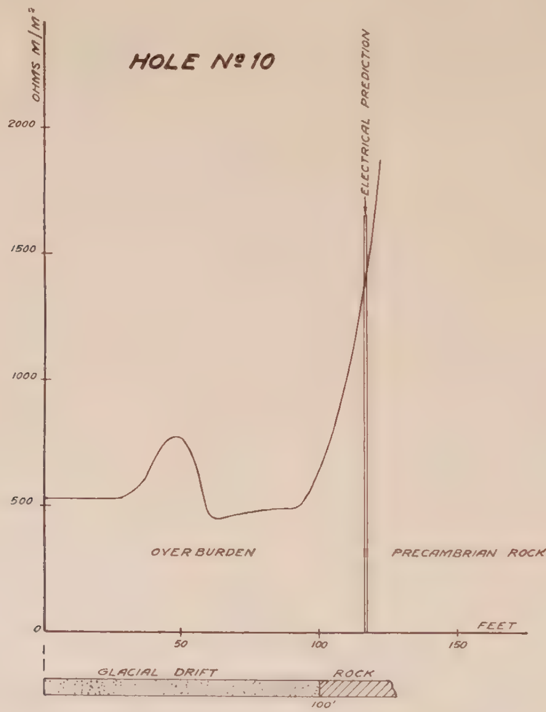

FIG. 8.—EXAMPLE OF A FAIRLY REGULAR OVERBURDEN WHICH CONTAINS A MORE RESISTANT LAYER.

while the electrical determination included some of the surrounding area where the rock is possibly lower. It is known that this hole is close to the edge of the buried gorge, which makes the foregoing explanation reasonable.

Holes 9 and 10 are also in a conductive overburden composed of sandy clay, but there is included a resistant layer of sand and gravel, which is shown in the profiles.

The profiles at holes 11, 13 and 14 are of a third type. At the surface there is a sandy, resistant layer, of which the thickness can be determined. Next there is an argillaceous, more conductive series of deposits, which rest on the schist. This arrangement of a conductive layer between two

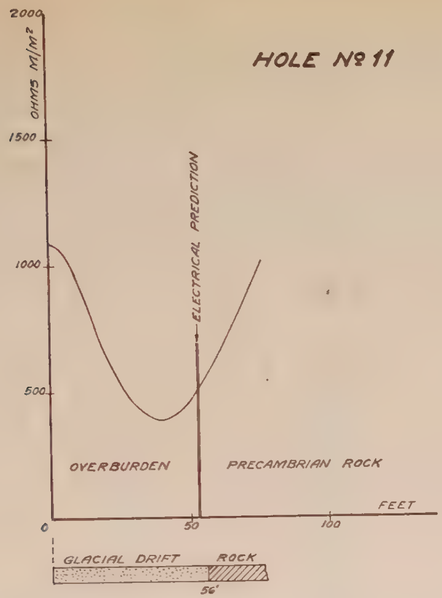

FIG. 9.—OVERBURDEN COMPRISES SANDY SURFACE LAYERS UNDERLAID BY MORE CONDUCTIVE (CLAYEY) MATERIAL.

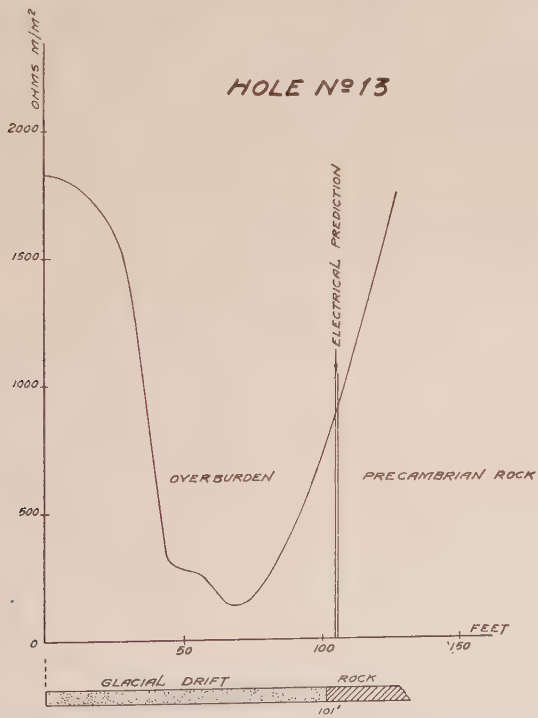

FIG. 10.—FIRST LAYERS ARE HIGHLY RESISTANT. THE VERY GOOD CONDUCTIVITY OF THE DEEPER MATERIAL GIVES A CLEAR POINT OF TRANSITION TO THE BED ROCK.

highly resistant masses is favorable to the interpretation of the electrical observations. The situation is the reverse, however, and may even be insoluble if the resistant sand or gravel deposit is in contact with bed rock. Differentiation of the two materials requires a ratio between their conductivities appreciably different from unity. When this does not occur the top of the resistant sand may be mistaken for the rock.

Such a difficulty arose in one place at the dam site when the electrical determinations there indicated a rock hill in the middle of the buried

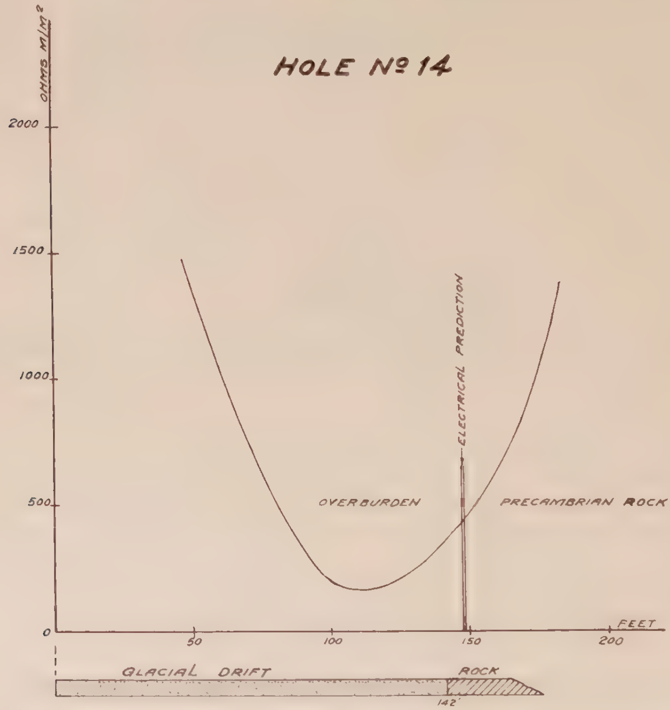

FIG. 11.—THE DEEPEST DETERMINATION. NO SPECIAL DIFFICULTY IN INTERPRETING. ACCURACY SATISFACTORY.

gorge. This would be a very abnormal condition, which could be caused only by the presence of very hard rock. The country rock is resistant to erosion and outcrops indicated that a belt of phyllite existed at the place in question, making the existence of such a hill highly improbable. Therefore the geologist requested a boring, which proved that the hill consisted of gravel and boulders and not of bed rock. This demonstrated the desirability of close cooperation between the geologist and the electrical observer. Probably in the future such mistakes can be avoided if the geology is well understood and close cooperation exists.

It is interesting to learn the average accuracy obtained in the 10 electrical determinations that were directly checked by the drill (Table 1).

These represent what may be expected under the disadvantageous conditions of irregular surface and bed-rock topography and discontinuous, changeable glacial deposits.

TABLE 1.—*Results of Electrical Determinations Directly Checked by the Drill*

Hole Number	True Depth to Rock, Feet	Predicted Depth, Feet	Predicted Depth in Relation to the True Depth, Per Cent.
1	29	24	83
7	27	41	151
9	108	108	100
10	100	118	118
11	56	53	95
12	50	43	86
13	101	105	104
14	142	148	104
20	167	115	69
21	147	142	97

The probable explanation of the error at hole 7 has been given. It may be mentioned that 40 ft. of steel casing were left in hole 10, which may have influenced the electrical results. Hole 20 penetrated the gravel hill, already described, and the error in the electrical determination was due to encountering the highly resistant gravel, which was mistaken for rock. This gravel was 121 ft. deep, which agrees well with the determined depth of 115 ft. to the resistant layer.

Five out of the 10 determinations are accurate within 5 per cent. or less, three more are within 20 per cent. of the facts and two are bad, but these errors can be explained as due to special conditions.

Conclusions

The results that can be obtained by electrical exploration in solving problems concerned with measuring the thickness of overburden have been given. This type of work can be useful on many occasions in civil engineering, such as investigating the foundations of dams or other important structures, and studying tunnel lines to learn whether the tunnel will have everywhere a safe cover of solid rock. The success of the work depends on differentiating unconsolidated formations from solid rock and, as there is usually a great contrast in their conductivities, favorable results can commonly be expected.

Without going into the question of exploration at great depths, or of tectonic studies over large structures, it is evident that electrical exploration at shallow depths can also render useful service in mining work. Mineralization often is known to be related to buried river valleys, and

these can be located in all cases where the electrical characteristics of the beds are sufficiently differentiated. Under such circumstances electrical studies are capable of economically supplementing a drilling campaign, which is usually a costly affair.

Electrical exploration is a tool quite different from geology or diamond-drilling. It does not supplant these but, on the contrary, supplements them and permits the rapid gathering of data to give a perspective of the situation, which then can be made more precise by subsequent exploration work. This latter exploration can be oriented more economically as a result of the electrical survey.

Close collaboration between the geologist, the driller and the geophysicist, such as was realized in this case, leads to the best results. Combining information from different sources leads to a more complete solution of the problem being attacked.

Discovery of Salt Domes in Alsace by Electrical Exploration

By G. Carrette* and Sherwin F. Kelly,* New York, N. Y.

(Boston Meeting, August, 1928)

Drilling in the Oligocene potash basin of Alsace prior to 1927 had shown important differences of level in the salt beds thus encountered. To explain this a somewhat unsatisfactory hypothesis of faulting had been advanced.

During the latter part of 1926 and the beginning of 1927 our firm was engaged by two organizations[1] for the purpose of trying to supplement, by an electrical survey, the geological data thus far collected. As a result of this exploration it was possible to affirm that salt dome structures were present in the Oligocene basin of the Upper Rhine, something heretofore absolutely unknown in that region. Two salt domes were actually outlined: one, near Meyenheim, was made the subject of a communication to the Academy of Sciences, by G. Friedel;[2] the other, near Hettenschlag, was briefly discussed by C. and M. Schlumberger[3] before the same academy.

The purpose of this paper is to indicate the stratigraphic and structural conditions of the area where these salt domes were found; to discuss the results of the electrical survey which led to the location of the domes; and to indicate the new geological outlook which such a discovery suggests.

Stratigraphy

In the southern part of the Rhine graben the Oligocene rests almost directly on the Middle Jurassic and consists, in ascending order, of:

1. Green marls, not salt-bearing, composed of fresh and brackish water deposits.

2. Middle stage, salt-bearing, consisting of lower, schistose marls and upper, variegated marls.

3. Blue marls, not salt-bearing, but including gypsum deposits; hence sediments laid down in a bay more or less connected with the sea.

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* Schlumberger Electrical Prospecting Methods.

[1] Mines Domaniales de Potasse d'Alsace and Société des Mines de Blodelsheim.

[2] G. Friedel: Sur l'existence d'un dôme de sel dans le bassin potassique oligocène du Haut-Rhin. *Comptes Rendus* de l'Académie des Sciences (1927) 1028.

[3] C. and M. Schlumberger: Découverte près de Hettenschlag d'un deuxième dome de sel sous la plaine d'Alsace. *Comptes Rendus* de l'Académie des Sciences (1928) 445.

This series of sediments was deposited principally in salt and brackish waters; therefore it follows that the group, as a whole, possesses a good electrical conductivity.

Following the Oligocene, the region was emergent, and much more recent clays (Miocene and Pliocene) filled the hollows of the erosion surface. These clays were particularly well developed in the downthrown compartments. They also are conductive.

Finally, Quaternary alluvium blankets the whole with sands and gravels whose highly variable thickness may even exceed 100 m. The Quaternary sediments, in distinction from the underlying marls, are characterized by a low electrical conductivity.

Tectonics

The region under discussion has suffered a number of tectonic movements. The resulting evidences thereof (folds and fractures) follow two principal directions, namely:

1. The Hercynian direction, to which there corresponds a phase of orogenic movements in the Permo-Carboniferous.

2. The Rhenan direction. The forces producing these folds and fractures reached their maximum contemporaneously with the Alpine movements, causing the downthrow of the median portion of the Vosges-Black Forest massif.

In addition to these paroxysms, other orogenic movements of lesser intensity have taken place at later epochs, even down to the Recent.

In fact, aside from the Hercynian movements which have left grandiose evidences of their magnitude throughout Europe, it must be remarked that there exists here a series of anticlines and synclines oriented in the Hercynian direction, northeast-southwest, but of an entirely different age. Already in process of formation during the period of emergence at the beginning of the Tertiary, these folds continued to form during, and probably after, the Oligocene. It was in this region, in process of folding, that the Oligocene salt formation was laid down. The salt was deposited in the zones of submergence, particularly along the axes of the synclines where it attained its greatest thicknesses; on the anticlines it was laid down only on the plunging portions. These constituted but limited areas of submergence where the sediments formed were much thinner. This remark concerns not only the salt, but also the material covering it.

The Rhenan tectonics consist of a principal system of faults oriented north-south and a secondary set at right angles thereto. The maximum of this orogenic disturbance was contemporaneous with the deposition of the Oligocene. It is the age which should be ascribed to the great bordering faults as well as to the inception of movement on the lesser faults which furrow the Rhine graben, and on which dislocations have continued down to our own times. As a result there has been created a

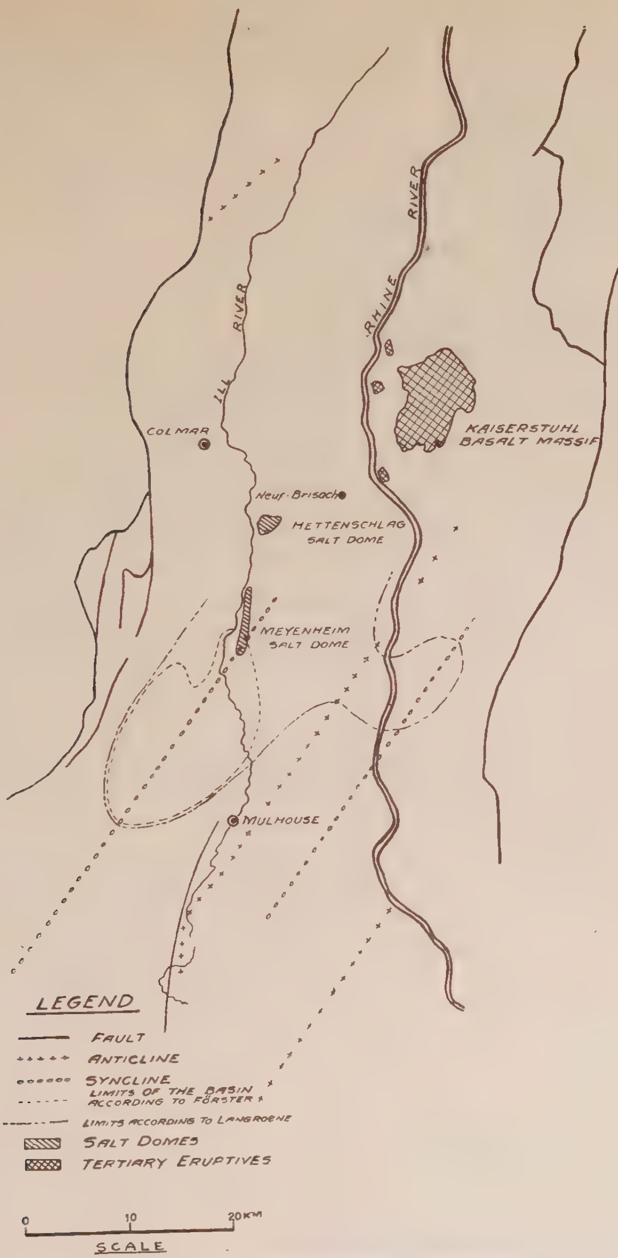

FIG. 1.—SKETCH OF THE UPPER ALSATIAN PLAIN, SHOWING THE MEYENHEIM AND HETTENSCHLAG DOMES AND THE MAJOR TECTONIC LINES.

compartment far from simple and composed of secondary horsts and grabens. It seems that the maximum depression is found beneath the Rhine river. This fact is quite natural since, in consequence of the continuity of the orogenic movements, the river has established its bed along the lowest segments.

Fig. 1 shows the principal tectonic lines mentioned above, namely, the axes of the Oligocene folds and the faults bordering the Rhine graben. The two salt domes are also marked on this map.

The Meyenheim Dome

The first dome, discovered in 1926, is located east of Meyenheim, a village about 20 km. north of Mulhouse. It is remarkably elongated, measuring 8 km. in length by 1.2 km. at the widest point, so that the appellation "salt anticline," or "salt ridge," would be more appropriate than "salt dome." Its major axis is oriented north 18° east.

Near the axis of the ridge two drill holes encountered the salt at 180 and 200 m. respectively, while 1500 m. farther east it was found at a depth of 1100 m. Evidently it may be granted that the elevation of the salt has suffered a marked change, of the order of 1000 m. At the same time there is observed an important diminution in the Oligocene marl covering. The alluvium, too, is thinner over the dome, whose only surface indication is a slight bend in the Ill River near Ensisheim. These considerations should be noted, since they indicate the remarkable persistence of the action causing the intrusion of the salt. The same effects, progressively diminishing, are observed in the salt, in the marls, in the alluvium, and at the surface of the ground. This point will also be put in evidence during the discussion of the second dome.

Fig. 2. shows the dome as outlined by curves of equal electrical resistivity. In the discussion of the stratigraphy it has been pointed out that the Oligocene marls are highly conductive, whereas the covering alluvium is comparatively very resistant. Therefore, values of the resistivities depend in a simple manner on the thickness of the alluvium. From this fact and the above-noted effects of the persistence of intrusive action, it follows that these equiresistivity curves will also be contour lines of the salt plug.

Both the curves and the drill results are shown in Fig. 2, and abundantly confirm this conclusion.

Before the electrical study was undertaken it was already known that the region from Oberentzen to Ensisheim presented a tectonic anomaly. This resulted from the data given by the drill holes 1 to 11, plotted on Fig. 2; in particular drill hole 1 encountered the salt at 180 m., whereas drill holes 2 and 3 entered it at only 600 and 610 m. On the contrary, holes 4, 5 and 6, respectively 600, 730 and 890 m. deep, did not find the salt at all.

In order to explain these results the hypothesis had been proposed of a horst of closely spaced step-faults, oriented north-south and running from Oberentzen to Battenheim, some 3.5 km. south of Ensisheim. This theory was rendered untenable by the results of the electrical survey,

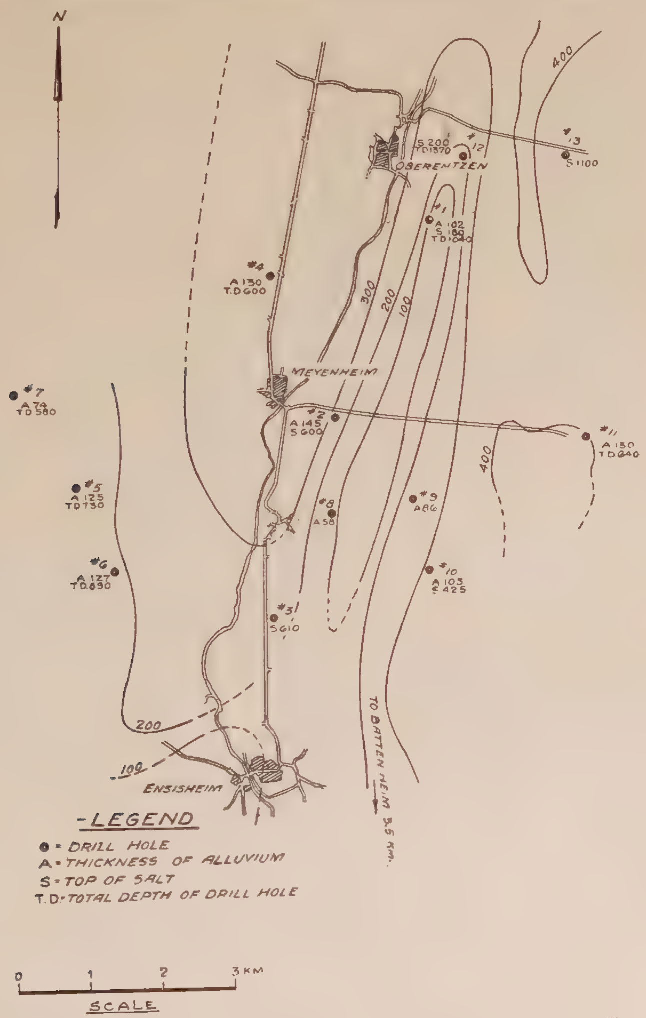

FIG. 2.—MEYENHEIM DOME AS OUTLINED BY EQUIRESISTIVITY CURVES. VALUES OF RESISTIVITIES ARE GIVEN IN OHMS PER METER CUBED.

as just noted, and the more recent drillings have proved the correctness of the new interpretation. For instance, drill hole 12 entered the salt at 200 m. and was still in it when stopped at 1370 m. It encountered beds almost vertical and in all cases dipping at an angle of 70° or more. On the contrary, drill hole 13 cut a great thickness of horizontal, Oligocene beds (marls and sands) and entered salt only at 1100 meters.

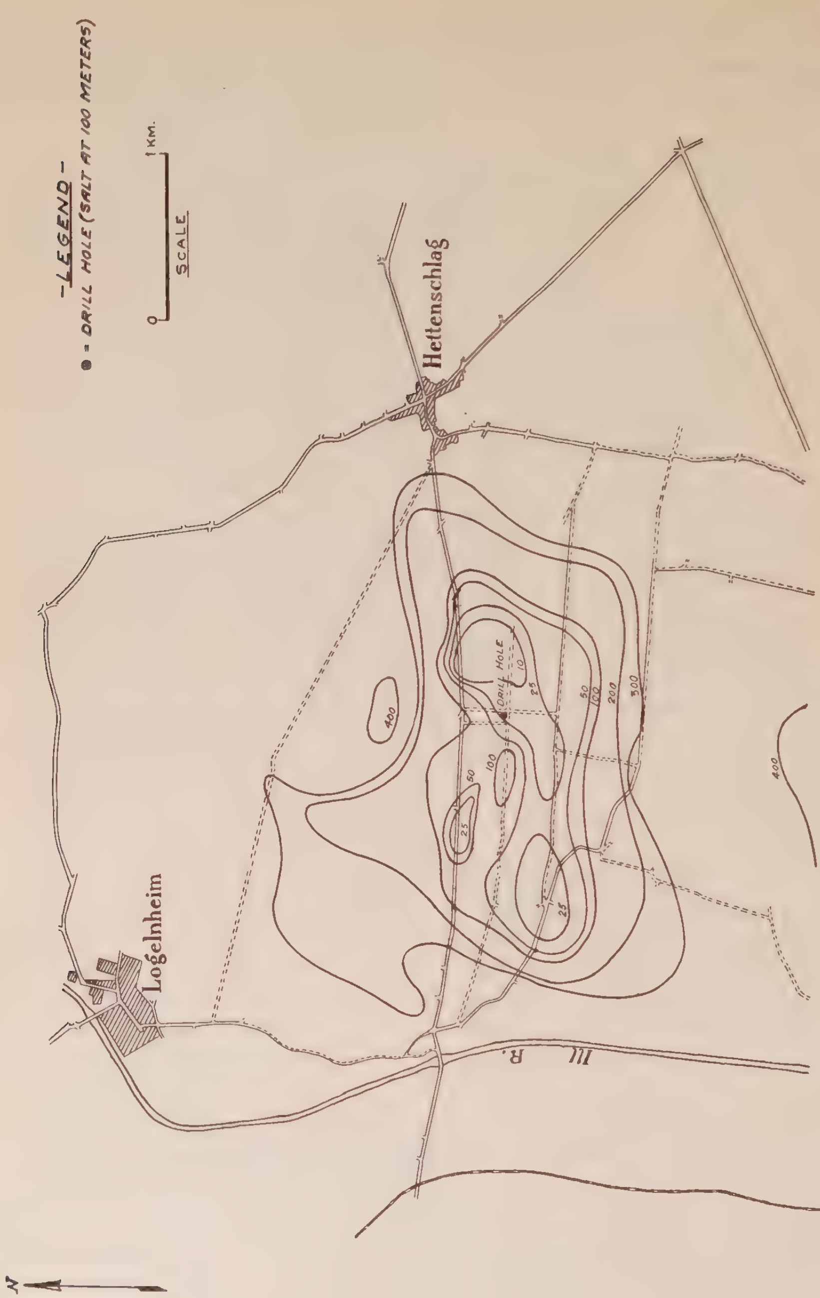

FIG. 3.—STUDY BY RESISTIVITIES OF THE HETTENSCHLAG SALT DOME. VALUES OF RESISTIVITIES ARE GIVEN IN OHMS PER METER CUBED.

The Hettenschlag Dome

If some prior geological indications existed which might have led to the idea of the occurrence of a dome near Meyenheim, this is not the case with the second one which lies farther to the north and is about 2 km. west of the town of Hettenschlag. Its discovery was due entirely to the electrical survey.

The dome is approximately elliptical, measuring 2.5 by 1 km., with the major axis east-west. Fig. 3 shows this structure as outlined by equiresistivity curves.

The electrical survey determined in advance the principal characteristics of the dome and showed a local diminution of the alluvium.[4] Generally, in this region, the thickness of these sediments is at least 100 m., whereas at one particular place, on the top of the dome (where a drill hole was later put down) it was electrically determined not to exceed 20 m. The depth to the salt was predicted to be 100 m. Actually, after passing through 10 m. of alluvium the drill entered the Oligocene marls and at 65 m. encountered gypsiferous marls, finally attaining the salt at 100 meters.

Although the only drilling carried out is far distant from any other, certain considerations, pertaining either to the particular tectonics of the region or to general geological laws of salt domes, give the impression that the upward movement of the salt was even more pronounced at Hettenschlag than at Meyenheim. It is not unreasonable to suppose that it amounted to 1500 meters.

If it be added to these geological and geophysical data that the topographic relief above the dome presents a slight rise of 2.5 m., the resulting ensemble of phenomena strikingly supports the theory of the persistence of the orogenic disturbances; a theory which has served as a guide in attempting to establish the relationship between the tectonics of the Alsatian Plain and the formation of the salt domes.

Geological Deductions and Conclusions

It now remains to place this picture of the domes in the frame of the Alsatian Plain tectonics.

First to be noted is the fact that the Meyenheim dome is situated on the axis of a syncline running in the Hercynian direction toward Kaiser-

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[4] We will not describe here the technique used to study formations disposed approximately horizontally; it has already been set forth by E. G. Leonardon and Sherwin F. Kelly in Some Applications of Potential Methods to Structural Studies, which appears in this volume. On page 192 is the graph of an "electrical drilling" carried out over the flat top of the Hettenschlag salt dome which gives the electrical prediction concerning the thickness of the alluvium and the depth to the salt.

stuhl (Montbéliard-Dannemarie-Kaiserstuhl syncline). Next, it will be observed that this dome is oriented in the Rhenan direction, as given by the fractures bordering the Alsatian graben. Also, the Hettenschlag dome is approximately on the prolongation of the Rhenan line which constitutes the axis of the Meyenheim dome.

There is every reason to believe that the plug of Hettenschlag is in the same syncline as the one of Meyenheim, or at least that it is located in a syncline separated from the preceding one by only a small ridge. This syncline would be bounded on the north by an anticline paralleling the granitic axis of the Vosges, but whose exact location cannot now be fixed.

The salt was deposited in Oligocene time, in the Montbéliard-Kaiserstuhl syncline. Since this axis seems to be a direction of weakness (as is further demonstrated by the basalt massif of Kaiserstuhl) the gradual sinking with increasing load of sediment should have been greater here than elsewhere. The salt in the Montbéliard syncline was consequently covered by a great thickness of sediments. For some time the German geologists have established, from their studies of the German salt basin, that the salt has a greater tendency to form domes the more deeply it is buried beneath later sediments. There is no point in entering here into a discussion of the theories attempting to explain salt-dome intrusions either by isostasy, or by tectonic movements acting on plastic matter. It will suffice to note that, in the zone of the Alsace potash mines, where the top of the salt formation is at an average depth of 300 to 400 m., it is merely strongly folded; farther north, at a place where the covering sediments have a thickness of about 1100 m., it assumes the form of an acute anticline (Meyenheim); then, still farther north, in a region where the salt-bearing formation lies, in all probability, even deeper, it gives rise to a quasi-circular dome shown to exist at Hettenschlag.

It is quite impossible to date the beginning of the upward push of the salt, but only to affirm that it progressed throughout the Quaternary and certainly is going on now. The sole action opposed to this one is the dissolving of the salt by water from the alluvium. This probably has little effect, however, since there is no true "cap rock" on top of the domes to give evidence of it, and also because there is no continuous sandy horizon in the Oligocene. Finally, such solution would have given rise to certain characteristics of electrical resistivity; observations showed, however, that these were absent.

DISCUSSION

Member.—Am I correct in my conclusion that the anomalies in the electrical measurements are not due to some difference in the thickness of the alluvium covering the salt?

S. F. Kelly.—There are three factors entering into the question of anomalies in these electrical measurements: (1) the covering alluvium is resistant; (2), it is under-

laid by conductive marls; (3) these are, in turn, dislocated by the highly resistant salt. The measurements graphically depicted were concerned with the resistivities of these formations, and put in evidence the fact that there had been a pushing up of the conductive marls into the resistant alluvium. The cause of this was deemed to be a salt dome.

E. G. Leonardon, New York, N. Y.—Over the top of the dome measurements can be obtained which indicate that the resistivities are increasing with depth, since rock salt is one of the most resistant substances encountered in nature. Such a phenomenon was observed in an "electrical drilling" over the Hettenschlag dome. This is given in a previous paper, which was cited in the course of the presentation of the one under discussion.

As the measurements were taken to greater and greater depths there was at first a rapid decrease in resistivity, corresponding to the marls. This was followed by a moderate increase, due to the gypsiferous beds, and at a depth of 100 m. an abrupt rise in resistivity was found, indicating the salt. In the exploration work that followed, this was the depth at which the salt was actually found, 100 meters.

K. Sundberg, Houston, Texas.—Is it possible also to indicate the presence of the salt in the case of the Meyenheim dome, which I understand is at the depth of about 180 m.? The depth in the other instance mentioned is about half of that. Is it possible from the electrical results to see any increased resistivity due to the presence of salt; also in the case of the Meyenheim dome where the depth is greater?

E. G. Leonardon.—The salt itself was not determined by electrical measurements on the Meyenheim dome, but was investigated only on the second dome discovered.

Member.—The map showing the lines of equal resistivity (Fig. 2) is, then, based on the thickness of alluvium and not on the salt, and the measurements that indicated the presence of salt were taken in another manner; but in both cases the map of equal resistivity is a map whose anomalies are determined by the thickness of the alluvium; is that correct?

E. G. Leonardon.—No, that is not correct. If you make a map of resistivities it must be based upon a certain depth of investigation and 1, 2, 3 or more maps of resistivities can be made for a given area. If an average depth of investigation of 50 m. is taken for the first map, an idea will be obtained of the resistivities at a depth of about 50 m. If I make a second map at a depth of 100 m., it would not be absolutely the same as the first one, and if I make a third map at a still greater depth, it might be quite different.

Suppose I make a map of a shallow dome in Texas. There are many things to be considered. The top soil is conductive, the salt is resistant—there are series of phenomena which must be taken into consideration and which permit making determinations of reasonable accuracy, and several maps at different depths can be made to get the necessary results.

In the case of the Hettenschlag dome, a shallow map takes in only the alluvium and the marls. In fact, one does not investigate the salt dome, but does investigate the block which has been pushed up by the salt. If a greater depth of investigation is undertaken, then of course the salt dome would not be so accurately outlined, although it would appear distinctly as a resistive mass.

Member.—That is just the point I was trying to bring out. In the equal-resistivity map shown, the salt dome is resistant and the surrounding marl is conductive, but it is my understanding that the alluvium plays the greatest part in the

determinations. And further, that the resistivity map shown must be controlled by the thickness of the alluvium.

S. F. Kelly.—That depends entirely on the depth at which the map of resistivities is established. Suppose a salt dome lies at 100 m. deep. Then if a resistivity map is made for 50 m. depth it will show a block of the overlying conducting marls shoved up into the resistant alluvium. In other words, there will be a conductive island in a sea of high resistivities. If, now, a map is made for 150 m. depth, the salt will show as an island of much higher resistivity in the *comparatively* less resistant marls.

Field Observations of Electrical Resistivity and Their Practical Application

By J. G. Koenigsberger,* Freiburg i. Br., Germany

(Boston Meeting, August, 1928)

The electrical specific resistance of rocks in the field is measured by sending a current through a medium of great volume, compared to the electrodes, whose resistivity should be measured. The whole resistance can be determined by the Wheatstone bridge and the specific resistance calculated by a well-known formula. The resistivity of some rocks in place in Germany and in Switzerland is given; first, of a rock near the surface, and especially near electrodes.

Practical applications are made in the determination: (1) of the continuity of a conducting orebody; (2) of the surface area of an orebody; (3) in the detection of a water-bearing fault in a salt mine.

Preliminary observations are described: (1) on the influence of the depth of ground-water table; (2) on the detection of gas under high pressure in coal seams.

The observations in this paper of the electrical resistivity of rocks and soil in place are along the lines of the very interesting method and results of the Messrs. Gish, Rooney, Hotchkiss and Fisher.[1,2] All the work described has been done since 1920.

Method Used

The very simple method consists in the application to the rock of two plane circular electrodes of iron or brass of a suitable diameter, with the aid of a contact substance, and in measuring the resistivity in a Wheatstone bridge with alternating current of 100 to 400 Hertz frequency per sec.). The general formula is known to be

$$R = R_o + \frac{\sigma}{4}\left(\frac{1}{a} + \frac{1}{b} - \frac{1.7}{l} + \frac{0.8(a+b)}{l^2}\right). \tag{1}$$

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* Mathematical-physical Institute, University of Freiburg.

1 W. J. Rooney and O. H. Gish: *Terrestrial Magnetism* (1925) **30,** 161; (1927) **32,** 49.

2 W. O. Hotchkiss, W. J. Rooney, J. Fisher: Earth-resistivity Measurements in the Lake Superior Copper Country. See page 51.

in a homogeneous infinite[3] medium, in which a (and b) is the radius of the electrode plate, l is the distance between the centers of the two electrodes, σ is the specific resistivity of the rock in a specified region. R_o is the resistance of the cables to the electrodes; R is the total electrical resistance between the two electrodes observed in the Wheatstone bridge. The region near each electrode approximately for a distance fivefold the diameter $2a$ of the electrode (or $l = 10a$) has the principal influence (about 90 per cent.) on the value of the resistivity. This can be deduced from a graphical representation of the current lines or from the formula given. The direction of the current lines near the electrode is perpendicular to the electrode. The electrodes are of iron 5 mm. thick with a screw on one side for attaching wire. The contact substance is hematite powder or clay, or earth with an aqueous solution of $FeCl_3$ and $FeSO_4$ (or simpler, also of NaCl, 10 per cent.). The resistance of such a paste is very low compared with that of the soil or rock. Change in the concentration of the salt does not affect the observations. If large electrodes are to be employed an iron fence such as is used for gardens or tennis courts is often sufficient, but then the quantity of salt will have an influence (up to about 30 per cent.) on the measured value. An induction coil or transformer for 1 watt is sufficient, which gives 1 to 50 volts; in coal mines a little alternating-current dynamo driven by hand, which gives no sparks, is used. For observation of the minimum a good telephone of 2000Ω is sufficient.

The usual resistance box, which does not need to be very accurate, can be used as a Wheatstone bridge. Two parts have each 10^1, 10^2, 10^3, 10^4, 10^5, 10^6, 10^7 ohm and a third part with 10, 20, 20, 50 . . . to 1.10^6, 2.10^6, 2.10^6, $5.10^6\Omega$. Capacities (the same as for radio) can be used for compensation of phase differences. Self-inductions are mostly not necessary. For connecting cables copper wire 1 mm. dia. is used; sometimes for long use, iron-copper or bronze cable.

Values of Resistivity in Place

Some values of resistivity σ were observed in the Alps beneath the Alvier Mountain, Sargans, Switzerland. The diameter[4] of the electrode was 26 cm. The distance of the two electrodes was 3 to 50 m. The resistivity is given in ohms per cubic centimeters. The rocks were in place at natural outcrops, as shown in the accompanying table.

The temperature was about 7° to 18°. The observations were made on dry days (it rains every 3 to 10 days, as usual in the middle and west

[3] "Infinite" means large compared with l. If the observations are true about 10 per cent., the dimensions of the "infinite" region must be larger than $10l$.

[4] 1000 m. = 1093 yd. = 3280 ft.; 1 U. S. A. mile = 1609 m.; 1 m. = 1.093 yd. = 3.28 ft.; 1 cm. is 0.394 in.; 1 mm. is 0.039 in.; 1° C. = 1.8° F.; 0° C. = +32° F.

European climate; approximately one-third to one-quarter as much rainy as dry time). The resistances in the subsoil in the mine are sometimes 10 to 30 per cent. higher than at the outcrops, because at the surface the rock contains moisture with humic acids and salts of the soil, while in tunnels or mines the water in the porosities is usually purer. The ventilation and the absence of rain make the rocks in the mine dryer than ordinary at the surface. But they are always wet, and in holes the relative humidity is near 100 per cent. Rarely is the resistance of the same rock in the mine lower than above. In a dry summer week the resistivity at the surface increases. Sometimes also, after much rain, the resistance increases a little and decreases afterwards slowly in good weather, because the rain water is purer than the water that remains long in the soil.

The potential drop near one electrode was 3 to 50 volts and therefore about 0.3 to 5 volts per centimeter.

Electrical Resistivity of Rocks in Place

	Ohms per Cu. Cm.
1. Wet dense humus grass soil (0.5 to 2 m. on Jurassic limestone)	4,000
2. Schiltkalk (dense Jurassic limestone)	350,000
3. Quinten-kalk (dense Jurassic limestone)	250,000
4. Mergeliger kalk (dense Jurassic limestone with clay)	140,000
5. Dogger (ferrugineous dense Jurassic sandstone)	400,000
6. Portlandshales (dark Jurassic shales) parallel to the schistosity	50,000
7. Portlandshales (dark Jurassic shales) perpendicular to the schistosity	12,000,000
8. Quartz porphyry in a wet mine, about 50 m. deep, Black Forest (Germany) Badenweiler near Freiburg (Baden). The mean annual temperature there would be about 10° C.	40,000
9. Quartz-barite lode, same locality and conditions	40,000
10. Galena in the lode, same locality and conditions	30
11. Sandstone (coarse Carboniferous sandstone of Segeu gottes grube, Waldenburg, Lower Silesia, Germany). On the irregular vertical sidewall of the gallery in a coal mine some 300 m. under the surface dry, ventilated, in winter, 11° to 16° C.	25,000
12. Sandstone (denser than 11), same locality, etc.	51,000
13. Schieferton (Carboniferous argillaceous shale) same locality (perpendicular to the bedding)	77,000
14. "Flammkohle" (coal)	15,000
15. Like 14, but with fissures	60,000
16. Rock salt, very pure, in the mine of Deutsche Solvay Werke, Borth near Wesel, Niederrhein, Germany at 22° C.	10^6 to 10^7
17. The same, but not pure, with inclusions	3.10^3 to 5.10^5
18. Humus soil of a grain field, very wet (near Borth)	18,000

The data for soil and rocks in the table are of the same order of magnitude as those found in the United States for a similar climate by McCollum and Logan,[5] by Rooney and Gish,[6] by Hotchkiss, Rooney and Fisher,[7] and for Sweden by K. Sundberg.[8] It is interesting to note that the values of the resistance of the soil calculated for the use of radio telegraphy in Germany by J. Zenneck are higher, about 2.10^4 for wet; $1.10^6\,\Omega$ per cu. cm. for dry soil. The electrical engineers in Germany assume for currents of low frequency the value 10^4 ohms for soil. The season, rain and temperature, and the depth under the surface alter considerably the resistance of the soil near the surface.

The anisotropy of resistivity in stratified or schistose rocks should be observed in place, while dependent on the amount and quality of moisture. Argillaceous Devonian slates of Siegen near the mine Grube Ziethen, Seligenthal by Hennef (Rheinland, Germany) have the resistance R of 3400 ohms when the two electrodes are applied on the plane of schistosity and of 3000 with electrodes perpendicular to the schistosity. (Referring to equation 1, $a = 10$ cm.; $l = 100$ cm.) The current lines were through a distance of about threefold the electrode diameter approximately perpendicular to the electrodes. This outcrop of the slates lay in a ground that was a little wet. For slates with normal moisture, such as there would be underground, the resistivity σ parallel to the plane of schistosity was 66,000 and perpendicular to the plane was 75,000. A steep wall of a little old stone quarry of the same slates, dried by air and sun, was 7 m. away and 3 m. higher. The resistance R was 4500 to 5000 for electrodes perpendicular to and 7300 for electrodes parallel to the schistosity ($l = 200$ cm.). For these dry slates the resistivity σ and their difference between perpendicular and parallel were therefore higher than in the first case; 110,000 for parallel and 165,000 for the perpendicular. That was to be expected, as the moisture would first vanish from the larger and deeper cavities which extend parallel to the schistosity, but would not vanish quite so easily from the small porosities, which are distributed irregularly.

Observations of resistivity in the laboratory, of rocks removed from their place without the natural moisture, can not give a sound basis for deductions on electric methods in geophysics. The large majority of dry rocks, except some ores,[9] graphitic schists or rocks

[5] B. McCollum and K. H. Logan: Electrolytic Corrosion of Iron in Soils. U. S. Bur. of Stds. *Tech. Paper* 25 (1914). The average data for soil in St. Louis (warm, wet, saline) are lower, for Albuquerque, New Mexico (dry) higher than the data for Philadelphia, Pittsburgh and those given here.

[6] *Op. cit.*

[7] *Op. cit.*

[8] K. Sundberg, H. Lundberg and J. Eklund: Sveriges Geologiska Under-sökning. *Arsbook* 17, No. 8 [C] (1923).

[9] P. F. Kerr and C. K. Kabeen, *Econ. Geol.* (1925) **20**, 729; H. Reich, *Jahrbuch* d. Preuss. Geologischen Landesanstalt (1925) **46**, 627.

impregnated with ore, are good insulators. In moderate climates the rocks in the field are never dry; also in deep mines, which seem to be dry, the rock pores have a humidity of about 90 to 100 per cent. Current lines go from the surface as deep as 1000 m. and more. The conductivity is given by the volume, form, number of porosities[10] and fissures of the rock and by the concentration of ions in the water contained therein.

Calculation of Single Resistance, Approximate Evaluation of Discontinuities in the Ore and of the Ore Surface

If the space for the current lines (perpendicular to the lines) is not infinite compared with electrode distance, the resistance would be larger than given by equation 1. Such a case is discussed later.

The whole resistance of a circuit with two electrodes in infinite space can be separated into several parts. Let p_1 be an iron electrode applied to medium I, p_2 to medium II. The mediums I and II are bedded in medium III. Then I and II (Fig. 1) can be considered as

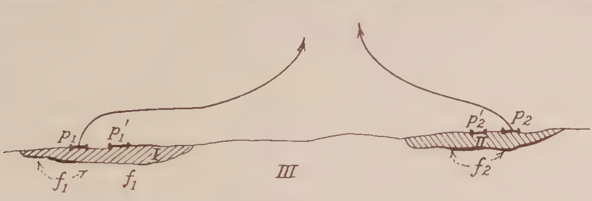

FIG. 1.

secondary electrodes with boundary surfaces f_1 and f_2, for which equation 1 is valid but with other constants.

The observation and calculation can be made in the following way. First, the resistance R_1 between p_1 and p_1^1 the center distance l_1 in medium I and R_2 between p_2 and p_2^1 with distance l_2 are to be measured. That gives the value for $\frac{\sigma}{4}\frac{1}{a}$ and $\frac{\sigma_2}{4}\frac{1}{a}$ for the mediums I and II.

Then

$$R = R_o + \frac{\sigma_1}{4}\frac{1}{a} + \frac{\sigma_2}{4}\frac{1}{a} + \frac{\sigma_3}{4}\left(\frac{c_1}{A_1} + \frac{c_2}{A_2} - \frac{1.7}{L}\right) \tag{2}$$

L is the center distance of f_1 and f_2. The bounding surfaces f_1 and f_2 separating I from III and II from III are to be considered as circle planes. c_1 and c_2 are constant depending on the situation and extension of f_1 and f_2. If f_1 and f_2 are near the surface of III, and surrounded everywhere below and on the side by III (like Fig. 1) $c_1 = c_2 = 1$. R_o is the resistance of the cables from the Wheatstone bridge to the two electrodes. If

[10] H. Hlauschek, Geologische Grundlagen der geoelektrischen Erdölsuche. *Ztsch. f. prakt. Geol.* (1927) **35**, 22.

p_2 is applied directly to III, $\frac{\sigma_2}{4}\frac{1}{a} = 0$, $A_2 = a_1$ and L is the distance between the center of f_1 and p_1. $\frac{\sigma_1}{4}\frac{1}{a}$ and $\frac{\sigma_3}{4}\frac{1}{a}$ are determined by separate observations; a is known, σ_1 and σ_3 can be calculated, also A_1, first without the correction $\frac{1.7}{L}$. With the value of A_1 and f_1 it is possible to evaluate, with the aid of the geologist and mining engineer, the value of L.

The resistance of the cables R_o was measured for 100-m. length and could therefore easily be calculated for any other length. The length of the cables in difficult pathways in the mountains were sometimes more than 5000 m. The cables were of different quality and resistance and were laid during the experiments in different ways, although of given length.

We can also make use of an approximate formula for two circular plane electrodes immersed in an infinite medium at the distance h_1 or h_2 under the surface:

$$R = \frac{\sigma}{8}\left(\frac{c_1}{a_1} + \frac{c_2}{a_2} - \frac{1.7}{l} - \frac{1.7}{l^1} + \frac{1}{2h_1} + \frac{1}{2h_2}\right) + R_o \text{ for } h_1, h_2 > a_1, a_2 \quad (3)$$

where a_1 and a_2 are the radii of the electrodes, l their center distance, h_1 and h_2 the distance of center of the electrodes from the surface, and $l^1 = l^2 + (h_1 + h_2)^2$.

If the orebodies have rather a rectangular form with sides e and d, the equivalent radius could be calculated approximately from $\sqrt{\frac{ed}{1.15\pi}}$. If an orebody of the height h and length y is in two parts, distant from one another only by a short distance x, the whole resistance R' after deducing the resistance of the cables and the expanding resistances of the electrodes, is a combination of the two expanding resistances R_1 and R_2 of the orebodies I and II against the medium III and of $R'' = \frac{\sigma_3 \cdot x}{hy}$. It is $\frac{1}{R'} = \frac{1}{R''} + \frac{1}{R_1} + \frac{1}{R_2}$.

Applications of the Method[11]

Connection of Two Orebodies

There was an outcrop (Figs. 1 and 2) of hematite in a small old abandoned mine on the mountain slope, at a height of 1245 m. Modern

[11] The first application of a resistivity method for practical scope was made by Brown and McClatchey in 1900 (Patents U. S. A.). Afterwards Daft and Williams in 1901 (Patents England and U. S. A.), used the qualitative observation of potential differences with a four-point method for finding ores. On the instigation of W. Petersson [*Glückauf* (1907) **20**] experiments with this method were made in Sweden by W. Slade Olver. But there was not a theoretical basis; quantitative data were not determined; therefore the results of these earlier experiments are somewhat vague. C. Schlumberger (Prospection électrique par les Procédés Schlumberger, 17. Paris, 1927. Soc. de prosp. electr.) has recently detected with resistivity measurements salt domes in Alsace. See *Min. & Met.* (1928) **9**, 398.

mining is proceeding at $O2$ about 1060 m. above sea level. The question to be answered is whether the orebody went from $O2$ to $O1$ without interruption, and to evaluate the quantity of the ore.[12]

The orebody at $O2$ was about 2 m. thick; at $O1$ about 1 m. An electrode was applied on the ore at $O1$ another at $O2$ and connected with cables. The observation center was in all the following cases near $O1$.

The whole resistance was 147 ohms, the same ($\pm$ 3 ohms) at two different periods of the year and also the same during the daytime as at night. There must be deducted for the cables in this case R_o to $O1 = 28$; R_o to $O2 = 16$; therefore R_o total $= 44$ ohm $\cdot \frac{\sigma}{4} \frac{1}{a}$ and was for the two equal electrodes in $O1$ and $O2$ each $= 42$ ohms. There remains a residue of $147 - 128 = 19$ ohms.

Tubes for water and compressed air are extended into the mine to $O2$, fastened by hooks to the hematite, and therefore in close contact with it.

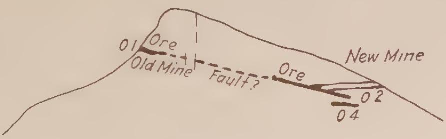

FIG. 2.

The whole resistance R_B of the tubes with their expanding current lines in the ore was 3 to 5 ohms. The resistance between $O1$ and $O2$ with the connection to $O2$ by the tubes (instead of electrode and cable) and the resistance R_o of the cable to $O1$ and of one electrode R_1 on the ore $O1$ was 98 ohms. As R_1 was 42; R_o to 0.1. was 28, $R_B = 5$, the resistance of the ore lode was therefore 23. That found before was 19 ohms, therefore the average is 21.

Let us take first the supposition that the ore goes without interruption from $O2$ to $O1$. The specific resistivity of the ore was 2000 and of the limestone surrounding the ore was about 330,000 to 700,000 (average value about 450,000) observed in the mine. The value for the limestone may be too low; in the undisturbed mountain it is perhaps higher than the average in the mine. The limestone around the ore also has a resistivity much greater than that of the ore; and the greater number of the current lines would remain in the ore from $O2$ to $O1$ if there is no interruption. The distance between the two electrodes l is 290 m. $= 2.9 \cdot 10^4$ cm. and therefore of no influence (see equations 1 and 2). If there was no interruption, the resistance must therefore be $\sigma x : hy$, in which h is the height of

[12] Observations by Prof. O. Hecker and the author.

the ore. The ore gradually diminishes from 2 m. in the new mine $O2$ to 1 m. in the old $O1$; the average value of h would be 1.5 m. The distance x between $O1$ and $O2$ was 290 m.

The lateral dimension y, according to observation in the new and old mine, was about 250 m. The resistance of the ore between $O1$ and $O2$ therefore must be 15 ohms. This value differs from the observed by 6 ohms. The difference lies a little beyond the limit of accuracy, taken on the assumption that the data given for σ, h, x and y are known with an error of only ± 10 per cent. If the two orebodies at $O1$ and at $O2$ were widely separated, the current lines must diverge in the limestone. The resulting resistance would be larger than the observed value of 121 ohms, namely at minimum $2.\sqrt{2.60} = 170$; yet the observed value of 121 ohms is much too low, as the remainder found here (see above) is only 21 ohms, from which first must be subtracted the resistance of the parts of the ore itself, about 10 to 15 ohms, at minimum. So there remain only about

FIG. 3.

6 to 10 ohms instead of 170. Therefore the orebody can not be widely interrupted, but must extend from $O2$ nearly to $O1$.

We can now recalculate the resistance in the ore. A slight correction must be made for the expanding conductivity (see equation 3) which is, for the total orebody with two surfaces surrounded on 0.6 of the circumference by limestone, about 60 ohms (see next section). That would give, as mentioned above, approximately a correction of $2.\sqrt{2.60} = 170$ ohms, or a conductivity by diverging current lines of 1:170. Therefore $\frac{1}{21} = \frac{1}{170} + \frac{1}{R^1}$. Therefore R^1 is about 23 instead of 21 ohms, and the remainder is now larger; namely, 8 ohms.

If a fault has made a discontinuity in the ore, the resistance of the separating limestone would be $\sigma x^1 : hy$, where h is the average height of the ore, y the lateral dimension of the ore and x the unknown thickness of the limestone fault separating two parts of the ore. This gives for x' about 1 m.; but considering also the conductivity by diverging current lines, x' could be larger to a maximum of 3 m. The orebody must therefore have an average height of 1.5 m. and extend from $O1$ to $O2$ with only a very little interruption of at most 3 meters.

Several years after these experiments and deductions, the mining work had advanced so far that the management of the mine was able to see that our conclusions were correct. There were only a few small faults observed in the ore between $O1$ and $O2$, which divided the ore, but not continuously, by about 1 to 2 m. of limestone. The whole orebody, during the experiments in 1923, had a surface of about 250,000 sq. m., as has been proved by subsequent mining operations.

Determination of the Surface Area of Ore Layers

There were other outcrops of ore (see Fig. 3) in the mountains—$O2$ $O5$, $O6$, and some ore $O4$ observed in another part of the new mine. The question was whether these orebodies were connected with the principal one, and if not, how large was each separate orebody. Outcrop $O5$ to $O2$: 550 ohms, after much rain 540Ω (±10Ω). The resistance for the electrodes to the ore was the same at $O2$ and $O5$, that is 42 each. R_o to $O2$ = 16; R_o to $O5$ = 69. There remained also about 375Ω to explain. That value is much too high for an ore connection between $O2$ and $O5$.[2]

The resistance of 375 is the resistance of the medium between the O ore as one electrode and the ore at $O5$ as another. It can be divided approximately into the resistance of current lines from the ore in limestone near $O2 = x$ and the resistance near $O5 = y$. Therefore $375 = x + y$. A second set of observations gives the resistance between a point on the surface (soil), on which an electrode was put, and (1) the outcrop $O2$, with the other electrode or (2) $O5$. The ores $O2$ and $O5$ were in the mountains, bedded in limestone, and about 400 m. under the surface. Their center distance was about 1000 m.

1. Point Al to $O2$ gives 250Ω. After subtracting the resistance of the cables 32 + 16 and the electrode on the ore resistance = 42, there remained $R_{Al} + x = 160$, where x is the resistance caused by the divergence of the current lines out of the ore $O2$ in the limestone and R_{Al} is the resistance of the electrode (a short water tube) in the soil.

Point $Al - O5 = 370$. There remains $R_{Al} + y = 227$. Therefore $y - x = 227 - 160 = 67$ ohms.

2. Point G_o to $O2$ = 250. There remains $R_{go} + x = 146$.

Point G_o to $O5$ = 470. There remains $R_{go} + y = 313$. The difference $y - x$ is here = 167 ohms.

3. Point Wa to $O2$ = 190. Res.: $R_{wa} + x = 127$.

Point Wa to $O5$ = 350. Res.: $R_{wa} + y = 237$. The difference $y - x = 110$ ohms.

4. Point La to $O2$ = 235. Res.: $R_{La} + x = 141$.

Point La to $O5$ = 410. Res.: $R_{La} + y = 273$. Therefore $y - x =$ 132 ohms.

5a. Point K to $O2$ = 450. Res.: $R_k + x = 300$.

5*b*. Point *K* to *O*5 = 800. Res.: $R_k + y = 597$. Difference $y - x = 297$.

For the preliminary calculation we take the middle of $y - x$ for cases 1, 2, 3, 4.

Therefore $(67 + 167 + 110 + 132): 4 = 119$.

With $y + x = 375$, this gives $y = 247$, $x = 128$.

Now the topographical conditions (see Fig. 4) must be examined for a thorough explanation of the differences of $y - x$ in cases 1, 2, 3, 4. Point *Al* is situated above near the center of the orebody *O*2. Therefore only the current lines of the upper part, half of the conductivity, are here observed. In case 1 the orebody of *O*5 can send current lines in all directions except to the south, where the line of outcrops gives the southern ore border. In case 2, the point G_o lies nearer *O*2 than to *O*5, but the resistance value for *O*2 is also larger than in paragraph 1, while the current lines of the underlying surface of the ore *O*2, contribute not so much as the lines of the lower surface of the short ore *O*5. In cases 3 and 4, the value for *O*2 is approximately the same as that first given in this section. In paragraph 4, the upper current lines of *O*5 are working comparatively less than those of *O*2.

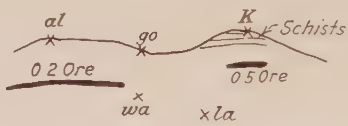

FIG. 4.

The value in case 5 must be considered separately. Between point *K* and the ore schists with a very high specific resistivity are lying uppermost perpendicular to the cleavage (about 1,200,000 ohms). This would first change the value of the res. 5*b* for point *K* to *O*5, which therefore is much too large. But the current lines from *O*2 to *K* must pass the cleavage planes at a greater angle, therefore this resistance is also too high.

If we take the average of two best values, according to the first observations recorded, we have $(110 + 132):2 = 121$, about the same as before. y would be about 235 and x, calculated by the best value (paragraph 4), about 120.

If we calculate the resistance for the different points, which consists of electrode-soil resistance and the soil-limestone resistance, we find, point $Al = (0 + 33):2 = 17$; point *Go* $(86 + 19):2 = 52$; point $La = (14 + 46): 2 = 30$; point *Wa* = about 5 ohms.

These differences are easy to explain. Point *Wa* consists of a net of water tubes in good contact with limestone and soil. During these later experiments it was separated from the ore *O*2. Point *Al* is a short water tube in good electrical connection with the limestone by water and by a

spring. Point *La* is a little mountain river, in which a long iron wire was laid as electrode. Point *Go* is a long iron wire in the humus soil. The resistances between the different points were also observed directly. Point *Al* – *Wa* = 60 ohms. Subtracting that of cables, 26, instead of 17 + 5 = 22 there remain:

Point *Al* – *La*:90; res. 42, instead of 17 + 30 = 47
Point *La* – *Wa*:50; res. 32 instead of 30 + 5 = 35
Point *Go* – *Wa*:110; res. 62 instead of 52 + 5 = 57
The error is about ± 10 per cent.

Outcrop O6

Point *La* to *O6*: 525 ohms, subtracted for cables: 76. The resistance of the electrode on the ore is 42, there remains the res. 407. For point *La* the resistance is 30. Therefore remains for *O6*:377 ohms. Point *Go* to *O6* gives 530; subtract for cables and electrode on the ore at *O6* together 149, res. 381.

Ore in *O2* to ore *O6* = 670; subtracted for the two electrodes on the ore in *O2* and in *O6*: 2 × 42 = 84, for the cables 76; res. 519. There was therefore no electrical connection between the two ores. Subtracting for *O2* the resistance 120, there remain 399 ohms for *O6*. The average value, taking into consideration the influence of topography on the current lines, is about 380 ohms for *O6* in limestone.

Outcrop O3

This outcrop has to *O2* the resistance 830; subtracting 136 for cables and resistances on the ore, there remains 694. Therefore there was no connection between *O2* and *O3*. The latter has the resistance of 694 – 120 = 574 ohms.

Outcrop O4

This outcrop, separated by a great overthrust fault from the principal orebody *O2*, has to *O2*, the resistance 670. Deducting resistance of cables etc., there remains for *O1* alone 410. It has therefore no connection with *O2*.

Calculation of Surface Area of Flat Orebodies

These calculated resistances for the orebodies give a possibility of calculating approximately the surface area of the layers as explained earlier in the paper. It must first be determined whether (1) only one surface or (2) two or (3) the nth part of one or of two can send current lines into the surrounding medium. The constants C_1 and C_2 are then 1, 0.5, or $n \times 0.5$, or $n \times 1$. The mountain slope always cuts limestone and

ore through a great part of the circumference; *e. g.*, for ore $O1$–$O2$, for 0.40 of the circumference of the orebody. Therefore the resistance of ore $O1$–$O2$ would be lower if the ore were totally surrounded by limestone: $R' = 0.6.R$, where R is the value given afterwards. Taking account of this consideration, of equations 2 and 3 and of the resistances of the orebodies themselves, we have for the resistance value deduced for only one surface of the ore sending current lines in all directions in an infinite medium for the ore $O1$–$O2$, approximately 70 ohms ($\pm$ 25 per cent.), and 35 ohms for two surfaces sending current lines. The ore $O5$ gives about 200 ohms for one, and 100 for two surfaces; ore $O6$, 500 for one and 250 for two; ore $O3$, about 700 for one and 350 for two.

For ore $O4$, the evaluation is not easy. The ore is near $O2$. The border contact is very good, on account of tectonic pressure of a great overthrust. It is not possible to calculate directly the radius of an orebody by formulas, when every point of the ore surface is not in good contact with the limestone. The slight temperature changes in the year during millions of years since Jurassic time, the epigenetic metamorphism of the ore and great tectonic pressures, have partly separated the ore from the limestone. The cavities 0.1 to 5 mm. thick are not filled with water. Observation and calculation shows that for 100 parts of the area of the border plane between ore and limestone only about 5 to 10 parts have a good contact. This average value is sufficiently constant for a larger surface. The resistance of the limestone far from the surface and from the mine is also not known exactly; it would be larger than 400,000; perhaps 700,000 ohms per centimeter.

The surfaces f of the orebody are proportional to the square of the radius or of the diameter or, according to the formulas 1 or 2, inversely to the square of the resistance. We put f of the ore $O2 = 100$. In square meters, it was about 250,000. For ore $O5$, $f = 12$; for ore $O6 = 2$; for ore $O3 = 1$; for ore $O4$, the surface is < 1. These last data have a possible error of $\pm$ 50 per cent. The thickness of the ore layer, which also decides the possibility and probability of exploiting the different orebodies, is not given by this method. But the thickness is known at the outcrops and varies very little and very slowly. Therefore the approximate determinations of the surfaces given were sufficient to decide the question as to which orebody was to be mined or whether other orebodies not mentioned here should be utilized.

Detection of a Water-bearing Fault in a Rock-salt Mine by Resistance Measurements

In a gallery (see Fig. 5) of a rock-salt mine in Germany, one corner was suspected to be near a water-bearing fault. Such faults occur there sometimes in connection with Tertiary layers of quicksands full of water, some

hundreds of meters higher than the rock salt. Therefore by piercing through the wall of the gallery, which is 600 m. under the surface, into the fault, the whole mine might be drowned in a few hours or days. The task was to detect by geophysical methods whether there was really a water-carrying fault in the neighborhood. Several methods were applied.

The radioactivity of the rock salt in the corner at C was nearly double that elsewhere in the gallery.[13] The temperature of the salt in boreholes 10 m. long, in the corner, probably on account of water evaporation through fissures, was about 1.6° C. lower than in other parts and corners of the gallery. The equipotential lines of Schlumberger[14] are rotated to come parallel to the strike of the water-bearing fault, more and more as point C is approached. The salt in this gallery is not pure.

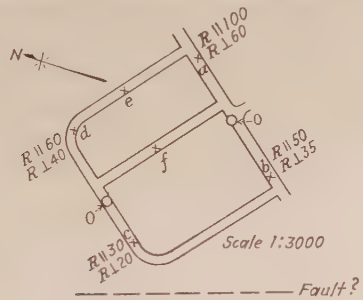

FIG. 5.

The resistance of the rock salt was determined. The resistance of rock salt is high, therefore conduction on the surface might give serious errors in the result. To avoid that, the surface was cleaned with a broom and by a compressed-air current. The electrodes were put in fresh hewn niches of the salt. If the electrodes are not very far from one another, surface conduction increases more with center distance, as room conduction does. This gives a criterion to distinguish the two kinds of conduction. The surface conduction can be neutralized by a ring circle of

[13] See *Ztsch. f. Prak. Geol.* (1926) **34,** 151.

[14] It is interesting to note that the differences of potential (method of Schlumberger; see pages 180 and 199), and not only the magnetic field of the current, could be observed with an amplifier in the poorly conducting rock salt 606 m. under the surface, when an alternating current of lower frequency was sent into the earth by means of two electrodes applied on the earth surface at about 4 km. away from each other. The minimum was not very sharp and has ±15° uncertainty for the direction of potential lines, which demonstrates the influence of secondary capacity and inductive effects. But the principal effect of about 70 per cent. was nevertheless that of the direct current. When the two so-called "tübbinge" (two iron ring cylinders forming two shafts and going from a little below the surface down to about 500 m. deep) were used as one electrode, and when the other electrode was some thousand meters away on the earth surface, the effect was stronger and the angle of the not very sharp minimum of equipotential lines was less, only about ±5°, so that the effect was almost given by the primary current.

the kind that William Thomson first employed in electrostatics, or can be eliminated by a method analogous to the Thomson bridge. But the observation in the purest rock salt of highest resistivity where the surface conduction could lower the resistivity by 100 per cent. or more, has not so much practical interest for the given problem. It is only rock salt with low resistivity that matters; there the surface conduction has very little influence compared to the inner conductivity.

The rock salt in the mine in the neighborhood of the presumed fault has an anisotropy of conduction. The lower resistivity was in a plane parallel to the line west to northwest. This direction is nearly perpendicular to the strike of the fault. A second line of the fissure plane was not ascertained; perhaps the plane is vertical. These planes are capillary fissures with old, now inclosed, infiltrations of water from the fault in the rock salt. This water is now a concentrated solution of salt. The average diameter of the electrodes was 22.4 cm. ($a = 11.2$). The total resistance between point a and point e was 3900 Ω; between point d and point e, 2300 Ω; between point a and point f, 1900 Ω. The errors are about ± 10 per cent. These results demonstrate that the surface conduction has no notable influence. The way along the surface from point e to point f is about four times longer than from point d to point e, but the resistance is lower, because the direct way, point e to point f, lies more in the direction of better conduction, which is vertical to the strike of the fault, than the way point d to point e. The data given in Fig. 5 are relative. The resistance in point a in the direction of the higher resistivity and of the current lines, nearly parallel to the strike of the fault, is put equal to 100. It is about 410,000 ohms per cubic centimeter.

In a second gallery in rock salt, at a depth of 718 m., the salt was much purer; therefore surface conduction and imperfect isolation of cables, of Wheatstone bridge and observer from the wet salt soil seriously affected some measurements. That is shown by the following data. There were points 1, 2 and 4 in niches of the wall with dry surfaces. The values were 1 to 2 = 378,000 ohms; 1 to 4 = 450,000; 2 to 4 = 360,000. The distance l was about 40 m. to 60 m., the radius of the electrodes only 11 cm.; the correction for l could therefore be neglected. That gives $3.78.10^5 =$

$$\frac{\sigma_1}{4a} + \frac{\sigma_2}{4a};\ 4.5.10^5 = \frac{\sigma_1}{4a} + \frac{\sigma_4}{4a};\ 3.6.10^5 = \frac{\sigma_2}{4a} + \frac{\sigma_4}{4a};\ \text{or}$$

$$\sigma_1 = 10.3.10^6;\ \sigma_2 = 6.3.10^6;\ \sigma_4 = 10.10^6.$$

Point 3 was on the wall near the sinking shaft to the gallery, which was wet from water coming from above. The wall near 3 was also wet and 3 had electrical connection with the soil, which is always a little wet and dirty. The resistance 1 to 3 was only = 28,000 ohms; 3 to 4 = 32,000 ohms, which would give with the values of σ_1 and σ_4 for $\sigma_3 =$

$-9(10)^6$; but this negative value for σ is impossible and demonstrates the effect of surface conduction.

Point 4 was situated below point C in the 606-m. level, which there has low resistance, high radioactivity, etc., and is situated near a water-bearing fault. But the specific resistivity at point 4 is about the same as near points 1 and 2. This and the optical purity of the salt indicate that the water-bearing fault was and is not continuous down to the 718-m. level, but was closed in the plastic salt between 606 and 718 meters.

Changes in the Depth of Ground Water (Water Table) Given by Resistance Observations

The influence of the specific resistance of different conducting layers on the whole resistance between two electrodes on the surface can be calculated theoretically and proved by experiments. A very good method was studied thoroughly in field experiments of Gish and Rooney. Another method was used several years ago by H. Katz on the advice of the writer and is given here as an illustration of the practical application of resistance observation, but not as the best method for this purpose.

Iron garden fences of 2 by 2 m. used as electrodes were embedded in the earth about 10 to 30 cm. deep, with salt and slag, and were moistened. The quantity of the salt affects the value of the resistance so that it is different for equal electrode diameters. The diffusion of the salt going on with time changes also the resistance and makes it finally increase. It would perhaps be better to take a mixture of salt (NaCl) and of gypsum. An approximate formula gives the observed resistance $R =$

$$\frac{1}{-4k}\left[\left(\frac{1}{a_1}+\frac{1}{a_2}-\frac{1.7}{l}-\frac{1.7}{\sqrt{2h^2+l^2}}-\frac{k_2-k_1}{k_2+k_1}\left(\frac{1}{2d}+\frac{1}{2\sqrt{l^2+2d^2}}\right)\right]\right.$$

in which a_1 and a_2 are the radii of the electrodes,[15] l their center distance, h their depth under the surface, d the distance of the ground water from the electrodes, k_1 the conductivity of the upper layer, k_2 that of layer with ground water. The relative resistances are given in Table 1.

Table 1.—*Resistances for Different Depths of Ground Water*

Depth of Ground Water Meters	Centre Distance of Electrodes, Meters				
	12	10	5	3	2
	Observed Relative Resistances, R				
2.2	100	100	100	100	100
1.9	97	95	100	98	97
1.8	83	81	82	80	82
1.5	66	65	73	71	69

[15] For a quadratic with side C, a = C:1.4.

The resistance for a depth of ground water of 2.2 m. was put equal to 100 for each center distance. The influence of the depth of the ground water is clearly shown in the table.

Change of Resistivity in a Coal Containing Gas under High Pressure

In the coal mines near Waldenburg (Lower Silesia, Germany) there are seams noted for dangerous sudden eruptions of CO_2.[16] The number of these so-called CO_2 seams is now increasing while the mining operations are proceeding to greater depth. Observations of mining engineers and geologists indicate that it is probable that the CO_2 is the pneumatolytic result of recent basaltic eruptions. With the aid of the Commission for the Study of CO_2 Eruptions in Lower Silesia, I measured some resistivities there in 1923. The resistivity (spec. resistance) of the rocks, sandstone etc., is of the order of 2.10^4 to 8.10^4 ohms. A solid coal seam has $1.6.10^4$ per cu. cm.; a seam with fissures, $5.7.10^4$; another seam 5.10^5. But the CO_2 seams have given unexpectedly high values, greater than 1.10^7.

The resistivity of different coals of the same locality, which have laid several months in the dry air of the laboratory, was[17] about 10^7 to 10^8. Made wet inwardly by pure drinking water and the water on the surface removed, the resistance of the same coal decreased to 1.10^4 – 4.10^4. In the laboratory there was no systematic difference in dry or wet state in the resistance between the coal of a CO_2 seam and another.

The increase of resistance of the CO_2 coal seam may be explained in the following way. The coal surrounded by or developing gases under high pressures, according to the physicochemical law of mass reaction, was slowly freed from water by these gases.

The adsorbed gas takes the place of water, but as it can not dissolve and ionize any salt in noticeable quantity, it remains a nonconductor like the coal itself. Therefore the CO_2 seam has a resistance much higher than other seams, particularly in the interior. On the surface the coal is discharging the gas and absorbing moisture in its place, and shows therefore initial conductivity.

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[16] A valuable publication on this topic appeared in *Ztsch.* für das Berg-Hütten-und Salinenwesen im Preussischen Staat. (1927) **75.**

[17] Anthracite coal of the other localities has, wet and dry, sometimes a noticeable conductivity, perhaps effected by graphite elements which are demonstrated by Röntgen interferences. Brown coal of other localities might sometimes be an insulator and also when in place in the mine, when it contains fat and resin. H. Reich (*op. cit.*) has also found that coal of the Wealden (lower Cretaceous) in a dry state is an insulator, but when wet has much lower resistance. Tertiary brown coal of Lausitz when wet also has low resistance.

There was no occasion for more extensive and more accurate experiments in coal mines. It would be not improbable that coal seams with methane of high pressure could be detected in the same way by their much higher resistance through qualitative observation. A simple resistance-measuring device could be used inclosed in a tight box, with the galvanometer on the box, an instrument similar to that used for measuring cables of higher resistance with stationary or alternating current of a few volts. All sparks must be avoided. As electrode there could be used a half-open box or pieces of iron fénce of a constant diameter with salted clay for contact substance. The larger the diameter and the center distance of the electrodes, the farther the current lines reach into the interior of the coal seam. The surface conduction can be avoided by cleaning the coal with a broom or with air current. The electrodes can be applied to opposite surfaces of the coal seam so that the current must go through a larger coal volume. The electrodes should be not too near to the adjacent siliceous rocks, which mostly are moist and therefore have good conductivity, because the affinity of siliceous compounds is much greater for water than for carbonic acid or methane, as is proved by the behavior of the zeolites. The coal behaves otherwise.

Certain Aspects of Magnetic Surveying

By L. B. Slichter,* Madison, Wis.

(New York Meeting, February, 1928)

It has been estimated that rock exposures in most mining districts aggregate less than 1 per cent. of the total surface area.[1] Conclusions concerning the hidden 99 per cent. necessarily have been based on knowledge of only a small fraction of the surface, and from this limited information the geologist has been obliged to deduce the approximate and general nature of the underlying formations. In the search for ore it is necessary to supplement this general picture with greater detail. Fairly accurate knowledge of relatively small zones is required. This knowledge is carefully sought in many different ways—detailed interpretations of geological history, the tracing of float, the use of test pits, of trenching, of drilling, and of other methods. Practical ingenuity, scientific brilliance, great perseverance and patience, and the expenditure of large sums of money have all been a part of this search. The problem of exploring beneath cover with economy, and of obtaining essential information which this cover hides, is obviously a difficult one. It is also a very important one, for geologists inform us that already the story of the 1 per cent. exposed outcrops has been largely read.[2] There remains the nearly virgin field comprised in the hidden 99 per cent., and the importance of this zone is bound to increase continually.

In recent years, the problems connected with the exploration of this "99 per cent. zone" have attracted a constantly enlarging group of investigators and at present greater efforts than ever before are being made to resolve them. This work, in its various special aspects, has enlisted, beside geologists, the cooperative efforts of chemists, physicists, and mathematicians. The progress made in this development should be kept before those directing explorations in order that due advantage may be taken of a changing situation.

My subject concerns certain aspects of one of the older physical methods of exploration, the magnetic method. This method is familiar in principle to most of us, and because of its age, is especially suitable for

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* Physical Exploration Corpn.

[1] W. O. Hotchkiss: Magnetic Methods for Exploration and Geologic Work. *Trans.* (1923) **69**, 36.

[2] J. F. McClelland: Prospecting Development and Exploitation of Mineral Deposits. Peele's *Handbook*, 2d Ed., 407.

illustrating present trends and progress. It lacks, perhaps, the somewhat dramatic interest which has been aroused by the newer electrical methods, but this too is an advantage, as conducive to a saner picture of the situation.

Surveys of local variations in the earth's magnetic field for the purpose of locating magnetic ores have long been made. The simple pocket dip needle, which is really nothing but a compass mounted on a horizontal axis, is a standard field instrument which has proved of great utility in many localities. The dip-needle surveys of the Lake Superior iron district are examples of such work done on a large scale.

The aspect of magnetic explorations has been changed, however, in recent years by the introduction of reliable field instruments having an enormously greater sensitivity than those formerly available for practical work. At least tenfold to fifteenfold the sensitivity of the pocket dip needle is now obtainable in practice, and one may readily perceive that this large factor has greatly expanded the possible scope of explorations by this method. Changes in the earth's magnetic field of the order of only 5 parts in 10,000 are readily measured by present methods. This high sensitivity is significant and the magnetic method is capable of detecting a smaller amount of the appropriate material at a greater distance than any indirect method now existing. Not only this, but, unlike the dip needle, the newer magnetic balances furnish reliable information about both the direction and intensity of the magnetic distortion. For this reason interpretations of results are greatly facilitated.

Magnetic Susceptibilities of Rocks

With a dip needle, or with any type of magnetic balance, one measures the differences in the earth's magnetic field which exist between various points in the area of the survey. The earth's normal field is uniform over the small areas usually comprised in a survey for ores, but when differences exist in the magnetic character of the underlying rocks this field is distorted by an amount which depends, among other things, upon a factor called the magnetic susceptibility of the rock. The meaning of this term and an idea of its basic role in explaining geological structure in terms of magnetic measurements may be gained from a simple example. Fig. 1 shows in section a vertical contact between two uniform rocks, A to the east and B to the west. Let us assume that rock A is more magnetic than B, and for the present purpose consider only the vertical component of the earth's field, shown by the arrows. A rapid change in the strength of the vertical component will be found in passing across the contact, with uniform higher values to the east, and uniform lower values to the west. This is shown by the curve on the figure, which represents values of the magnetic intensity along an east-west traverse. If the

susceptibility be small, as is the case for most rocks, it is found that the change in the vertical intensity which occurs in passing over such a contact is directly proportional to the difference in the magnetic susceptibilities of the two rocks. Magnetic susceptibility should be conceived, then, as a multiplier which expresses the amount by which a material is capable of boosting the magnetic field above the normal value. The way in which this boosting action takes place is familiar to all geologists. The rocks, in fact, are magnetized by the earth's field. The strength of the magnetization induced is found to be equal to a constant times the strength of the earth's field, and it is precisely this constant which is the magnetic susceptibility of the rock in question. Magnetic susceptibility is a physical constant of material which may be accurately

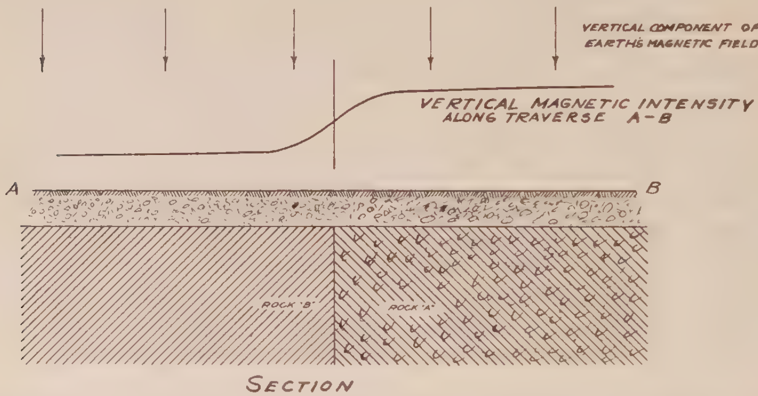

FIG. 1.—DIAGRAM SHOWING CHANGE IN VERTICAL INTENSITY OF EARTH'S MAGNETIC FIELD CAUSED BY CONTACT BETWEEN TWO KINDS OF ROCK.

measured in the laboratory, and it plays the same basic role in magnetic surveys as does electrical conductivity in electrical ones, or density in gravitational measurements. We are interested, then, first in what values of the magnetic susceptibility[3] may be assigned the various rock formations, and, later, how the combined factors of form, depth of cover, and of magnetic susceptibility influence the ability to recognize the characteristics of the particular hidden structure through magnetic measurements.

A simplifying generalization may well be noted at the outset. Often many rocks and minerals may be eliminated from consideration in a given district because they are too inert, magnetically speaking, to have noteworthy influence relative to other factors in the problem. Just what should be regarded as a minor influence will depend on the size

[3] Throughout this paper, "susceptibility" indicates the effect per unit volume rather than per unit mass.

of the effects being sought, but it is easy to see that rocks of susceptibility below a certain limiting value can never produce appreciable influence, no matter how abundant or how favorably placed. The detectability of a given rock formation is in general favored by the presence of large amounts of the rock, and of a sharp boundary between it and its neighbors. The most favorable possible situation is in fact shown by Fig. 1, for here the entire space to the east is filled by rock *A* and an abrupt transition occurs between rock *A* and rock *B*. If, then, the magnetic change which rock *A* produces is below the sensitivity of present instruments, this particular kind of rock may always be disregarded. Changes of about 1 part in 1000 may be considered the limit for useful interpretations in the field. A susceptibility of rock *A* equal to 0.00015 will just about produce this change, and we may thus rule out all rocks having a susceptibility less than this value. What rocks and rock materials are thus barred? Strictly speaking, we are without sufficient data to make a sweeping generalization on this question, especially in view of the fact that magnetism at times depends in a touchy manner upon relatively minor changes in the chemical composition of a substance. Nevertheless, in so far as measurements upon minerals are available, all (except some half dozen containing the ferric metals and manganese) appear to fall into the inert classification. The evidence for this conclusion is of two kinds—the statement accords with our knowledge of all the 500 odd inorganic compounds upon which measurements have been made, as prepared by the chemist in the pure state. In this list are rock minerals such as quartz, hydrophilite, fluorite, anhydrite, calcite, hematite, melanterite, bieberite, graphite, cuprite, tenorite, chalcocite, covellite, pyrolusite, and the native metals. None of these artificially prepared substances, except compounds of the ferric group and manganese, has appreciable susceptibility in our sense of the term, and it is reasonable to suppose that the more complex rock-forming minerals are similar.

In addition, measurements by Stutzer[4] and others on natural and therefore usually impure specimens of the substances listed below show

Natural Mineral and Rock Specimens Having a Susceptibility below 0.00015

Tourmaline	Ankerite	Sphalerite	Hematite
Beryl	Amphibolite	Galena	Pyrolusite
Chalk	Diorite	Iron pyrite	Augite
Dolomite	Greenstone	Marcasite	Anthracite
Apatite	Gypsum	Chalcopyrite	Rock salt
Hornfels	Basalt	Limonite	Blue slate
Hornblende schist			

[4] F. Stutzer, W. Gross and K. Borneman: Über magnetische Eigenshaften der Zinkblende und einiger anderer Mineralien. *Metall und Erz* (1918) **15**, 1.

them to be below our limiting value. Of course, the fact that these minerals have been measured as inert does not prevent other samples of the same mineral from showing appreciable activity, for in point of fact an impurity of only 0.1 per cent. magnetite is sufficient to produce easily observable effects.

The following minerals, all compounds of iron, are strongly magnetic: magnetite, franklinite, ilmenite, pyrrhotite, and to a less extent, specular hematite. As we shall see, magnetite has a susceptibility about 10,000 times the lowest detectable value of 0.00015, and is by far the most important of the magnetic minerals.

Between these few magnetic minerals and the large inert group there is a long list of eruptive rocks which show extreme variations from one specimen to the next. These variations are twofold to tenfold. The basic igneous rocks generally are distinctly more magnetic than the acid types. Since sufficiently exact mineral analyses of these specimens are almost always lacking, it is impossible to correlate definitely their magnetism with their mineral analysis, but the variations are usually due entirely to differences in magnetite content. As an illustration, two gabbros from northern Wisconsin measured by the writer were found to have a susceptibility of about 0.00043 and 0.00068. After pulverizing and separating with a Dings magnetic separator, the respective samples were found to contain 0.15 and 0.24 per cent. magnetite by volume, values which fairly well account for the observed susceptibility. It seems probable that all rocks and minerals, except the ferric group, derive whatever value they may have of the susceptibility in excess of 0.00015 from the presence of impurities from the ferric group, especially magnetite. Whether or not this is a correct hypothesis, field experience shows the advisability of assuming that the susceptibilities of the country rock are unknown, and cannot always be correctly obtained from experience with other similar samples from a different district. Careful tests are recommended in order to establish whether, in the first place, a given formation is sufficiently uniform magnetically to form a magnetic unit, and, secondly, to establish the proper values of the susceptibilities in the case.

Susceptibility of Magnetite

The value for the susceptibility of magnetite which we find applicable exceeds by many hundred per cent. the accepted ones of 0.04 to 0.097 commonly given by such writers on magnetic surveying as H. Haalck, R. Krahman, E. Pautsch, and R. Ambronn. In view of this large discrepancy in so fundamental a constant, a word of explanation is required. The writers mentioned cite for their data either the experi-

ments of Stutzer[5] or of Koeningsberger.[6] While Stutzer points out that his work is inaccurate for materials of so high a susceptibility as magnetite, due to the unusually large value of the end effect in his short samples, this is apparently not the chief cause of the discrepancy. The magnetic susceptibility of a material is by no means a fixed constant, but depends on the strength of the magnetic field used in the measurements, and on certain other factors. In magnetic surveys we are interested, of course, in the value of the susceptibility which applies when the material is acted upon by the weak magnetic field of the earth. Stutzer did most of his work at a field strength over 300 times that of the earth, but the measurements on magnetite were made at lesser fields, of value not stated. It seems probable that the high field strength, and the fact that his specimens were pulverized, are the chief causes of the discrepancy. Hotchkiss[7] in 1922 emphasized the need of magnetic data obtained at fields as weak as the earth's, and Koenigsberger in 1923 remarked that no such data appeared to exist in the case of magnetite. Haalck[8] in his valuable treatise on magnetic surveying also remarks that a change of field strength, and the pulverizing of the specimens, may influence the results of measurements, but he believes such errors small compared to the normal uncertainties in field conditions. There is much evidence, however, to show that the values so generally adopted are much too small for solid magnetite in weak fields.

P. Weiss,[9] in 1896, made classic studies upon magnetite crystals at fields nearly as weak as the earth's. From six samples from different localities he selected three uniform specimens, and cut from them rectangular prisms, with faces parallel to the binary, tertiary and quaternary axes, and measured the susceptibilities when the specimens were magnetized in the directions of these axes. Fig. 2 shows his results with three such prisms, each cut from the same crystalline unit obtained from Brozzo, northwest Italy. The values along the three axes are shown by the three curves, which are substantially the same, but the rapid manner in which the susceptibility increases as the field strength is decreased from 500 Gauss to 20 Gauss, is interesting. The earth's field is about 0.6 Gauss, or off the figure at the left. It is noted that a value of about 8 occurs at a field strength of 20 units, or about one-hundredfold the value 0.097 quoted. Fig. 3 is similar, and refers to specimens from the Tyrol.

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[5] F. Stutzer, W. Gross and K. Borneman: *Op. cit.*

[6] J. Koenigsberger: Fortschritte der Magnetischen und Gravitometrischen Aufschlubverfahren. *Glückauf* (1923) **59**, 992.

[7] W. O. Hotchkiss: *Op. cit.*

[8] H. Haalck: Sammlung Geophysikalischen Schriften 7. Berlin, 1927. Borntraeger.

[9] P. Weiss: Recherches sur l'aimantation de la magnétite cristallisée. *L'Éclairage Électrique* (1896) **7**, 487.

The curve for the tertiary axis was obtained from a specimen cut from a different crystal, but from the same deposit. The curves show a maximum value of about 10 at a field of 8 Gauss.

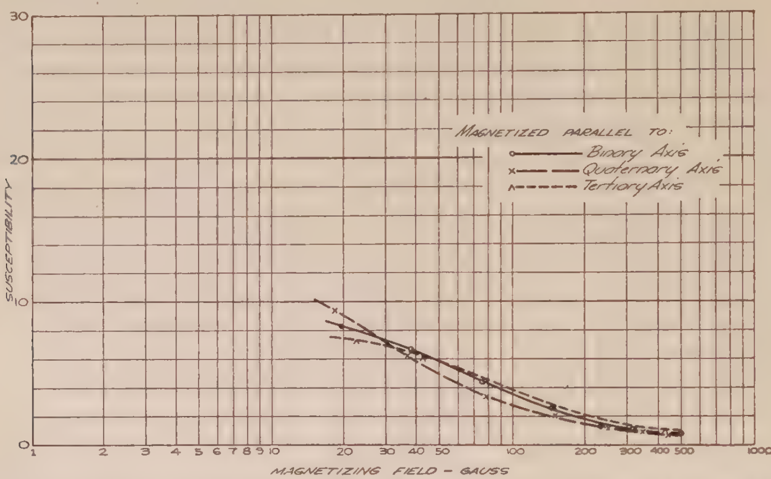

FIG. 2.—MAGNETIC SUSCEPTIBILITY OF MAGNETITE FROM BROZZO, N. W. ITALY, (AFTER P. WEISS).

Fig. 4 shows the high susceptibility of a specimen from Traverselle, northwest Italy, magnetized parallel to the binary axis. At a field strength of 3 Gauss, the susceptibility is about 15 times the value obtained

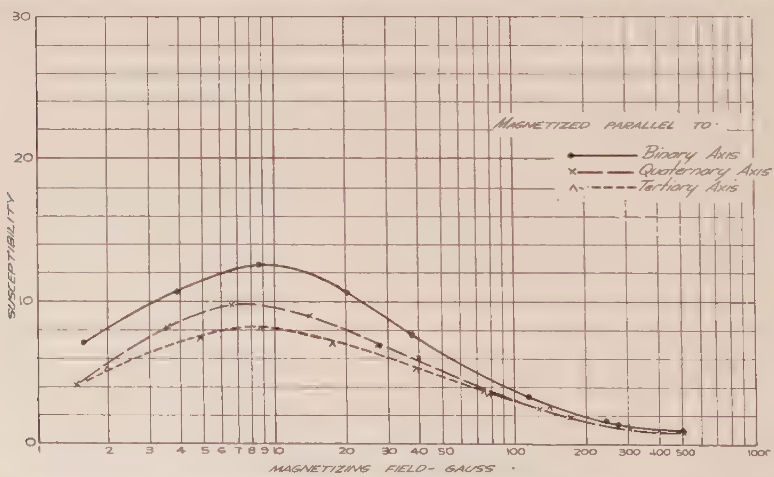

FIG. 3.—MAGNETIC SUSCEPTIBILITY OF MAGNETITE FROM TYROL (AFTER P. WEISS).

at 200 Gauss, and at 0.9 Gauss, which is only 50 per cent. greater than the earth's field, a value of 24 was obtained. This is about 250 times larger than the accepted value of 0.097 previously mentioned.

The work of C. P. Steinmetz,[10] by a different method, is corroborative of Weiss's results in so far as it goes. His results on two solid prisms are

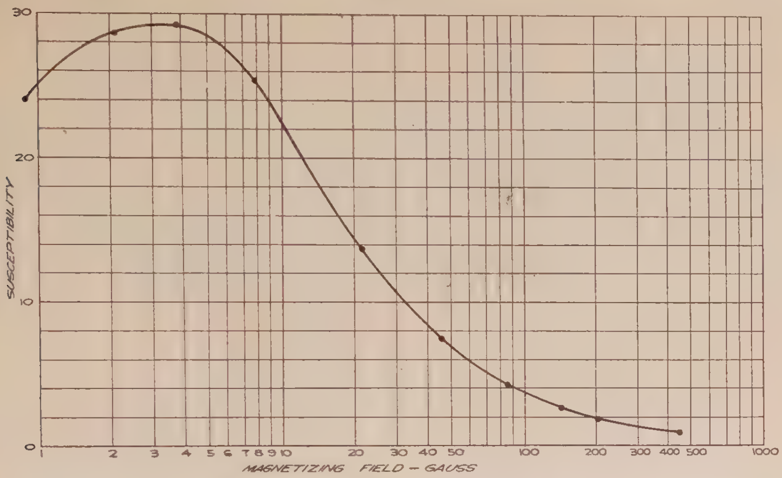

FIG. 4.—MAGNETIC SUSCEPTIBILITY OF MAGNETITE FROM TRAVERSELLE, N. W. ITALY (AFTER P. WEISS).

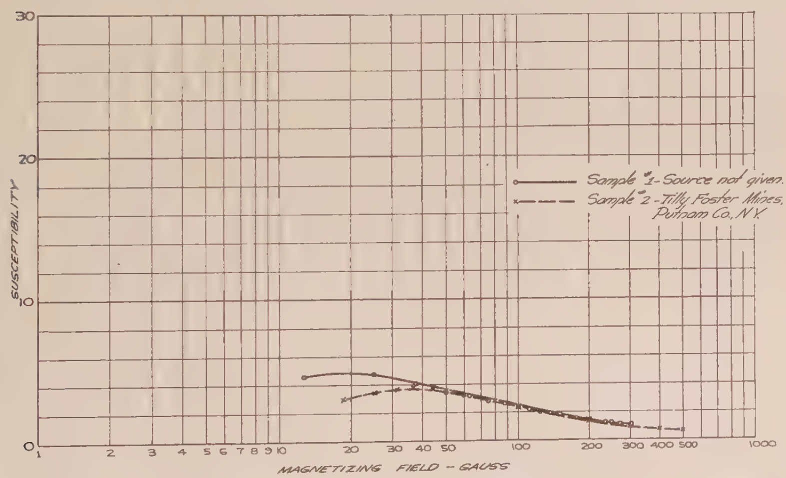

FIG. 5.—MAGNETIC SUSCEPTIBILITY OF MAGNETITE FROM PUTNAM COUNTY, N. Y. (AFTER C. P. STEINMETZ).

shown in Fig. 5, where it is seen that 15 Gauss is about the lower limit for the field strength in his work. A value of about four is here obtained, or 40 times that of Stutzer. Further confirming data on this subject were

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[10] C. P. Steinmetz: On the Law of Hysteresis. *Trans.* A. I. E. E. (1892) **9**, 671.

obtained by the interesting experiments of H. du Bois,[11] who determined the susceptibility of solid magnetite by a totally different method, through the rotation of the plane of polarization of a light wave reflected from his specimen. His value, as given in Smithsonian tables, is likewise large, and in general agreement with the previous authors. His minimum field, however, is 500 Gauss, where a value of 1.33 was obtained.

The sharp increase in susceptibility with decreasing field strengths which has been noted in the case of magnetite is also obtained with other crystals. For example, the recent measurements of Dussler and Gerlach[12] by essentially the same method as used by Weiss show a sharp decrease in susceptibility of iron crystals occurring at field strength of about 2 gauss. The work of Honda and Kaya[13] on iron crystals also confirms this general situation.

Effect of Pulverizing

Our measurements have been made in field strengths equal to the earth's and less, and like those of Stutzer, were made upon powdered specimens. For the rapid measurement of large numbers of samples the use of crushed specimens is economical and convenient. Moreover, by mixing the active substance with an inert material, a specimen approximating a disseminated occurrence of the mineral in nature may be readily obtained. However, the powdering of the specimen reduces the effective susceptibility by a large amount, as will be clear by reference to Fig. 6, which shows two curves, one for powdered iron, the other for crushed magnetite. The percentage of voids in these samples was changed by dilution with an inert powder, or by closely packing the crushed material. In the case of iron, the powder was a fine dust, but in order to reduce voids to a minimum, some of the magnetite mixtures were of assorted grain size. These mixtures are indicated by the points carrying flags in Fig. 6. The percentage of voids varies from 23 per cent. to practically 100 per cent., as shown by the horizontal scale, while the vertical scale shows the measured susceptibility. These susceptibilities are all reduced to the basis of a normal density; that is to say, allowance has been made for the voids so that were it not for other factors a horizontal line indicating a constant susceptibility would have been obtained. It is clear that both the materials become more highly magnetized, through the reduction of

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[11] H. du Bois: On Magnetization in Strong Fields at Different Temperatures. *Phil. Mag.* [5] (1890) **29**, 293.

[12] E. Dussler and W. Gerlach: Eisencinkristalle. *Ztsch. für Phys.* (1927) **44**, 279.

[13] K. Honda and S. Kaya: On the Magnetisation of Single Crystals of Iron. *Sci. Repts.* Tohoku Imp. Univ. [1] (1926) **15**, 721.

voids. This effect may be simply explained, on the basis that a slight decrease in the length of the relatively "high-resistance" air gaps results

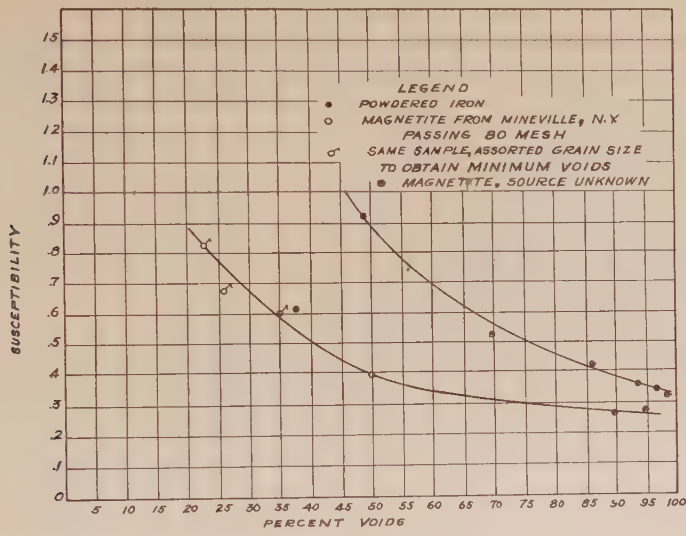

FIG. 6.—VARIATION OF MAGNETIC SUSCEPTIBILITY OF MAGNETITE AND IRON POWDERS WITH PERCENTAGE VOIDS IN FIELD OF 0.64 GAUSS.

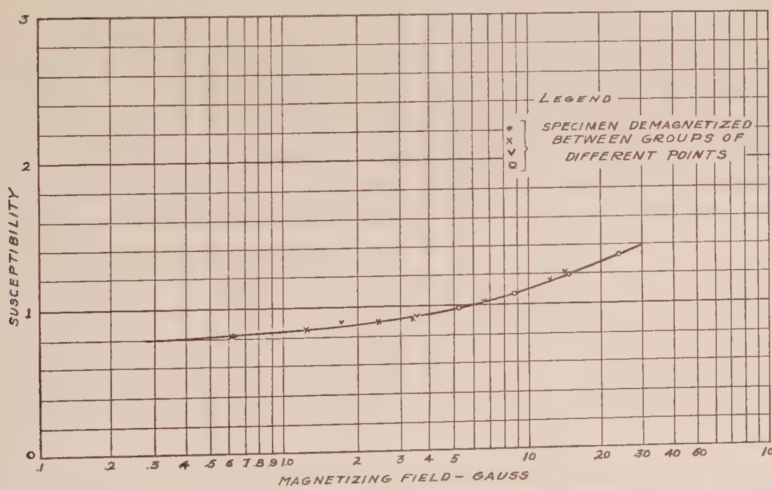

FIG. 7.—SUSCEPTIBILITY OF CRUSHED MAGNETITE FROM MINEVILLE, N. Y., AT WEAK FIELDS. VOIDS 23 PER CENT.

in a large increase in the magnetic conductivity of the specimens as a whole. The situation is quite analogous to the conduction of electric current between such particles. It is evident that close packing would

aid such a flow completely out of proportion to the increased density caused thereby.

The susceptibility for magnetite containing 23 per cent. voids is 0.82. Material reduction in voids below this figure is not practical, but it is clear from the course of the curve that one may expect solid uncrushed magnetite, corresponding to zero voids, to show a susceptibility of about 1.5, and possibly notably higher. These experiments, therefore, are considered in agreement with those of Weiss, and point to the explanation of his high results for solid specimens.

The curves showing the increase in susceptibility with density have been confirmed by magnetite and iron powders held in a matrix of paraffin. Our results on the other members of the ferric group measured at low field strengths also show higher values than those of Stutzer.

At low field strengths of between 0.3 and 25 Gauss, the change of susceptibility of crushed magnetite with field strength is somewhat different from that found by Weiss for solid specimens. The curve of Fig. 7 shows that the initial rise seen in Figs. 3 and 4 (after Weiss) is more gradual and that the peak (which we know must occur, because of the eventual saturation of the magnetite) is not obtained at field strengths less than 25 Gauss.

Since doing the above work the recent research of Welo and Baudisch[14] upon synthetic magnetite has come to our attention. Their specimens were of finely divided form, prepared by four different methods. These powders were measured at field strengths from about 10 Gauss up to nearly 1000 Gauss, and the values at low fields agree closely with those found by us. The susceptibilities in a field of about 10 Gauss varied between 0.23 and 0.63 for samples prepared by different methods. The authors explain the observed differences chiefly on the basis of differences in internal strains set up in the material by the methods of preparation. In fact, specimens showing the greatest difference were rendered strikingly similar as to magnetic characteristics, simply by annealing at 1000° C. *in vacuo*.

Their samples were highly diluted, containing, on the basis of a normal density of 5.05, between 64 and 79 per cent. voids. Nevertheless, despite this small range in density, a consistent trend of the susceptibility upward with decrease in voids was observed, in agreement with curves of Fig. 6. In addition, the variation in the position of maximum susceptibility, such as is exhibited by Weiss's results (Figs. 3 and 4) and indicated by our results, is discussed and explained on the basis of internal stresses. These experiments throw light on the causes of moderate differences in the magnetic susceptibilities which are customarily observed between different specimens of magnetite.

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[14] L. A. Welo and O. Baudisch: Studies on Precipitated Magnetite. *Phil. Mag.* [7] (1927) **3**, 396.

Apparently Excessive Observed Intensities

It is a common experience in the application of magnetic methods that higher responses are obtained than can be accounted for by the susceptibilities assigned. Various methods of explaining these observations have been advanced. In certain cases intense local anomalies seem clearly caused by lightning strokes and these may be easily recognized and disregarded. Haalck[15] is inclined to the opinion that the long geologic time during which the substances have been subjected to the earth's magnetizing field is a possible explanation. Welo[16] considers the possibility that the extremely high pressures obtained in the crust have produced higher susceptibilities. In testing this hypothesis he has shown that a specimen of magnetite, when subjected to a pressure of 1200 atmospheres and magnetized by a field of 30 Gauss, retains about 50 per cent. more permanent magnetism after removing from the field than is the case when similarly magnetized at atmospheric pressure. That higher magnetizations are often needed to account for field observations is unquestionable. Lasareff,[17] in discussing the great magnetic body near Kursk, Russia, says that this ore assumes a far higher magnetization than can be caused by the present earth's field, and for this reason a hypothesis was advanced of the existence of pure iron.

If, however, the magnetization is dependent on such uncertain factors as the pressure or stress conditions in the mineral, or on the geologic age of the magnetite, we are handicapped at the start by these additional complications. The postulated influence of pressure or age would certainly be difficult to evaluate *a priori*, and a great gain in simplicity, scope, and reliability of interpretations would result could we generally disregard hypotheses of this type, and base our analysis simply upon the magnetization which would be induced by the earth's field as it exists today, upon materials whose condition can be duplicated in the laboratory. We think it probable that the simple hypothesis of magnetization by the present field is adequate to explain responses which have been considered excessive, and that observed effects may often be satisfactorily accounted for by the larger values of magnetic susceptibility advocated in this paper.

Two magnetic bodies which produce anomalies so intense that explanation has been considered difficult are those at Krivnavaare, Sweden, and at Kursk, Russia. We do not know the complete evidence which may have been advanced to show that the magnetic response at these bodies is abnormal, and have been unable to obtain satisfactorily complete

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[15] H. Haalck: *Ibid.*, 113.

[16] L. A. Welo: Effect of Pressure on the Magnetisation of Magnetite. *Science* [N. Y.] (1926) **64**, 453.

[17] P. Lasareff: *Beiträge zur Geophys.* (1926) **15**, 101.

accounts of the arguments in each case.[18] Therefore we hesitate to discuss this question, but shall venture a suggestion, because the meager information which we have about these bodies does not appear abnormal. The permanent magnetization of these two deposits is listed by Haalck[19] as 0.78 and 0.7 respectively. This in itself is not disconcerting, for on the basis of a total intensity of 4.9 in Sweden, and of 4.5 at Kursk, the appparent susceptibilities[20] are 1.6 and 1.55 respectively, in agreement with each other and in accord with the higher determinations described in this paper. In each case, moreover, the direction of the magnetization is said[21] to coincide with that of the present earth's field, so that this circumstance is also consistent with the simple and desirable view.

The Kursk Anomaly

Complete magnetic maps of the Kursk anomaly have been published by Lasareff.[22] From his map we have taken the points plotted in Fig. 8,

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[18] Since writing the above, a paper by G. A. Gamburzeff and M. Polikarpoff has appeared. (Beitrage zur Frage nach der Ursache der Kursker magnetischen und gravimetrischen Anomalie. *Beitrage zur Geophysik* (1928) *19*, 219). In it the magnetization of the Kursk body is said to be *not* in the direction of the earth's field (H. Haalck, *Ibid.*, 43, 132), but in a direction dipping 65° to the east, or nearly the same as the dip (61°) of the vein. The original value of 0.7 is given for the intensity of the magnetization. On this basis, a good agreement is obtained between theory and the observations, provided a 40 per cent. increase in the dimensions of the ore section and depth of cover is assumed as an extrapolation from the drilled section.

[19] H. Haalck: *Ibid.*, 132.

[20] By "apparent susceptibility" is meant here simply the ratio of the permanent magnetization to the present existing earth's field. The estimation of the susceptibility of magnetite from the permanent magnetization of ore specimens is fraught with three uncertainties:

1. The value of the field which induced the original magnetization when the ore was laid down is not known.
2. The change in direction and intensity of magnetization of the ore during geologic time is problematical.
3. When diamond drill cores are used in sampling the influence of the steel drill bits may be appreciable.

Magnetite has an unusually high retentivity, and appears competent to maintain its magnetization throughout intervals even of geologic duration. Thus the physical and magnetic environment of a magnetite deposit during formation may be expected to find reflection in the present magnetic condition of the material. In this connection, the experiments of Honda and others on iron deposited from an electrolyte in a magnetic field are of interest. Under such circumstances, the retained magnetization is found substantially equal to the induced magnetization, and the iron is unusually retentive.

In view of the above considerations, the "apparent susceptibility" may well be far from the true value. However, it will tend in general to be *too low*, because the decrease in field strength in the interior of a magnetic body has been disregarded.

[21] H. Haalck: *Ibid.*, 43, 132.

[22] P. Lasareff: *Ibid.*, 80.

which represent values of the vertical magnetic intensity observed along a traverse crossing the vein at right angles, at a location where the disturbance is most pronounced. The orebody, as indicated by four drill holes, is shown in section below. The theoretical distortion of this ore computed on the basis of uniform magnetization,[23] and a value of 2.7 for the susceptibility, is represented by the solid curve. Here the contributions of both the vertical and horizontal components of the earth's field to the vertical anomaly are included, on the basis of a vertical and hori-

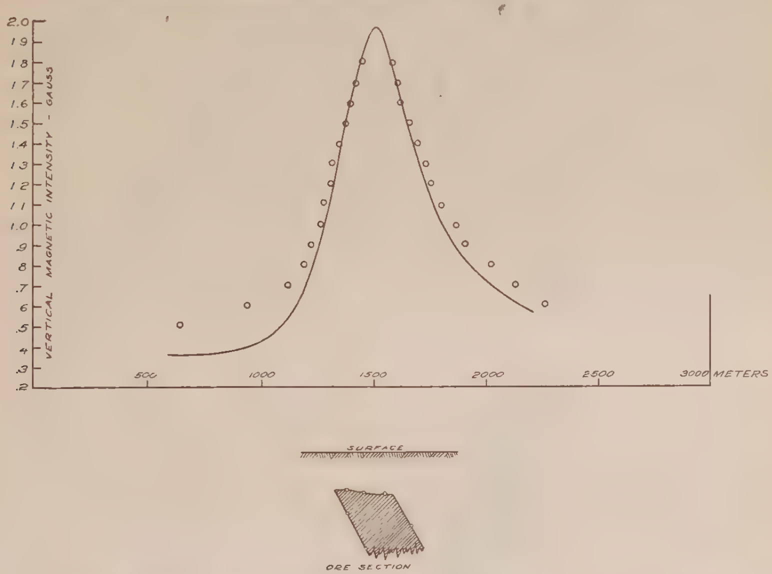

FIG. 8.—COMPARISON OF OBSERVED AND THEORETICAL MAGNETIC ANOMALIES AT KURSK, RUSSIA.

zontal component of the normal earth's field in this region of 0.4 and 0.2 gauss respectively. The curve is a good fit to the points, except at the lower portions, where the observed values are somewhat higher, perhaps due to the presence of disseminations of magnetite outside the limits of the orebody proper, as shown by the section. On the basis of this analysis a susceptibility of about 2.7 is competent to explain the vertical anomaly.[24] In the light of the susceptibilities experimentally determined and discussed in this paper, this value does not appear excessive.

[23] Because of the high susceptibility, a strictly uniform magnetization would not exist, but the discrepancy introduced by this assumption is small compared to those occasioned by irregularities in composition and shape of the ore.

[24] No consideration is given here to the observations on horizontal intensity, because the strike of the body is so nearly north that the component of the earth's field transverse to the vein is only about 18 per cent. of the vertical one. Thus small geometrical discrepancies and the inaccuracies in the analysis produce greater errors than are involved in considering the vertical component.

The value of 2.7 indicated is confirmed to some extent by measurements published by Lasareff[25] of the permanent magnetization of drill-core samples. Two drill holes near the section of Fig. 8 were sampled.

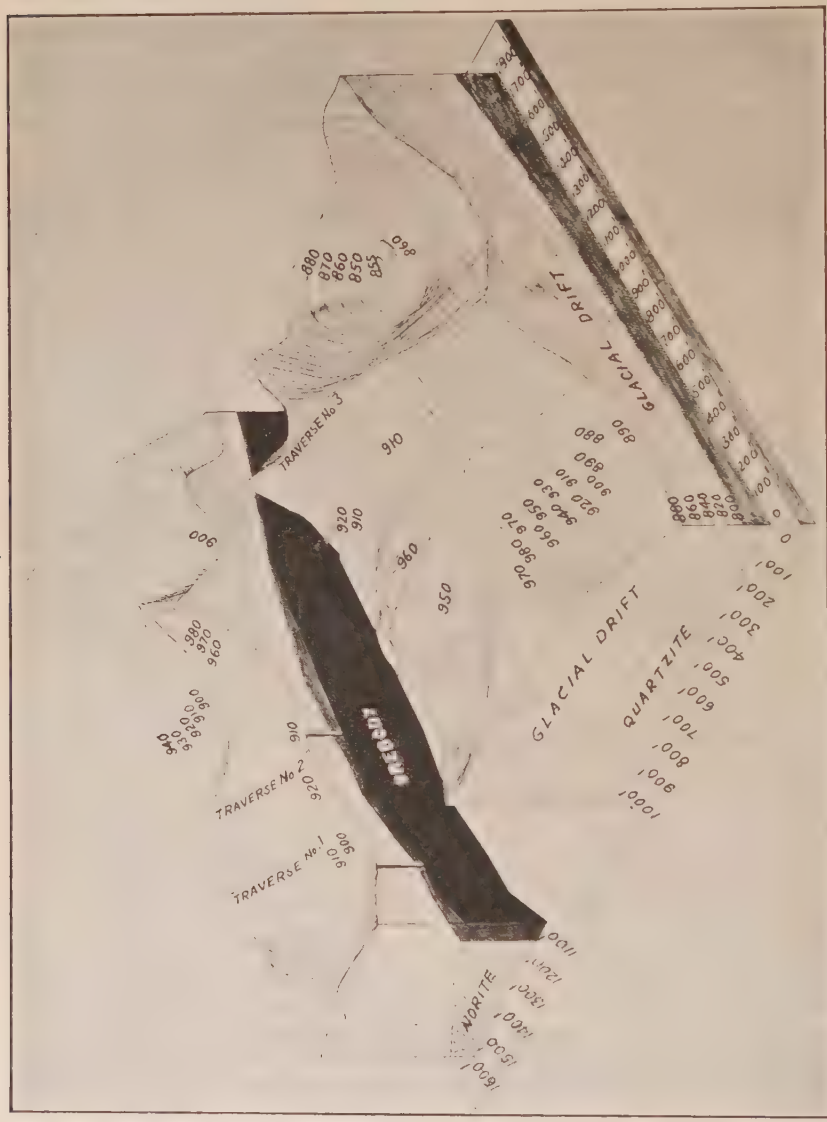

FIG. 9.—TOPOGRAPHY, GEOLOGY, AND LOCATION OF MAGNETIC TRAVERSES AT FALCONBRIDGE OREBODY, SUDBURY, ONTARIO.

Eight core samples were taken in one drill hole between depths of 500 and 1300 ft. and seven samples in the other between depths of 700 and 800 ft. The average magnetization of these 15 samples is 1.1, which corresponds

[25] P. Lasareff: *Ibid.*, 85.

to a susceptibility of 2.75, an unexpectedly close agreement with the previously calculated value. The highest susceptibility required for an individual sample is 8.5. This drill-hole evidence appears weak, however, because it is not known how much magnetization was possibly produced by the steel drill rods during drilling.

This brief consideration of the Krivnavaare and Kursk magnetite deposits indicates the possibility of explaining the magnetic anomalies at these ores in terms of the direct action of the earth's field, and on the basis of a susceptibility of between about 1.5 and 3.0. These values are in accord with the laboratory measurements on ordinary magnetite specimens which were discussed earlier in this paper.

Magnetic Response at Falconbridge Nickel Body

A similar need for higher values in the case of pyrrhotite is illustrated by our magnetic survey at the Falconbridge nickel body, near Sudbury, Ontario, Canada. Stutzer gives a value of 0.007 for pyrrhotite, whereas it appears that one of 0.125 is required.

The geological structure at this body is relatively simple; it is shown by the block diagram of Fig. 9. The orebody occurs as a vertical vein averaging about 35 per cent. pyrrhotite between norite to the north and quartzite to the south. The glacial gravel is pitted in places by deep kettleholes, but in other areas the surface is nearly level. The orebody has been well drilled, is of simple shape, is undisturbed by mining operations, and thus affords an unusually favorable opportunity for checking theory with practice. The analyses of the drill records were available to us and thus the magnitude of the response could be compared with the richness of the pyrrhotite shown by the drill. The location of three magnetic traverses across this vein is shown by the heavy dotted lines on the block diagram and the fourth traverse, which will be referred to, is located in level country some 2000 ft. to the east.

This body was readily detectable by the pocket dip needle, and it will be clear that a good qualitative agreement with the known conditions was obtained by the use of this simple instrument. In Fig. 10 are shown the results for four traverses, each of which crossed the ore nearly normally. The circles represent observed readings of the dip needle in degrees, which are plotted as profiles along the traverse. This dip needle had a sensitivity of about 200 gamma per degree deflection, as measured by a Helmholtz coil. Its normal position of rest was nearly horizontal; the normal earth's field dips at about 75° in this vicinity and it is sufficiently accurate, in view of the uncertainties in dip-needle measurements, to assume that the response is directly proportional to the changes in vertical intensity.

The theoretical response of this orebody due to a uniform vertical magnetization is shown by the solid curves in Fig. 10, which have been calculated in accordance with the equation

$$v = \frac{2kVt}{r} \sin \theta \qquad (1)$$

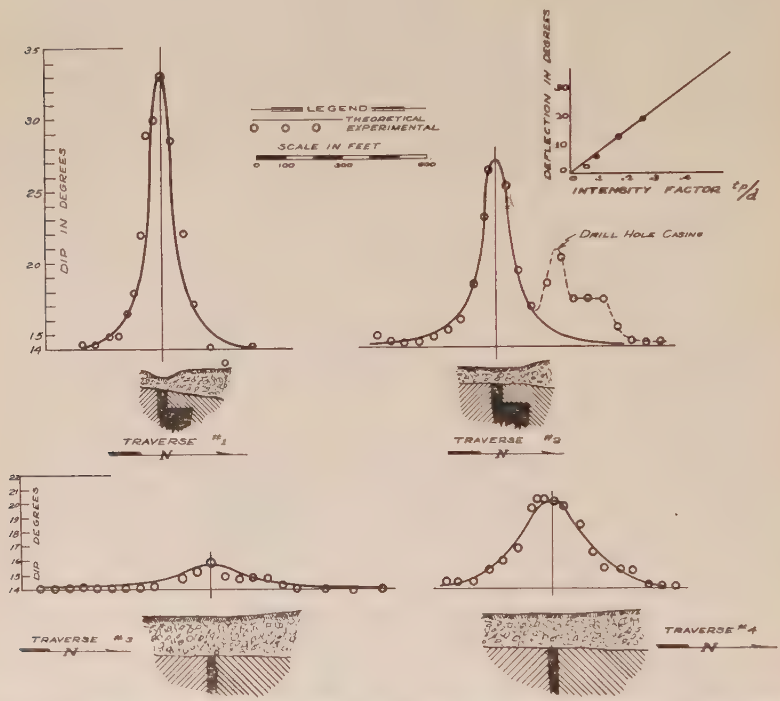

FIG. 10.—COMPARISON OF OBSERVED AND THEORETICAL MAGNETIC RESPONSE AT FALCONBRIDGE OREBODY.

where

- v = change in vertical field strength due to the vein
- V = normal value of vertical magnetic intensity
- k = susceptibility
- t = thickness of ore vein
- r = distance from top of vein to observing station
- θ = angle between r and the horizontal

It should be remarked that this expression disagrees with formulas recently published by H. Haalck[26] and based on similar assumptions. In Haalck's derivation an unjustified assumption concerning the symmetry was made whereby a formula containing an extra power of r in the denominator resulted. That is to say, his results show the variation of the intensity with depth of cover to depend on the inverse square law,

[26] H. Haalck: *Ibid.*, 56; *Ztsch. für Geophys.* (1926) **11**, 8.

whereas it seems clear that the correct result must be an inverse first power.

All four magnetic curves shown in Fig. 10 are drawn to the same scale and it is clear that there are large changes of both shape and intensity in the response curves. The shape of the curve for veins as thin as the present one is dependent only on the depth to the top of the ore, but the intensity depends on the richness of the ore and the vein thickness. For convenience in discussing these two factors of shape and intensity, we have first drawn theoretical response curves having the same maxima values as those measured in the field. These are the curves shown in Fig. 10. Clearly there is a very good agreement as to shape between the observed data shown by circles and the theoretical computations shown by the solid curve.

In accordance with equation I, the intensity of response should be proportional to the richness of the pyrrhotite multiplied by the thickness of the vein times the susceptibility of pyrrhotite and divided by the depth of cover. Table 1 lists all these factors except the susceptibility, as they exist under the four traverses, t representing vein thickness, d depth of cover, and P percentage of pyrrhotite. Here the percentage of pyrrhotite is taken from the geologist's records of the drill cores. To repeat, a linear relation is demanded by equation 1 between the factor tp/d of the last column and the maximum observed response. That this is actually the case may be seen from the graph shown in Fig. 10 which shows the peak deflections in degrees plotted against the factor tp/d. Here all the points except the first small value, which is subject to relatively large error, lie nearly on a straight line. This straight line thus confirms equation 1. The only factor remaining to evaluate is the susceptibility that must be assigned to the ore in order to account for the observed intensity. From the graph of Fig. 10, this value may be calculated readily and compared with laboratory measurements. The graph requires a susceptibility of 0.125, whereas measurements on pulverized specimens show values of 0.025 to 0.028, therefore we have to account for a discrepancy of about 500 per cent. The following are three plausible explanations: (1) A change in sensitivity of the dip needle between its use in the field and its laboratory calibration; a dip needle is not suited for measurements of absolute intensity, and is subject to changes in sensitivity from small causes; (2) the pulverizing of the specimens may reduce the effective susceptibility, in analogy to the effect pointed to in the case of magnetite; (3) the ore vein may possess a permanent magnetization in excess of the value which it would acquire from the present value of the terrestrial field. Pyrrhotite is exceedingly retentive of its magnetism, much more so than magnetite, and therefore, if subjected to strong magnetizing influences, would retain its magnetization for long periods.

Of these alternatives the most probable appears to be the reduced susceptibility observed when a sample is pulverized. Tests on pyrrhotite indicate a behavior in this respect analogous to the results observed with the iron and magnetite powders, and in view of the greatly higher results observed for solid magnetite, a factor of four or five between the powder and the solid is probable. The positive conclusion from

TABLE 1.—*Effect of Richness of Pyrrhotite on Deflection*

Traverse	Cover d, Feet	Vein Thickness t	Percentage of of Pyrrhotite p	Maximum Deflection, Degrees	tp/d
1	45	30	39	19	0.26
2	55	35	27	13	0.172
3	140	25	33	1.5	0.059
4	115	30	37	6	0.096

these experiments is that the position, depth of cover, form and relative richness of the vein are clearly indicated. The uncertainties concern the absolute richness of the ore and the vein thickness, and they persist chiefly because of the lack of data concerning the susceptibility and magnetization of the pyrrhotite. We hope to make further investigation of this question, and to repeat the field measurements with more accurate and sensitive magnetometers.

SURVEY IN SOUTHWEST WISCONSIN

The next example deals with the more usual case where ore was not found by the magnetic survey. The area covers about 500 acres in southwest Wisconsin, 1 mile long by ¾ mile wide. Readings of vertical intensity were taken at 300-ft. intervals along north-south traverses separated by 300 ft., and the results are plotted in Fig. 11 as profiles. These profiles, crosshatched in the figure, show a uniform increase from north to south, with a maximum difference of about 300 gamma, or 1.5° on the scale of the dip needle used at the Falconbridge nickel body. The profiles afford a consistent picture of the magnetic situation in this region. It is one of great magnetic regularity, for the difference between maximum and minimum values of the intensity is less than ½ per cent., and seems typical of this sedimentary district. The increase of intensity to the southwest seemed at first sight associated with the slight dip of the limestone beds to the south-southwest. This is not the explanation, however, for it would require a magnetite content in the limestone of at least 1 per cent. by weight, whereas it contains less than 0.1 per cent. The cause of the gradual variation is undoubtedly due to deep-lying igneous rocks. The survey illustrates the practical precision and economy of magnetic work in the field, for

the results consistently distinguish variations in the earth's field of 5 parts in 10,000 and only four field days with one man were required for the completion of this detailed survey. The absence of magnetic irregularities near the surface is evident, but deep-lying influences are indicated. These deep-lying influences could have been explored, were it advisable, by a survey on a larger scale, including large regions.

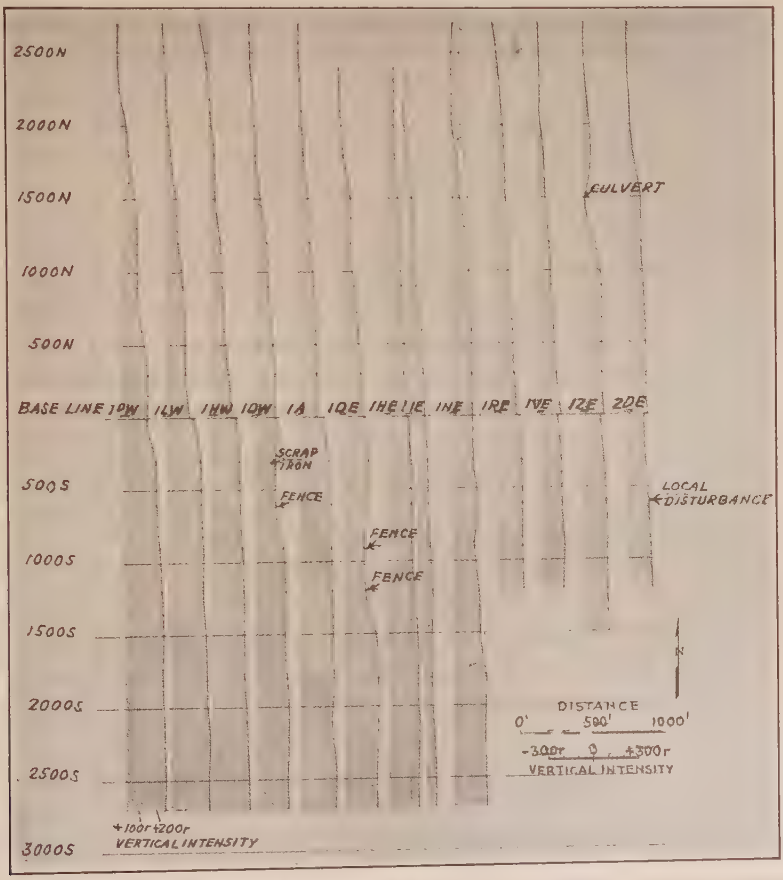

FIG. 11.—SMALL AND UNIFORM CHANGES IN VERTICAL MAGNETIC INTENSITY OBSERVED IN SOUTHWEST WISCONSIN DISTRICT.

INTERPRETATION AND APPLICATION OF MAGNETIC SURVEYS

Whenever formations present a reasonably uniform and definite magnetic structure, the interpretation of results is similar analytically to the interpretation of gravitational surveys by the Eötvös balance. The essential identity of gravitational and magnetic interpretations was early recognized by Baron Eötvös, but it will do no harm to emphasize this useful similarity. The similarity is particularly close, whenever

the structures in interest extend with little variation for long distances in a given direction, as was the case, for example, at the Kursk deposit and at the Falconbridge nickel body. The magnetic response in the case of such structures is easily formed through the addition of the familiar gradient and curvature terms of the Eötvös balance.

When it is known that structures are regular, magnetically speaking, a sound basis for the analysis of results is available. In many regions, however, the fact of greatest significance will be the irregular distribution of magnetite, often of such complexity that interpretations are impossible. The influence of minor amounts of magnetite is both the greatest strength and the greatest weakness of the magnetic method; for, on the one hand it enables formations to be traced because of small differences in their magnetite content, and on the other, it may produce so complicated a situation as to bar useful results. Little can be said *a priori* about the possibilities of a magnetic survey under complicated conditions. It should be noted, however, that combined with information gained from other methods, in particular the electrical methods, a magnetic survey will often acquire new significance. As Dr. Mason said,[27] it has been our practice to use magnetic methods whenever a physical survey is made, in accordance with the general principle of utilizing all the evidence, both geological and physical, which can economically be obtained. Additional information of value has often resulted, which in some cases served to corroborate and reinforce the findings of other methods, and which in other cases was unique.

Summary

In classifying rocks magnetically, the inert, highly magnetic, and intermediate types are briefly discussed, and the peculiar importance of a few of the ferric minerals, and especially magnetite, is explained. Experimental evidence shows that the susceptibility of magnetite changes greatly as the field strength is changed, and that it also depends greatly on the form in which the magnetite occurs. The hitherto accepted values appear far too low. For fields as weak as the earth's, a value of about 0.3 is obtained for disseminated magnetite, while for the solid material a value of 1.5 to 3.0 or higher appears correct.

Three instances are given of the fact that field observations of intensity are often much in excess of those which can well be accounted for by accepted values of the susceptibility. In each case the higher values here advocated appear adequate to account for the observed results – in two cases it is shown that there is a satisfactory quantitative agreement between observed magnetic data and the form and size of the known orebody which caused the magnetic response.

[27] Max Mason: Geophysical Exploration for Ores. See page 9.

DISCUSSION

M. C. LAKE, Duluth, Minn.—Have Mr. Slichter's researches explained the negative attraction that is often associated with magnetite deposits?

L. B. SLICHTER.—When a magnetized body lies in the earth's field, we find in its neighborhood certain regions of increased magnetic intensity, and others of decreased intensity. These zones of decreased intensity appear as "negative" attractions. Because of the fundamental nature of magnetism, they are always associated with the "positive" regions, but the exact position at which the lows and highs occur is dependent on the shape of the body, the nature and direction of its magnetization, and the value of the undisturbed earth's field. The picture is thus rather complex, and each case requires separate consideration. Sometimes (for example, in the Falconbridge magnetic profiles shown in my paper) a positive attraction only is obtained, but this should be regarded as the exception. The reason that no negative values were observed here is merely that they were confined to regions beneath the surface, where measurements are impossible. If, however, the dip of the Falconbridge ore were shallow instead of steep, we would obtain negative values. In other words, both positive and negative values are the normal expectation in magnetic work.

H. T. F. LUNDBERG, New York, N. Y.—I may have misunderstood Mr. Slichter, but there is one point that I would like to emphasize, and that is the importance of the earth's magnetic field and its direction as regards the axis of the orebody. In one instance in Northern Africa, the earth's magnetic field is almost parallel with the earth's surface and the long axis of the orebody is about north-south. We have there a maximum with north polarity on the north end and a very strong south, though negative, reaction on the south end, while at about the middle there is an absolutely neutral region. In other words, there is a zero line right to the middle.

H. A. BUEHLER, Rolla, Mo.—I think Mr. Lake has been rather modest in asking his question. He has been doing some magnetic work in Missouri on an iron orebody and has found some very beautiful magnetic highs and a magnetic low that would be just like a syncline, geologically. If he had not been doing any mining, he would have thrown out that area in the low as absolutely useless, but fortunately he got over there in the mining and found over 100 ft. ore in that low. I am very much interested in the matter of polarity, because under ordinary conditions he would have thrown out the entire low area as having no ores, because it looked unfavorable, but it has proved to be a region of very good ore.

L. B. SLICHTER.—I do not mean to imply that a high necessarily denotes the occurrence of ore beneath it. For example, I can mention a similar case out in Montana where the ore was indicated by a low. It is the relation of the highs and lows to one another, and to possible geological causes, which forms the clues from which consistent deductions must be attempted. One cannot explain your problem without knowing the exact conditions that are to be met there, but I see nothing paradoxical about what you have said; in fact, we have met the same sort of thing.

H. A. BUEHLER.—There is a decided difference in the polarity of the two orebodies.

D. C. BARTON, Houston, Tex.—Dr. Heiland published[28] a very interesting illustration of a bipolar anomaly in Germany. A mine in what turned out to be the negative side ran through a fault and lost the orebody. A magnetic survey by Dr. Heiland disclosed a big positive anomaly offset slightly from a big negative anomaly over the

[28] C. A. Heiland: *Bull.* Am. Assn. Petr. Geol. (1926) **10,** 1197; also *Engng. & Min Jnl-Pr.* (Jan., 1926) **121,** 9.

known orebody. A gallery over to the positive anomaly found the ore, as Dr. Heiland had predicted. The orebody of the old mine area had negative polarity and the offset mass positive polarity.

S. H. Hamilton, Philadelphia, Pa.—In carrying on magnetometric surveys by the Thalén-Tiberg method, so well described by Dr. Haanel,[29] I had occasion to make elaborate surveys at Cranberry, N. C., where the negative end of the orebody was the dominant end. This was very difficult to explain at the time with the information available, but after drilling and other explorations we found the structure to be an overthrust fault. In other words, we found a new orebody as large as the previous Cranberry body, directly on top—in the hanging wall of the Cranberry mine. This new orebody has been mined out during the past 10 or 15 years.

I dare say Mr. LeFevre will cite some excellent examples in northern New York from surveys I made for the Witherbee-Sherman company, in which large negative as well as positive orebodies were found, one on top of the other, in the nature of inbricated chutes. In fact, areas of negative attraction indicate areas of valuable ore as often as areas of positive attraction.

E. G. Leonardon, New York, N. Y. (written discussion).—On page 249 the author says that the response observed on orebodies is often much greater than it should really be according to the values of the magnetic susceptibilities measured in the laboratory. It is justly observed that these values for the magnetic susceptibility are generally obtained by using magnetic fields which are too intense, and that when the measurements are made with magnetic fields of the order of magnitude of the terrestrial field, higher figures are obtained which are in better accordance with the experiments conducted on mineral deposits.

To the above might be added the following observations: Magnetic minerals, under the action of repeated shocks, are capable of acquiring a magnetization much greater than the one they acquire when submitted undisturbed to the same magnetic field. These shocks decrease the hysteresis and facilitate the orientation of the molecules. A well-known example of this fact is found in mining work where it has been observed that drill steels, under the action of the terrestrial field, acquire a powerful magnetization. This magnetization is due to the repeated shocks which the steel receives while working in the hole and placed in a constant direction with respect to the magnetic field. On the contrary, the magnetization would be almost nothing if the steel were at rest.

It is probable that mineral deposits, under the action of terrestrial seisms, undergo a similar influence which might assume considerable importance when the length of geological time is taken into consideration. These remarks would complete the opinion suggested by Haalck, which is quoted in Mr. Slichter's paper, page 249.

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[29] E. Haanel: On the Location and Examination of Magnetic Ore Deposits by Magnetometric Measurements. 1904. Department of the Interior, Ottawa, Canada

Theory of Adolf Schmidt's Horizontal Field Balance

By C. A. Heiland,* Golden, Colo.

(Boston Meeting, August, 1928)

Some 15 years ago, Dr. Adolf Schmidt, director of the Magnetic Observatory in Potsdam, Germany, developed an instrument, which was a modification of Lloyd's balance, for the measurement of the vertical component of the earth's magnetism in the field. There was a keen demand for this because (1) it was desirable to have a field instrument that would measure the component of the earth's magnetism which stands in the closest and simplest relation to the configuration of the subterranean magnetic formations—the vertical component; (2) it was desirable to have an instrument that was not only more sensitive but also more reliable than the existing vertical magnetometers (the dip needle, for instance).

The application of the magnetometer in oil geology demanded such an instrument, because the magnetic disturbances produced by most types of magnetic formations associated with the occurrence of oil (salt domes, faults, anticlines with crystalline core, etc.) are too small to be detected with the ordinary dip needle. The writer can prove, by some investigations recently made for the U. S. Bureau of Mines, that these instruments may be used also to very great advantage in mining, because they are capable of measuring anomalies which are as great as twice the normal strength of the earth's magnetic field. Even for such anomalies, the field balance is more advantageous than the dip needle. First, the instrument rests on a firm support and can be adjusted in the proper azimuth, which insures measurement of the vertical intensity only, without a part of the horizontal intensity, which is very important for the interpretation. The readings give the vertical intensity in linear terms and not as a function of an angle; and the field balance is insensitive to temperature when used with scale values that are adequate for the determination of large anomalies. In another phase of mining, the gold placer work, the field balance has also shown excellent results for which the dip needle is not adequate. What has been said about the dip needle holds also, though to a lesser extent, for two other types of vertical variometers recently used, the Tiberg balance and the Thomson-Thalén magnetometer.

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* Professor of Geophysics, Colorado School of Mines.

To date, the successes of this instrument on this continent are most conspicuous in oil geology. There are now more than 200 instruments of this type in use around the oil fields of Texas, Kansas, Mexico, Louisiana, Oklahoma, New Mexico, Colorado, etc. Unfortunately, very little is published about the results; many areas have been surveyed three and four times by different companies on account of lack of cooperation. In many cases the operators of the magnetometers are not capable of handling these sensitive instruments, and fewer men can interpret the results. Many an operator believes that all he has to do to find the oil is to look for a magnetic high on a small portion of land. Few realize that successful interpretation of magnetometer surveys can not be made without a thorough theoretical understanding of the magnetic effect of certain types of magnetic bodies (that is, how it depends on latitude, orientation, dimensions, mineral content and geologic history of the stratum); also, that a small magnetic survey should, if possible, be evaluated only in the light of the magnetic character of the whole area and by comparison with other magnetic anomalies. The reason for this is that magnetic anomalies are closely associated with the geologic history of the deposit, because they depend on so many physical properties of the rocks (magnetic content, mechanical microstructure, previous temperature and pressure conditions, secondary or primary location, etc.).

The properties just referred to are more numerous than those affecting the interpretation of other geophysical results, so that magnetic anomalies are much more difficult to interpret than torsion-balance or seismograph results.

The interpretation of magnetic results is considerably facilitated if not only one but two components of the magnetic field are known. Theoretically, the most complete analytical information on the disturbing field is furnished if the three components determining magnitude and direction of the vector are measured. In practice, however, two components seem to be sufficient in most cases—the vertical and the horizontal intensity. Besides, the determination of the third component necessitates the measurement of the magnetic declination; even if this quantity is measured in the most expeditious manner (by relative observations without determining the astronomic meridian), it requires a definite arrangement of the stations (mutual visibility of consecutive stations).

Numerous types of instruments,[1] are available to measure the horizontal intensity in the field with a great degree of precision. The question may arise, therefore, why Adolf Schmidt has seen fit to design a

[1] A good description of other types of horizontal variometers is given by C. Hartnell: Horizontal-intensity Variometers. U. S. Coast and Geodetic Survey, *Spec. Pub.* 89.

new type of horizontal variometer. The answer is as follows: If a horizontal variometer is to be used with the vertical balance, it is advantageous to use the same procedure in reading both instruments and computing the results. From this viewpoint, the objection to all previously existing horizontal variometers would be that the readings are obtained in angles of deflection of a needle necessitating the use of trigonometric functions for the computation of the results. To conform to the practice of reading used in the vertical type, it was desirable to have a relative instrument that would furnish the results in the form of readings uniformly proportional to the horizontal intensity.

The horizontal field balance in principle is similar to the vertical balance, except that the magnetic system is balanced in a vertical position in the magnetic meridian. It was first described by the writer in 1925.[2] A brief description of the fundamental formulas was given by the writer in 1926,[3] H. Haalck[4] and G. Angenheister[5] in 1927, and O. C. Lester[6] in 1928.

Description of Magnetometers

The construction of the horizontal balance will be most readily understood by describing both vertical and horizontal balances together. The principle of the vertical instrument is as follows: A magnetic balance system is balanced on a knife-edge perpendicular to the magnetic meridian. Its deflection, the tilt of which is measured by a telescope with a Gauss eyepiece, is directly proportional to the vertical intensity. The mass distribution of the balance system is such that the center of gravity of the system is on the south polar side below the pivot, so that the balance magnet, which would stand vertical if the center of gravity coincided with the pivot, lies practically horizontal in the field of observation. The case that contains the magnetic system, the telescope, thermometers and level, is shown in Fig. 1. A special duofold casing is around this case to protect it from heavy temperature changes.

As a bearing for the magnetic system a double bridge serves, with two quartz pieces which are ground in semicylindrical form and the long

[2] C. A. Heiland: Instrumente und Methoden zur Ermittlung nutzbarer Lagerstaetten. *Ztsch. f. Instrumentenkunde* (1925) **9,** 424. English translation: Instruments and Methods for the Discovery of Useful Mineral Deposits. *Engng. & Min. Jnl.* (1926) **121,** 47.

[3] C. A. Heiland: Construction, Theory and Application of Magnetic Field Balances. Amer. Assn. Petr. Geol. *Bull.* (1926) **10,** 1189.

[4] H. Haalck: Die magnetischen Verfahren der angewandten Geophysik, 86. Berlin, 1927.

[5] G. Angenheister: Erdmagnetische Messungen. *Handbuch der Physik,* Kapitel 28 (1927) **16,** 785.

[6] O. C. Lester: A Simple Derivation of the Working Equations of Magnetic Variometers for the Vertical and Horizontal Intensity. Amer. Assn. Petr. Geol. *Bull.* (1928) **12,** 855.

axis of which stands perpendicular to the quartz knife-edge. For arresting the system, an arm to which three points are fastened is used. These points fit into three grooves on the under side of the balance system, thereby allowing the system to rest always on the same place on the bearings. The vertical up and down movement of the arm is controlled by turning the lever or knurled screw. By placing the lever upon "fest" the system is arrested. By the insertion of the clip the lever is

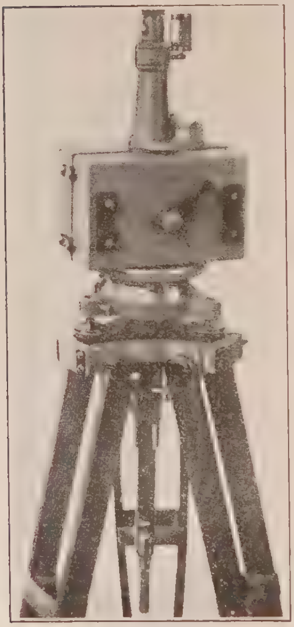

FIG. 1.—VIEW OF VERTICAL FIELD BALANCE.

FIG. 2.—OPTICAL SYSTEM OF A MAGNETOMETER.

restrained from spontaneous movement and the instrument is fitted for transportation; the balance is held firm by two springs fastened to the inside of the cover with a slight pressure from above.

For damping the oscillations two copper dampers, which can be taken out, are employed. On the back of the box a window is fixed for viewing the interior and reading the inside thermometers. The case is held on the tripod by means of three pegs which fit into three holes in the tripod top and is fastened by turning in three friction plates.

The observation of the magnetic system is made possible through the telescope by the following arrangement (see Fig. 2):

An illuminating mirror throws a pencil of rays through a frosted glass window upon a parallel glass plate fixed at an angle of 45° over a graduated plate in the telescope. This plate is in the focus of the objective lens. The light reaches the balance mirror through the objective lens of the telescope and is reflected by it. In observing through the adjustable ocular, two unnumbered scales appear, one seen directly and one reflected. The scale shows 40 divisions; the 0, 20, and 40 marks being especially strong. The marks on the fixed scale, as a

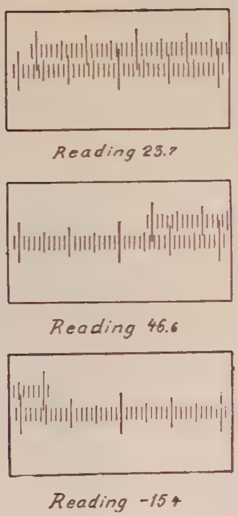

Fig. 3.—Method of reading a magnetometer.

Fig. 4.—View of horizontal field balance.

rule, are numbered from the south side of the case. In reading the 20 mark of the reflected scale is considered an index and its position upon the direct scale is observed. If this is no longer in the field of vision, the index 0, or 40, is reduced to 20. Thus it is possible to read -20 to $+60$ scale divisions (see Fig. 3).

The magnetic systems (Figs. 5 and 6) consist of two magnetized bars of tungsten steel fastened together by a cube-shaped piece of aluminum, which holds the quartz edge, ground to an angle of 120°. Two large lateral brass screws, tightened by little German silver bolts, are used to adjust the system to the field of survey. The sensitivity is regulated by a large brass screw, on the lower side of the system tightened by a

German silver screw. The temperature of the system is read by two thermometers, one of which is graduated from +15 to +35°, and the other from +30 to +55° C.

The mechanical arrangement of the horizontal balance is exactly the same as described for the vertical variometer. Only the principle of the measurement is different. Readings are taken from the north instead of from the south. The longitudinal axis of the magnetic system is vertical; it swings in the plane of the magnetic meridian. Figs. 4 and 7 show the instrument.

The instruments are set upon wooden tripods, the lower legs of which are adjustable. The tripod is the same as for the vertical variometer. The top can be leveled by three screws. By releasing the clamp screw, it is possible to turn the top and the positions can be read on a scale. A tube, which can be shifted and clamped by means of a screw, serves as a holder for auxiliary magnets in extending the field of survey; the distance from the middle of the auxiliary magnets to the axis of rotation can be read on an index. For adjustment in the meridian a compass is used. The compass is always held in a certain position with reference to the top by a little pin underneath, which fits into a hole in the top of the tripod.

Three auxiliary magnets and various accessories are added to the instrument; it can be carried on the back in a canvas sack, while the tripod can be hung over the shoulder in a case.

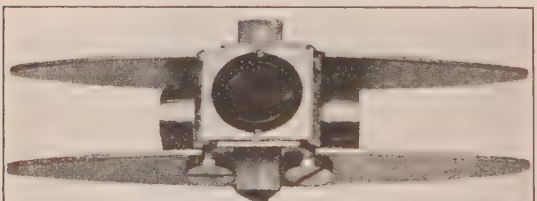

FIG. 5.—MAGNETIC SYSTEM OF VERTICAL BALANCE.

FIG. 6.—MAGNETIC SYSTEM OF HORIZONTAL BALANCE.

The first instrument was constructed in 1922. Since 1924, the author has not only been engaged in the testing and standardizing of this horizontal variometer, but has also applied it in several field surveys. The experience thus gathered gave rise to a detailed development of the theory, which is published at this time not because the writer feels that it is altogether complete but because there is a demand from the operators for a knowledge of the effects of various influences on the instrument (orientation, inclination, temperature, condition of knife-edge, etc.).

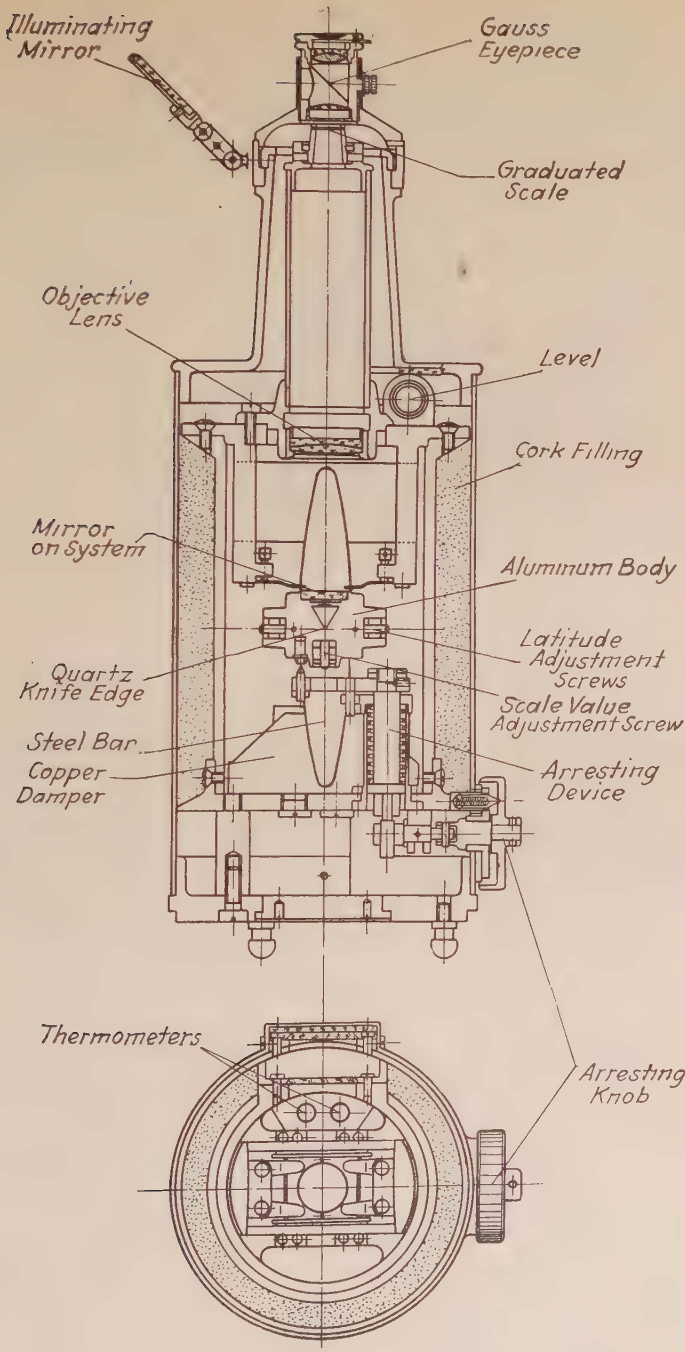

FIG. 7.—CROSS-SECTION OF HORIZONTAL FIELD BALANCE.

Fundamental Theory of Magnetometers

The theory of the horizontal field balance may be derived in exactly the same way as that of the vertical balance. Detailed derivations of the theory of that instrument have been given by Schmidt,[7] Heiland and Duckert,[8] and V. Spacek.[9]

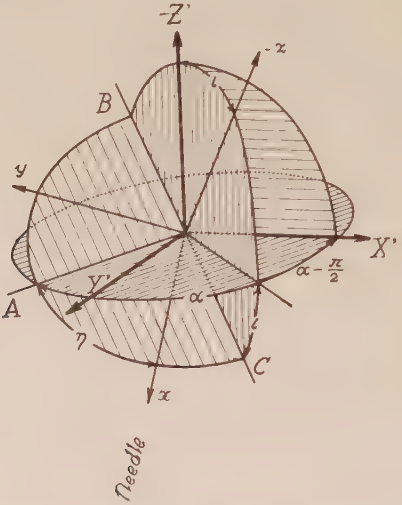

Fig. 8.—Effect of three orthogonal fixed forces on three orthogonal movable components.

We will go briefly through the chief steps of this derivation to facilitate the understanding of details of the theory of the other instrument. We commence with the investigation of the force that is produced by the earth's magnetic field upon a magnetic needle which is capable of free motion in space. This is an imaginary needle, suspended at its center of gravity by an infinitely elastic fiber.

The earth's magnetic force is a vector, which is defined by magnitude and direction. Analytically, both of these data are in turn defined by the magnitude of three rectangular components X', Y', Z' (astron. north, east and vertical component). To determine the force that is exerted

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[7] A. Schmidt: Ein Lokalvariometer fuer die Vertikalintensitaet. *Taet. Ber.* des Kgl. Pr. Meteor. Inst. f. d. J. (1914) 109.

Ein Lokalvariometer fuer die Vertikalintensitaet. *Idem* (1915) **2,** 87.

[8] C. A. Heiland and P. Duckert: Beschreibung, Theorie und Anwendung einer Neukonstruktion von Ad. Schmidt's Feldwage. *Ztsch. f. angew. Geophysik,* (1924) **10,** 289, 321.

[9] V. Špaček: Měření Schmidtovým variometrem pro vertikální složku zemského magnetismu v okolí Řipu. Rozpravy II. Třídy České Akademie Ročnik XXXVI, Cislo 10.

upon the magnetic system with which the three coordinates x, y, z are assumed to be connected, we determine the value of these components from Euler's geometric relations:

$$\left.\begin{aligned}
x &= \begin{cases} X' (\cos\alpha\cos\eta + \sin\alpha\sin\eta\cos\iota) + \\ +Y' (\sin\alpha\cos\eta - \cos\alpha\sin\eta\cos\iota) + \\ +Z' (\sin\eta\sin\iota) \end{cases} \\
y &= \begin{cases} X' (\cos\alpha\sin\eta - \sin\alpha\cos\eta\cos\iota) + \\ +Y' (\sin\alpha\sin\eta + \cos\alpha\cos\eta\cos\iota) + \\ +Z' (-\cos\eta\sin\iota) \end{cases} \\
z &= \begin{cases} -X' \sin\alpha\sin\iota + \\ +Y' \cos\alpha\sin\iota + \\ +Z' \cos\iota \end{cases}
\end{aligned}\right\} \quad [1]$$

where (see Fig. 8)

$\sphericalangle AOX' = \alpha =$ azimuth of the plane ABC in which the north pole of the system moves

$\sphericalangle zOZ' = \iota =$ inclination of this plane from the horizontal plane

$\sphericalangle AOx = \eta =$ angle designating distance of north pole of needle upon plane ABC from horizontal plane.

As all magnetic matter is polarized, and as we assume that north and south magnetism are of equal strength in our needle, the force acting in its longitudinal direction can exert no translatory force; hence, $x = 0$. The resultant force is thus $\sqrt{y^2 + z^2}$.

We make another simplification, as in practice it is always easy to determine the direction of the horizontal projection H (horizontal intensity) of the earth magnetic vector with a compass. Turning, thus, the original system of coordinates about the Z' axis by the angle of the declination, Y' will vanish; $X' = H$, $Z' = Z$ (= vertical intensity). Hence, the total force equals

$$\sqrt{[H(\cos\alpha\sin\eta - \sin\alpha\cos\eta\cos\iota) - Z\cos\eta\sin\iota]^2 + [Z\cos\iota - H\sin\alpha\sin\iota]^2}$$

A further simplication of this formula is possible because a needle suspended as described is not in actual use. All magnetic needles turn about a rigid axis. (Also the suspension of a compass is equivalent to such an axis, as the center of gravity is placed so far below the point of suspension that virtually only movements in a horizontal plane are possible.) Referring to Fig. 8, the rigid axis would lie in the z direction and confine the movement of the needle to the plane ABC. Then the force z cancels out, because it does not affect the lateral movement of the needle. Denoting by M the magnetic moment of the needle (or the total moment of a pair of needles), the moment D_1 of the force y is

$$D_1 = My = M[H(\cos\alpha\sin\eta - \sin\alpha\cos\eta\cos\iota) - Z\cos\eta\sin\iota] \quad [2]$$

This is the moment for a system which is suspended in its center of gravity. This not being the case, a second moment D_2 is produced by the gravity. The influence of gravity upon a magnetic system which is suspended in any direction in space may be found by means of Euler's relations again. Assuming that the deviation of the direction of gravity from the vertical is immaterial for our problem, we put $X' = 0 = Y'$ and $Z' = g$. For a system moving in bearings of a rigid axis, the components x and z can produce no lateral movement and merely different pressure on the bearings in these directions; thus

$$D_2 = emy = -emg \sin \iota \cos \eta \qquad [3]$$

where e is the distance of the center of gravity of the magnet from the axis of rotation and m the mass of the system in grams.

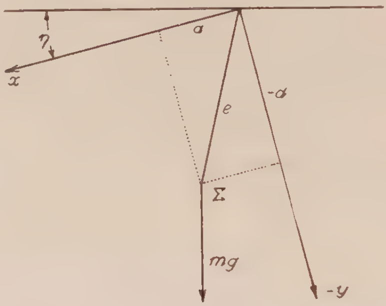

FIG. 9.—EFFECT OF GRAVITY ON VERTICAL FIELD BALANCE.

This formula holds only for the case that the center of gravity lies in the x axis. In reality, the center of gravity may have three coordinates in the system (x, y, z). Of these, the z coordinate may be disregarded as the system moves about an axis parallel to that direction. We then denote (Fig. 9) by a and d the projections of e on the x and y axes, so that equation 3 becomes

$$D_2 = -mg \sin \iota (a \cos \eta - d \sin \eta) \qquad [4]$$

In the position of equilibrium of the system, $D_1 + D_2 = 0$, or

$$\tan \eta = \frac{MH \sin \alpha \cos \iota + MZ \sin \iota + mga \sin \iota}{MH \cos \alpha - mgd \sin \iota} \qquad [5]$$

This is the principal formula for both vertical and horizontal balance for any azimuth and inclination.[10]

[10] The applicability of this formula is by no means confined to these two instruments. The formula holds for all magnetic systems capable of free movement in any direction in space as described.

The formula is so fundamental that not only the theory of the compass, the inclinator, and the dip needle may be based upon it, but also that of all deflection magnetometers.

For the vertical balance, this formula is considerably simplified, because in the positions of use:

$$\frac{\pi}{2} = \alpha = \frac{3\pi}{2} \text{ and } \iota = \frac{\pi}{2} \text{ or}$$

$\tan \eta = \dfrac{MZ + mga}{-mgd}$; that is, the magnetic system is stable only if a and d are minus in reference to the coordinates originally assumed. The center of gravity must be on the south side and below the axis of rotation. If so,

$$\tan \eta = \frac{MZ - mga}{mgd} \tag{6}$$

As the readings are not obtained in angles but in scale divisions s of a graduated scale, which is placed in the focal length f of a lens in an "autocollimational" optical system with a Gauss eyepiece, we write $\tan 2\eta = \dfrac{s - s_0}{f}$ where s_0 is the reading (usually 20) corresponding to $\eta = 0$, or disregarding the term $\dfrac{s - s_0}{f} \tan^2\eta$, as η is generally small.

$$\tan \eta = \frac{s - s_0}{2f}\text{; hence,}$$

$$s - s_0 = 2f\frac{(MZ - mga)}{mgd} \tag{7}$$

Putting $s_0 = 0$ and presuming that a reading s has been obtained at a location where the vertical intensity Z is known (or assumed to have a definite value) and that the reading s' has been obtained at a second location of which the vertical intensity Z' is sought, we have from [7]

$$s' - s = \frac{2fM(Z' - Z)}{mgd} \text{ or } Z' = (s' - s)\frac{mgd}{2fM} + Z$$

$$\text{or } Z' = \epsilon(s' - s) + Z \tag{8}$$

where $\epsilon = \dfrac{mgd}{2fM}$ is the scale value of the instrument.

Assuming that the first (or base) reading was made at a station where there was no local anomaly, or $Z = Z_{norm.}$, formula 8 expresses directly the use of the instrument as a relative detector of local disturbances or

$$\Delta Z = \epsilon(s' - s) \tag{9}$$

Theory of the Horizontal Balance

We will derive the theory of the horizontal balance directly from that of the vertical balance. A few substitutions are necessary, however, as the zero position of the magnetic system is no longer horizontal, but

vertical. Secondly, the two components a and d must be interchanged, because d is now parallel to the magnetic axis.

Substituting a for d and d for a and $\frac{\pi}{2} - \varphi$ for η, the principal formula is

$$\tan \varphi = \frac{MH \cos \alpha - mga \sin \iota}{MH \sin \alpha \cos \iota + MZ \sin \iota + mgd \sin \iota} \qquad [10]$$

In the positions for use, $\alpha = 0$ and $\iota = \frac{\pi}{2}$, or $\tan \varphi = \frac{MH - mga}{MZ + mgd}$. From this equation, we derive readily a conclusion as to what the position of the center of gravity must be to make the system stable. The center of gravity must be on the north side of the axis of rotation (to overcome the inclination and make the system stand vertical). As queer as it may sound, the center of gravity must also be above the axis of rotation in our magnetic latitudes, if the scale value is about normal (15.10^{-5} Gauss), to compensate for the pull of the vertical intensity, which is parallel to the gravity. As demonstrated later (equation 12), the center of gravity will be below the axis of rotation if $Z < 2 f\epsilon$. If the center of gravity is above the axis of rotation, however, it is without any practical significance, as the only factor that influences the behavior of the system in any respect is the resultant center of action of both vertical intensity and gravity combined.

We may write, therefore, as before

$$s - s_0 = \frac{2f(MH - mga)}{MZ - mgd} \qquad [11]$$

and by applying the same procedure as in equations 8 and 9, we obtain the formulas for use, indicating the application of the balance as a relative instrument:

$$s' - s = \frac{2fM\,(H' - H)}{MZ - mgd} \text{ or } H' = (s' - s)\frac{MZ - mgd}{2fM} + H$$

$$\text{or} \quad H' = \epsilon(s' - s) + H, \text{ where } \epsilon = \frac{MZ - mgd}{2fM} \qquad [12]$$

For determining local anomalies, as before,

$$\Delta H = \epsilon(s' - s) \qquad [13]$$

Theory of Various Influences

The various influences that affect the readings are:

1. Influence of false orientation.
2. Influence of improper leveling.
3. Influence of auxiliary magnets.
4. Influence of changes in vertical intensity (latitude and anomalies).

5. Influence of changes in gravity (latitude and anomalies).
6. Influence of temperature on readings in general, on the scale value and on auxiliary magnets.
7. Influence of the shape of the knife-edge on subsequent readings and scale value.
8. Optical influences.

1. *Influence of False Orientation*

To determine this influence, we put $\iota = \frac{\pi}{2}$ in the principal formula, No. 10, for simplification and obtain

$$s_\alpha - s_0 = 2f\left[\frac{MH\cos\alpha - mga}{MZ - mgd}\right] \tag{14}$$

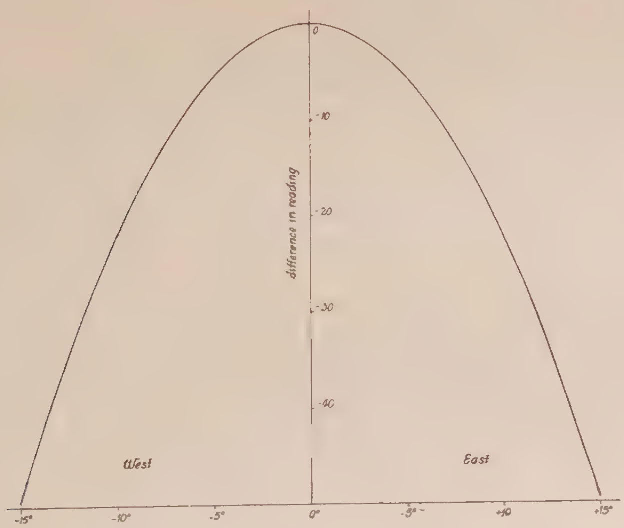

FIG. 10.—EFFECT OF MISORIENTATION.

Subtracting from this equation 11, we have the difference of false and correct reading

$$s_\alpha - s = \frac{2fMH}{MZ - mgd} \cdot (\cos\alpha - 1) \text{ or } s_\alpha - s = \frac{H}{\epsilon}(\cos\alpha - 1) \tag{15}$$

For the horizontal intensity 21,790γ in Golden, Colo., and a scale value of 15γ the curve shown in Fig. 10 has been constructed. The influence of a false orientation always consists in a decrease of the correct reading and is the same for readings above or below 20. It is very appreciable; an error of 40′ made in setting up the instrument produces an error in reading equal to the permissible optical error of one-tenth of a scale division. The influence is negative whether the misorientation be made toward east or west. The influence of improper orientation is much greater on the

horizontal than on the vertical field balance. It is not customary to apply a correction on account of misorientation; the instrument must be set up with great care before readings are taken. The graduated circle on the tripod head furnishes the required accuracy, because it is divided in divisions of 1° each and an accuracy of 0.5° and less in the meridian adjustment is readily attainable.

This pronounced relation of reading and azimuth suggests immediately two practical applications for emergency cases.

Suppose the auxiliary magnets are not available or their moment is unknown for some reason. The scale value can be determined by applying formula 15 on a location with no magnetic anomaly, the normal value of H for which may be taken from the government map. The ratio of $\frac{H}{\epsilon}$ is determined from a number of readings taken in different azimuths of which the angles may be read on the graduated circle of the tripod head. Precautions must be taken, however, to keep the temperature constant during this experiment, and to see that the instrument stays level in all azimuths. The accuracy in the scale value thus obtained is always satisfactory in an emergency; it depends, of course, on how close the actual horizontal intensity on the point of observation agrees with the value taken from an isodynamic map. If desired, the experiment may be repeated on a second location and thus the accuracy may be increased.

Another possibility of applying formula 15 is given if the compass of the instrument is not available or is out of order. Then the magnetic meridian may be determined with the same accuracy, as that is possible with the compass by taking a number of readings in any azimuths, plotting the results and determining the maximum reading. It is not even necessary to know the direction of the meridian approximately beforehand, because there is only one maximum reading throughout one complete revolution. The reflected scale is not in the field of vision in the whole hemicircle from $\alpha = \frac{\pi}{2}$ to $\alpha = \frac{3\pi}{2}$. (This is for the same reason that there is only one position of use, contrary to the vertical balance, as an inspection of formula 11 immediately reveals.) With an instrument of normal sensitivity the reflected scale will not appear until the meridian is as close as about 13° (for the value of $H = 21{,}790\gamma$ given above), if the system is so adjusted that the middle reading 20 corresponds to the intensity H of the locality. As $s_{min.} = -20$ is the minimum reading that may be obtained on the instrument, the angle $\alpha_{max.}$, in which the reflected scale will begin to appear, may be computed for any other horizontal intensity from the relation

$$\alpha_{max.} = \text{arc cos}\left(\frac{\epsilon(s_{min.} - s_H) + H)}{H}\right)$$

where s_H is the reading obtained in the meridian, which corresponds to the H of the locality (assumed to be 20 in the example as given above).

Instead of making a number of readings in several azimuths for the determination of the magnetic meridian from the shape of the curve (maximum reading, see above), only two readings may be made in two positions which differ 180°. From the difference of these two readings, the magnetic azimuth may be computed in which the first reading was made, as from formula 11

$$\alpha = \text{arc cos}\left(\frac{\epsilon}{2H}(s_\alpha - s_{\pi+\alpha})\right) \qquad [16]$$

The disadvantage of this short-cut method, however, is that an auxiliary magnet must be used to determine $s_{\pi+\alpha}$, whereby the accuracy is somewhat decreased.

2. *Influence of Improper Leveling*

A tilt of the instrument in any direction is composed of a component i_N in the north-south direction and another i_E in the east-west direction.

The influence of the former is merely of an optical nature; it is equivalent to an increase or decrease of the angle made by the system with a plane that passes through the graduated plate. Hence, if the tilt is i_N in radians, it is $2fi_N$ in scale divisions. Therefore, the change of the reading is

$$\Delta s = \pm 2f\rho(n_N - n_0) = \pm 0.2(n_N - n_0) \qquad [17]$$

where n_0 is the reading of the level bubble (mean of the two ends) in the horizontal position, n_N the new reading if the instrument is tilted toward north, and ρ the scale value of the level. As $2f = 1333$ scale divisions and $\rho = 30''$, the coefficient given above results. It is not customary to apply a correction for improper leveling; the instrument should be kept level within one-half a division of the bubble in the north-south direction; at right angles thereto, not so much care is necessary. From equation 17 it is seen that the readings become greater if the instrument is tilted toward north (bubble going south) and smaller if it is tilted toward south (bubble going north). In comparison with the vertical balance, greater care in leveling is necessary in the plane of oscillation of the system, because there are not two positions of use, in which this difference would cancel out. If in formula 17 the difference of two readings of the level is substituted which is obtained after revolving the instrument through 180°, the coefficient is $\frac{\rho}{2}$ instead of ρ.

The influence of a tilt in the east-west direction is much smaller. It changes only the influence of the gravity and vertical intensity by an amount proportional to the deviation of the position of the instrument

from the vertical. To derive this influence, we put $\alpha = 0$ in the principal formula, No. 10, and obtain

$$s_\iota = 2f\left(\frac{\frac{MH}{\sin \iota} - mga}{MZ - mgd}\right) \tag{18}$$

and, subtracting equation 11,

$$s_\iota - s = \frac{H}{\epsilon}\cdot(\operatorname{cosec} \iota - 1). \quad \text{As } \iota = \frac{\pi}{2} \pm i_E$$

$$s_\iota - s = \frac{H}{\epsilon}\cdot(\sec i_E - 1) \tag{19}$$

This difference is only 0.15 scale divisions, if i_E is as large as about 49′; but the level bubble is out if the tilt exceeds 5′. In any inclined position, be the inclination to the east or west, the reading in the tilted position is always greater than that in the horizontal position. The influence of this east-west tilt may be disregarded within the range of the level bubble.

In a recent publication,[11] Rieber discusses the influence of improper leveling and comes to the conclusion that it produces an error of about 15γ on a type of balance such as we deal with here. This value, however, is altogether too great if the observer operates the balance properly; that is, with the accuracy provided by the level bubbles. These permit the reading of differences in inclination of the instrument of about two-tenths of a bubble division; in other words, with an accuracy of about 6″ of arc.

The error produced by limitations in accuracy of leveling such as published by Rieber is not only contrary to everyday magnetometer experience but in error because it is not based on correct theoretical considerations. For instance, it is not possible to calculate this error by merely computing the difference in vertical intensity from the amount by which the axis of revolution of a vertical variometer deviates from the vertical. Not only the amount of this deviation, but also the direction of the inclination of the axis of revolution must be taken into account; thus the horizontal intensity becomes involved in addition to the vertical intensity. Furthermore, the change in the direction of the vertical intensity cancels out completely in the direction of oscillation of the balances, as a balanced magnetic system maintains its orientation in space and thus its position relative to the direction of the intensity when the case is tilted. Therefore, the only influence produced by the inclination in this direction is of an optical nature. This optical error even cancels on the vertical balance in the two positions of use. In other words, only the north-south component of the inclination of the

[11] F. Rieber: A New Micromagnetometer. See page 401.

axis of revolution producing an effect of the horizontal intensity becomes effective (while a north-south inclination of the instrument, when the axis of revolution is vertical, cancels in the average of the two positions). Hence, on most vertical magnetometers the level is in the north-south direction. The influence of the north-south inclination, being very small to start with, may be even reduced by noting the position of the bubble and applying a correction. Thus, actual experience with the vertical magnetometer shows that by making use of the accuracy furnished by the level bubbles, the total error introduced by limitations of accuracy of leveling, in careful operation, is not greater than about 3 gammas.

As to the horizontal balance, the optical influence mentioned before does not cancel, as there are no two positions of use; but it may be considerably reduced by taking into account the position of the bubble and by computing a correction. The influence of an inclination at right angles to the plane of oscillation is negligible, as set forth above, so that in this instrument only the north-south component of the total inclination becomes effective. Experience shows that in careful operation the total error introduced by the limitations of the level bubbles is also not much greater than 3 gammas.

3. *Influence of Auxiliary Magnets*

Auxiliary magnets are used for the determination of the scale value and to increase the range of the scale. The magnets are in the second principal Gauss position in reference to the deflected magnetic system. They must be used parallel to the plane of oscillation of the magnetic system; that is, in the magnetic meridian. If they deviate from it by the angle β, a false scale value is obtained, which is $\epsilon_0 \cdot \sec \beta$ instead of ϵ_0. If a magnet is used for redeflecting the disappeared scale by a difference in reading Δs and if the scale value has been determined correctly before, this difference will be $\Delta s \cos \beta$ instead of Δs, if the magnets are not in the meridian.

For the scale-value determination, generally an extension rod is used, which consists of a long vertical hollow graduated rail of square section. A slide with a horizontal pivot, about which a vertical disk may turn moves up and down this rail. Attached to the disk is a metal piece with a threaded hole to take the magnets. The disk has four stop notches, to place the magnet in two vertical or two horizontal positions. For the determination of the scale value of the horizontal field balance, the magnet is placed in one horizontal position (north pole north) and the disk is turned about a horizontal axis to the third notch, so that the north pole is in the south. If the disk with the stops should be somewhat out of line, the stops remain 180° reversed, but the magnet is no longer horizontal; the north pole will be first too high and later too low,

or vice versa. This will change the scale value to $\epsilon_0 \sec \gamma$ instead of ϵ_0 if γ is the angle of this vertical inclination of the magnet. The same formula applies if the threaded hole in the extension of the tripod normally used is not horizontal. Both misorientation of the magnet or its inclination can not cause a difference of the positive or negative deflection of the magnetic system; that is, $\frac{s_1 + s_2}{2} \gtrless s_0$, or $s_2 - s_0 \gtrless s_0 - s_1$ (if s_0 is the undisturbed reading, s_2 the reading when the north pole of the deflected system is in the south, and s_1 the reading when it is in the north.) Such an asymmetry may be caused by (1) the notches in the disk of the deflection-rod not being placed exactly 180° apart (rare); (2) the induction-coefficient of the deflecting magnet being relatively great (rare); (3) the shape of the knife-edge being unsymmetrical (most common).

If the magnet is not screwed in altogether in the threaded hole, or if the magnet is placed unsymmetrically in its casing, this will produce unsymmetrical deflections with the vertical field balance, because the vertical distance of the magnet from the system is a minimum in one position and a maximum in the other, which is 180° reversed. With the horizontal balance, however, this unsymmetry of the magnet does not produce a noticeable unsymmetry of the readings. The unsymmetry of the magnet is in the horizontal direction, and hardly affects its vertical distance from the system, aside from the fact that the departure from the recorded distance is the same in both diametrical positions. This holds, of course, only for the determination of the scale value with a deflecting rod underneath the balance in the second principal position of Gauss. If the deflecting magnet is placed on the side of the balance in the first principal position of Gauss, the unsymmetries of the magnet will have a maximum effect and produce the same unsymmetry of the readings, which may be observed on the vertical balance under such circumstances.

Using the same magnets, the influence of the coefficient of induction on the symmetry of the readings is greater with the vertical than with the horizontal magnetometer, under equal conditions, in latitudes where the vertical intensity is greater than the horizontal intensity. The asymmetry resulting from the induction coefficient is due to the fact that an additional magnetic moment is produced in the horizontal magnet by the horizontal component H; this induced moment adds itself to the natural moment of the magnet if the north pole is in the north, and decreases the moment if the north pole is in the south. The negative deflection is thus greater than the positive, or $s_0 - s_1 > s_2 - s_0$. Every magnet has two induction coefficients, one longitudinal and one transversal; for our purposes, the transversal coefficient may be disregarded, although it would have to be multiplied with the vertical intensity. The conditions described for the auxiliary magnet with the moment M_a in the distance r are thus expressed by the two equations

$$\left.\begin{aligned}\Delta s_1 = s_0 - s_1 = \frac{M_a}{\epsilon r^3}(1 + jH)\\ \Delta s_2 = s_2 - s_0 = \frac{M_a}{\epsilon r^3}(1 - jH)\end{aligned}\right\} \quad [20]$$

Hence, the asymmetry is $\frac{2MjH}{\epsilon r^3}$; as Δs can not exceed 40 scale divisions if both positive and negative deflections are observed, the maximum asymmetry is $2jH\Delta s_{max.}$ or about 0.2 scale divisions if the induction coefficient j is 1.10^{-2}. As this coefficient can become 5 and more times as great for magnets made of bad material, the asymmetry may be as great as one scale division.

The most common cause of the asymmetry of positive and negative deflection, however, is the shape of the knife-edge. We will analyze its influence later (p. 296).

Another physical property of the magnet which modifies the deflection is its temperature coefficient, the influence of which will also be discussed later (p. 295).

As all these effects (induction, knife-edge, temperature) increase in proportion to the deflection, scale values computed from great deflections are always more inaccurate than those computed from average readings. Since most observers will not want to go to the trouble of applying corrections for these effects, it is advised that the scale values be computed from deflections of average magnitude. On the other hand, the deflections must not be taken too small, as otherwise the inaccuracies in the readings produced by the limitations of the optical system as well as by the friction on the bearings enter into the results with too great a weight.

If the scale value is being determined on different parts of the scale, it must be expected that different values are obtained. It may be assumed that the varying induction of the vertical intensity might produce it (which varies as the cosine of the angle of inclination of the system), but a rough calculation shows that this influence is not nearly great enough to account for the actually observed differences. Two other influences are the reason. The first is an optical effect, which results from the fact that an optical system corrected in the usual manner does not furnish correct results when used in conjunction with a mirror of close distance and varying inclination. The error thus produced is composed of astigmatic and coma errors and increases as distance from the optical axis, but not linear. This error, however, has been materially reduced in the most recent types of balances, so that virtually the shape of the knife-edge is the only reason why scale values as determined from different parts of the scale do not agree. This influence will be dealt with in detail later (p. 295).

The method used in determining scale values of magnetic field balances is so well known that little need be said here about its principle.

With the deflecting rod, the auxiliary magnets are placed in the second Gauss principal position. Thus

$$\left.\begin{array}{l}\epsilon = \dfrac{2M_a \cdot k}{(s_2 - s_1) \cdot r^3}, \text{ where } k \text{ is the "deflection constant"} \\ \qquad\qquad\qquad \text{or } 1 + \dfrac{n}{r^2}, \ n \text{ being } -\dfrac{3}{8}L^2 + \dfrac{3}{2}l^2;\end{array}\right\} \qquad [21]$$

L, the pole distance (about five-sixths of the geometrical length) of the auxiliary magnet and l that of the deflected system (p. 305).

Auxiliary magnets are used not only for the determination of the scale value but also for compensating magnetic anomalies which have deflected the movable scale beyond the field of vision. If the scale has disappeared toward the south (too great readings), the auxiliary magnet must be used so that its north pole is in the north until the scale is again visible; the correction to be applied is positive, as the effect of the magnet is negative. The opposite is to be done if the scale disappears toward the other end. The correction in scale divisions is

$$\Delta s = \mp \frac{M_a k}{\epsilon r^3}; \ k, \text{ as before } = 1 + \frac{1}{r^2}\left(\frac{3}{2}l^2 - \frac{3}{8}L^2\right) \qquad [22]$$

It is not advised, however, that the corrections be computed in such a manner. Checking the computed values by experiment, it will be found that they disagree appreciably. The greatest discrepancy is found between positive and negative corrections. This is, first, due to the influence of the induction dealt with above; second, it is due to the fact that if we have such great deflections that they carry the scale beyond the field of vision, the shape of the quartz edge is not the same on the left side as it is on the right. Numerical discrepancies of computed and observed values on one side (either positive, or negative) are caused also by the shape of the knife-edge, because, as said before, the scale value usually depends on the deflection. Speaking of discrepancies between observed and computed values, it is assumed that the moment of the magnet and its distance is correct. Such discrepancies are frequently caused by carelessness in pushing the deflection rod against the top of the tripod. Discrepancies of observed and computed values of deflections are also due to differences in temperature.

The following procedure, therefore, is recommended. At a field station where the movable scale is about to disappear, an auxiliary magnet is inserted in any distance, the correct pole in the north, until the scale reappears, and the amount of deflection in scale divisions at that particular distance is noted. At the next station, where the scale may be completely out, the magnet is used in the same distance. After passing the magnetic high there will be a station where the scale just begins to appear again. The experiment described is repeated and the average of both

values is taken. Thus the correction is applied only for the sign for which it has been determined, and for approximately the same temperature. If the first magnet is not sufficiently large the procedure is continued with a greater magnet, etc.

4. *Influence of Changes in Vertical Intensity*

This influence must be taken into account as a correction. As the horizontal balance is generally used in conjunction with the vertical balance, the vertical intensity is known; if both instruments are not used together, the correction may be computed approximately from the government maps giving either the vertical intensity or the horizontal intensity and inclination.

To derive the correction for changes in vertical intensity we take formula 12 for the scale value and substitute two vertical intensities Z and Z'; as $\epsilon' - \epsilon = \Delta\epsilon$ and $Z' - Z = \Delta Z$, we obtain

$$\Delta\epsilon = \frac{\Delta Z}{2f} \text{ or } \Delta\epsilon = \frac{\Delta Z(\text{in } \gamma)}{1333} \qquad [23]$$

However, it would be false to take any scale reading and multiply it with this change in the scale value. This would be permissible if Z were parallel to the force of which the alteration is measured by the constant ϵ, the horizontal intensity. It will be readily seen that the influence of the vertical intensity increases as the departure of the system from the vertical position, and that this influence is positive for deflections to one side and negative for deflections to the other side. For our purposes it will be satisfactory to disregard a small deviation of the mirror vertical from the magnetic axis of the system and to assume that the reading of 20 corresponds to the vertical position of the system. Thus, if the readings are greater than 20, they will be too small on a second station when the vertical intensity increases from the first to the second point; if the readings are less than 20, the opposite will happen under the same circumstances. Hence, in the first case the correction must be positive, in the second negative. We write, therefore, for the correction ΔH:

$$\Delta H = \Delta\epsilon(s - 20) = \frac{\Delta Z}{2f}(s - 20) = \frac{\Delta Z}{1333}(s - 20) \qquad [24]$$

As the differences in H are figured from the base station, the difference in Z should be taken in the same way, provided that the scale-value determinations (which incorporate the original Z) are made also at that station.

The correction on account of changes in vertical intensity can not be neglected. From formula 23 it is readily seen that an anomaly in Z equal to $2f$ (which is frequently observed above granite ridges buried as deep as 4000 ft. and is therefore nothing extraordinary) produces a correction of 40γ in H if the movable scale is on the edge of the field of

vision. The diagram shown in Fig. 11 represents the vertical intensity correction as it depends on both reading and difference in vertical intensity. As this graph is represented in the isometric projection, it permits interpolation for any intermediate values of s and ΔZ.

As is well known, in any field-balance work the normal alteration of the measured intensity as latitude must be subtracted from the readings in order to obtain the disturbance produced by geologic causes only. This normal change is taken from isodynamic maps supplied by government institutions (U. S. Coast and Geodetic Survey).

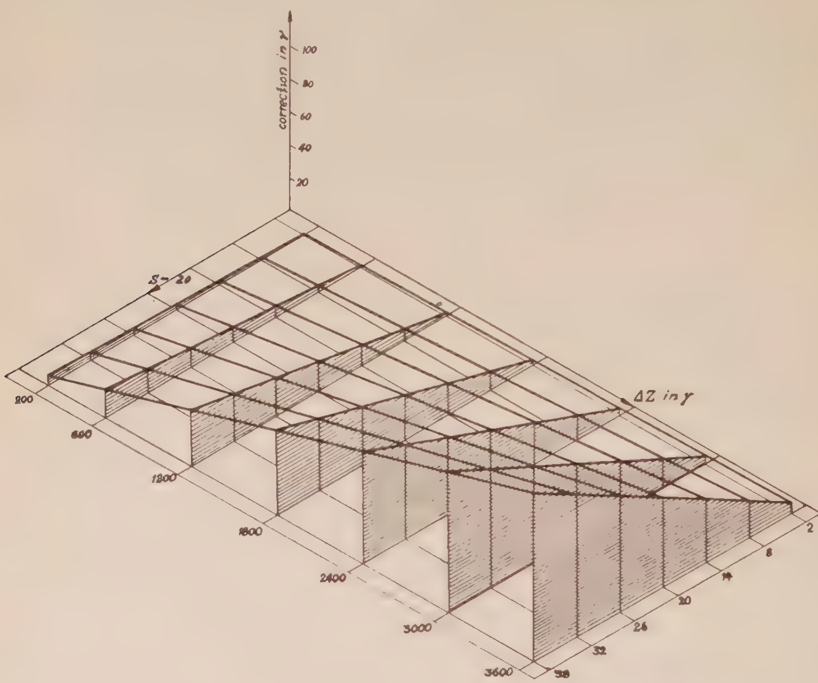

FIG. 11.—ISOMETRIC DIAGRAM SHOWING INFLUENCE OF VERTICAL INTENSITY.

The question arises whether the normal change of the vertical intensity as latitude must also be considered for horizontal intensity measurements to compute a correction such as that described. As the normal change of the vertical intensity, in most oil fields of this country, is about 15γ per mile toward north, it follows that the correction for a station which is about 18 miles north of the base station is only 8γ in H for the most extreme reading. This correction is tolerably small and may be disregarded, bearing in mind that for the distance assumed above it is advisable to establish a subsidiary base station, to determine the scale value there and thus incorporate the change in Z in the new scale value for the locality farther north.

The normal change of the vertical intensity as latitude may not be disregarded in another instance; that is, the comparison of the scale value by the observer with that given in the certificate of standardization of the observatory in Potsdam. If the certificate gives, for instance, 15γ as the scale value and the instrument is shipped to Venezuela, the observer will obtain only a scale value of 12.2γ, because the vertical intensity and the gravity are less. The instruments, therefore, become always more sensitive if shipped to lower latitudes. In the southern hemisphere the gravity becomes greater, it is true, but the vertical intensity is negative and its influence is much greater than that of the change in gravity. It may even happen that an instrument that has been adjusted to 15γ in Germany can not be used in other latitudes, unless the vertical screw is changed, because the system is unstable. The latitude in which this will happen may be determined from the equation for infinite sensitivity; the corresponding critical vertical intensity is $Z_l = \frac{mg_l d}{M}$, g_l being the gravity in the same latitude as the critical vertical intensity. It is advisable, therefore, that the manufacturers of the instruments adjust the horizontal balances not only for the horizontal intensity but also for the vertical intensity of the locality where the instruments are to be used.

TABLE 1.—*Proof of Theory of Vertical Intensity*

No.		Field in Z	ϵ	$\Delta\epsilon$ Determ.	$\Delta\epsilon$ Comput.
0		$\pm$ 0γ	14.93γ		
1	a	−1862	13.41	−1.52γ	−1.40γ
	b	+1862	16.37	+1.44	+1.40
2	a	−2758	12.69	−2.24	−2.07
	b	+2758	17.11	+2.18	+2.07
3	a	−4317	11.45	−3.48	−3.24
	b	+4317	18.57	+3.64	+3.24
4	a	−7232	8.99	−5.94	−5.43
	b	+7232	20.84	+5.91	+5.43

Table 1 shows an interesting experiment which proves that the theory of the influence of the vertical intensity is correct. In the second column, there is the additional vertical intensity which was applied; in the third, the scale value which was determined while the balance was subjected to the additional field; in the fourth, the difference of the determined

scale value, and in the fifth this difference as computed from the theory. The differences of observed and computed scale values are $+0.08\gamma$, $+0.13\gamma$, $+0.32\gamma$, $+0.49\gamma$, which is an excellent agreement indeed and sufficient proof of the theory. Besides, there is a small systematic error involved, as the determination of the moment of the magnet used for the vertical field was not very accurate; its moment is probably somewhat greater. Taking this into consideration, the agreement could possibly still be somewhat improved.

In describing the general theory of the instrument, some authors[12] disregard the influence of the vertical intensity altogether. Table 1 shows that this is not correct and may lead to appreciable errors.

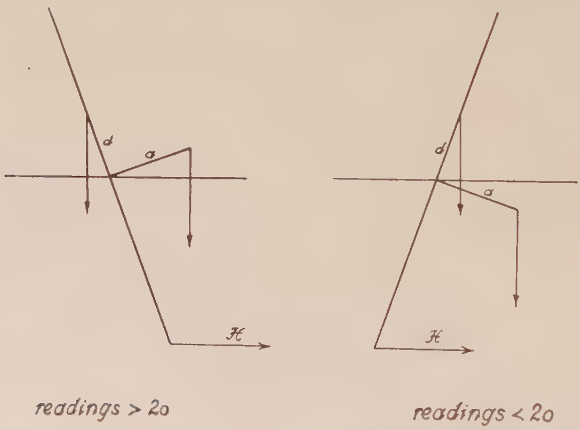

FIG. 12.—EFFECT OF GRAVITY ON HORIZONTAL FIELD BALANCE.

5. *Influence of Changes in Gravity*

Similar to the influence of changes in vertical intensity, the influence of gravity anomalies is different on both sides of the vertical position of the system; that is, on either side of the reading 20. Contrary to the effect of the vertical intensity, the positive effect is not numerically equal to the negative effect. If we have a reading smaller than 20, an increase in gravity will tend to make this reading still smaller; as the center of gravity is above the axis of rotation in our latitudes, the gravity produces a clockwise motion on both d and a (see Fig. 12). If we have readings greater than 20, the gravity produces a clockwise motion on a, but a counterclockwise motion on d. As frequently the effect on a is greater, an increase in gravity will also tend to make readings above 20 smaller. This decrease of a reading above 20, as said before, is not as great as the decrease of the same reading below 20. The exact conditions depend on the horizontal intensity for which the system is adjusted and the

[12] G. Angenheister: *Op. cit.*

vertical intensity affecting the scale value. In any case, the effect of a change in gravity may be derived by substituting the respective values in formula 11.

The gravity anomalies above large geologic disturbances seldom exceed 100.10^{-3} dynes. For a definite magnetic system ($H_1 = 0.19275$, $H_2 = 0.17925$, $s_1 = 65.041$, $s_2 = -25.041$, $Z = 0.428$, $\epsilon = 15\gamma$) the effect of an increase of gravity by 100.10^{-3} dynes has been computed. The new readings are $s_1 = 64.928$ and $s_2 = -25.165$. Both readings are smaller, as said before, but the difference is -0.113 scale divisions in the first case and -0.124 scale divisions in the second.

These figures are quoted not only as a proof of the theory but also to show that the influence of even relatively great gravity anomalies is negligible.

Consequently the normal change of gravity may also be neglected. In about 45° latitude, where the gravity gradient is at its maximum, a distance of about 70 miles corresponds to 1° change in latitude, or to an alteration of gravity of 87.10^{-3} dynes, which is still smaller than the assumed anomaly.

It has been said before that a scale value determined in one locality changes with change in latitude, on account of the variation of the vertical component and of the gravity. The change of the latter, however, is altogether immaterial as compared to that of the former as far as their influences on the scale value of a horizontal balance are concerned. If one would take a balance of which the scale value had been determined on the pole and would redetermine this value on the equator, the difference would be only 0.13γ on account of the change of gravity, if there was no change in the vertical magnetic intensity.

6. *Influence of Temperature*

The effect of temperature on any magnetic instrument is generally the reason why its sensitivity cannot go beyond a certain limit. This is not so much due to the fact that the temperature cannot be read in the field with much more accuracy than 0.5° C.; in fact, this accuracy would be sufficient for instruments compensated to have a very small temperature coefficient, if there were no other disturbing influences. The difficulties are produced by a thermic and elastic lag, which is different for different parts of the magnetic system. In addition to this, the mathematical treatment of these phenomena is a rather difficult matter, so that it is almost impossible to determine the true temperature correction in the field where the temperature gradients vary rapidly. The principal aim of the manufacturers of the instruments, therefore, is to decrease the amplitude of rapid temperature changes as much as possible by providing the balance with a double casing (see Fig. 7) and cork filling. In addition

to this, the magnetic system is compensated so as to have only a small temperature coefficient.

The temperature correction of the instrument furnished with the certificate of standardization holds for one temperature gradient only, which is usually small. If the operator attempts to check this value, he may not expect to obtain the same, unless he uses small temperature gradients. The temperature correction is not a constant quantity. It varies with the mechanical, magnetic and elastic condition of the system. It usually increases as the system grows older, but may be decreased again by an adequate treatment in the factory or a geophysical laboratory.

In determining a temperature correction, it will be frequently observed, for the reasons outlined, that in a set of observations the temperature correction found at the beginning is different from that obtained at the end. If the correction is small it is not unusual for the correction to change from a positive to a negative sign, and vice versa.

To my knowledge there has been only one publication that has taken into account these phenomena of lag in giving methods for the derivation of the correct temperature correction for observatory variometers—the article by O. Venske.[13] He describes the resultant lag (thermic and elastic combined) of a balance by a naturally rather involved exponential function and derives with it the correct temperature correction for temperature variations which are conceived as a system of superimposed waves.

In this paper, I will attempt for the first time to derive mathematically the actually observed irregularities of the temperature correction from the mechanical condition of the magnetic system. The following derivations hold for the horizontal balance only, but may be applied, with certain modifications, to the vertical balance also.

The resultant reading s_θ of a horizontal field balance at a temperature θ is influenced by three factors:

A. The drop of the magnetic moment M as temperature. We characterize this decrease by the coefficient μ, so that $M_\theta = M_0\,(1 - \mu\theta)$.

B. The expansion of the metals in the system. It produces a change of the horizontal and vertical distance of the center of gravity from the axis of rotation, which we designate by the coefficients p and q respectively so that $a_\theta = a_0\,(1 \pm p\theta)$ and $d_\theta = d_0\,(1 \pm q\theta)$.

C. The thermic and elastic lag of the system. It takes some time to warm up the various parts of the system, so that they do not have the temperature T of the box as indicated by the thermometer. At this temperature, their expansion is not strictly in proportion to it, but the elastic deformation lags behind. Formulas for this thermic and elastic lag will be given later.

[13] O. Venske: Thermische Nachwirkung bei erdmagnetischen Variometern. *Anhang zum Taet. Ber.* des Pr. Meteor. Inst. (1917–19) 80.

6A. *Influence of Drop of Magnetic Moment on Scale Value and Reading, Disregarding Expansion of Metals*

From equation 12, $\epsilon_0 = \dfrac{M_0 Z - mgd}{2fM_0}$; at the temperature θ,

$$\epsilon_\theta = \frac{ZM_0(1 - \mu\theta) - mgd}{2fM_0(1 - \mu\theta)} \tag{25}$$

Substituting for $\mu = 0.00048$, we obtain for a temperature change of 10° C. a decrease in scale value of only 0.12γ, which would mean for a deflection of 20 scale divisions (from the center) a change of only 2.4γ. This difference would have to be subtracted from any reading below 20 and to be added to any reading above 20.

If the metals in the systems are so adjusted that the readings are compensated for temperature, the influence of the temperature on the scale value also becomes so small that it may be disregarded. The influence of temperature on the readings is much greater than on the scale value only, if we do not consider again the expansion of the metals. Equation 11 was

$$s_0 = 2f\frac{(M_0H - mga)}{M_0Z - mgd}; \text{ hence, at the temperature } \theta,$$

$$s_\theta = 2f\frac{[HM_0(1 - \mu\theta) - mga]}{ZM_0(1 - \mu\theta) - mgd} \tag{26}$$

Substituting the values given above, we obtain, instead of a reading 20, the reading 13.10 if the temperature was increased by 10° C. That is a difference of 104γ; as emphasized before, the influence of the temperature on the reading (on the lateral center of gravity—component a) is much greater than on the scale value.

6B. *Influence of Drop in Moment and Expansion of Metals*

For equation 11 we write

$$s_\theta = \frac{MH(1 - \mu\theta) - mga(1 \pm p\theta)}{MZ(1 - \mu\theta) - mgd(1 \pm q\theta)} \tag{27}$$

Obviously, it must be possible to create a magnetic system which is compensated against (infinitely slow changes of) temperature. For such a system we note from equation 27 that

$$-MH\mu = mgap \text{ and } -MZ\mu = mgdq;$$

as $MH = mga$ on account of the latitude adjustment, we have

$$-\mu = p \text{ and } q = \frac{-\mu MZ}{mgd} \tag{28}$$

In other words, a must become smaller to compensate the apparent decrease in H, and d must also be smaller to compensate the apparent decrease in Z if the temperature rises.

Of course it is also possible to compensate the decrease in H by a decrease in d, but it is not advisable to arrange the metals in the system so that this comes true, because the compensation will hold only for readings on one side of the reading 20, while the effect is opposite on the other side. In other words, the temperature correction will be different for readings less than 20 and for readings greater than 20. In testing the temperature correction, therefore, it is always advisable to determine it twice, on either side of 20. If necessary, such type of compensation may be permissible for stationary instruments only, such as are used for the observation of variations, provided that records are taken in the same range for which the correction has been determined. Returning to equation 28, the question comes up as to how to arrange the metals in the system so that p and q will be negative. As we have two metals with quite different coefficients of expansion, we can arrange them unsymmetrically with respect to one another so that the center of gravity of one metal moves more to the left than the other moves to the right, and vice versa. The decrease of a may thus be obtained by arranging the steel blades toward the positive side and having, therefore, more aluminum on the negative side. The decrease of d can be produced by arranging more aluminum below the knife-edge. Of course, the shifting of the steel bars can not be done without a corresponding change in the distribution of the aluminum, because the correct position of the center of gravity in reference to a and d (latitude and ϵ adjustment) must be always maintained.

We assume such an unsymmetrical distribution of the masses of aluminum m_{al} and of steel m_{st}. We designate the distances of their centers of gravity from the axis of rotation in the horizontal direction by y_{al} and y_{st} and in the vertical direction by x_{al} and x_{st} respectively. Then p and q will be negative if we represent a and d by the following equations:

$$\left.\begin{aligned} a &= \frac{m_{st}y_{st} - m_{al}y_{al}}{m_{al} + m_{st}} \\ -d &= \frac{m_{al}x_{al} - m_{st}x_{st}}{m_{al} + m_{st}} \end{aligned}\right\} \qquad [29]$$

These equations express the distribution of masses in one plane only; namely, the plane of oscillation of the magnet. With sufficient approximation we can assume that the masses are symmetrical at right angles to this plane—an expansion parallel to the knife-edge would be without effect on the position of equilibrium of the system.

In deriving the equations for the relation of reading and temperature, we may substitute the linear coefficient of expansion for the cubic coefficient.

The linear coefficient depends, strictly speaking, upon the temperature. This relation is usually expressed by the equation

$$\beta = \frac{1}{l_0}\frac{dl}{d\theta}(\beta_1 + 2\beta_2\theta + 3\beta_3\theta^2 + \cdots$$

In addition, the expansion coefficient of some alloys (nickel steels particularly) varies irregularly as temperature, and is sometimes not even a conversible function of the temperature. The tungsten and cobalt steels used for magnet bars have thus far not been reported to show such irregularities within the temperature ranges that occur in the field.

For these temperature ranges we may also neglect the influence of the coefficients, which depend on the first or higher powers of the temperature in the equation given in the last paragraph, and designate the expansion of steel by the coefficient γ and that of aluminum by β, so that

$$\left.\begin{aligned} a_\theta m &= m_{st}y_{st}(1 + \gamma\theta) - m_{al}y_{al}(1 + \beta\theta) \text{ and} \\ -d_\theta m &= m_{al}x_{al}(1 + \beta\theta) - m_{st}x_{st}(1 + \gamma\theta) \end{aligned}\right\} \qquad [30]$$

which is

$$\left.\begin{aligned} am(1 - p\theta) &= m_{st}y_{st}(1 + \gamma\theta) - m_{al}y_{al}(1 + \beta\theta) \\ -dm(1 - q\theta) &= m_{al}x_{al}(1 + \beta\theta) - m_{st}x_{st}(1 + \gamma\theta) \end{aligned}\right\} \qquad [31]$$

From equation 31 we subtract equation 29, and obtain

$$\left.\begin{aligned} -apm &= m_{st}y_{st}\gamma - m_{al}y_{al}\beta \text{ and} \\ dqm &= m_{al}x_{al}\beta - m_{st}x_{st}\gamma \end{aligned}\right\} \qquad [32]$$

These equations permit an interesting practical application. We may compute from them how much we have to move the steel bars to obtain a complete temperature compensation. This displacement is figured from the position where the center of gravity of aluminum and the center of gravity of steel coincide. The results thus obtained theoretically may be expected to be in close agreement with the practical results if in the undisturbed condition of the system these two centers of gravity are close together. This comes true for the horizontal component of the center of gravity a, but not for the vertical d. The center of gravity of aluminum lies usually below that of the steel on account of the fact that there are three grooved bolts of german silver attached to the bottom of the system, and that there is also the small aluminum extension which holds the bronze screw for adjusting the scale value. On either side of the knife-edge, however, the lateral distribution of masses is very regular.

According to the theoretical considerations given in the last paragraph it should be expected that an adjustment of the lateral screws for latitude and a change in the position of the vertical screw for scale value should alter the temperature correction. These changes certainly affect

the correction, but the difference is usually smaller than the error with which the correction can be determined in practice.

We now proceed to a derivation of the lateral displacement of the steel bars which is necessary for the compensation against temperature. This derivation will be given for a only and may be carried out the same way for d, without, however, having much practical value, as said before.

Obviously, the necessary lateral displacement of the steel bars for compensation for the temperature effects is y_{st}. It may be determined from equation 32, in which a, p, m, m_{st}, γ, m_{al} and β are known, but not y_{al}. This second unknown may be eliminated, because there is only one value of y_{al} which corresponds always to one value of y_{st}. If the steel bars are moved, the aluminum must be moved accordingly (or, as that is technically not possible, aluminum must be removed on one side). In any case the latitude adjustment must always be maintained or $am = y_{st}m_{st} - y_{al}m_{al}$. We substitute, therefore, in equation 32, $y_{al}m_{al} = y_{st}m_{st} - am$ and obtain for the displacement of the steel bars, as $-\mu = p$,

$$y_{st} = \frac{am\,(\beta + \mu)}{m_{st}\,(\beta - \gamma)} \tag{33}$$

Substituting $\beta = 2\gamma$, the displacement is

$$\left.\begin{aligned} y_{st} &= \frac{am}{m_{st}}\left(2 + \frac{\mu}{\gamma}\right). \quad \text{The corresponding change of the center of gravity of the aluminum is} \\ y_{al} &= \frac{am}{m_{al}}\left(1 + \frac{\mu}{\gamma}\right) \end{aligned}\right\} \tag{34}$$

Hence, the amount by which we have to move the steel bars laterally is approximately

$$y_{st} = \frac{\mu}{10\gamma} \text{ in millimeters} \tag{35}$$

$$\text{as } \frac{am}{m_{st}} \text{ is very nearly } \frac{1}{100} \text{ and } \frac{\mu}{\gamma} \gg 2.$$

Substituting the numerical values of a magnetic system in equation 35, we find that the bars must be moved laterally about 4 mm. for complete compensation. The experiments that the writer has made so far with magnetic systems showed that the steel blades had to be moved 3 or 4 mm. The agreement of theory and practice is indeed remarkable.

Equation 35 proves clearly that the amount by which the steel bars must be moved depends on the quality and magnetization of the steel.

Similar computations may be made for y_{al} as well as x_{st} and x_{al}. The latter have no practical significance, as the metals are not symmetrically arranged from the beginning. It is better to determine x_{st} for each system by experiment.

The compensation of a magnetic system is a rather difficult matter and requires some experience. The user of a balance should not take a

magnetic system apart. If he feels that the temperature correction has become too great, after some time, the system should be returned to the factory or a geophysical laboratory for readjustment.

6C. *Thermic and Elastic Lag of the System*

It requires some time for steel and aluminum to take the temperature of the surrounding air, this time depending on their absolute conductivity for temperature, their ratio of exposed surface to mass, the state of motion of the surrounding air, and other factors. As primarily the ratio of surface to mass for the aluminum cube differs greatly from that of the steel, their temperatures usually differ. The expansion lags behind the temperature, and this lag is different for aluminum and steel on account of their different elasticity. The object of the following paragraphs is

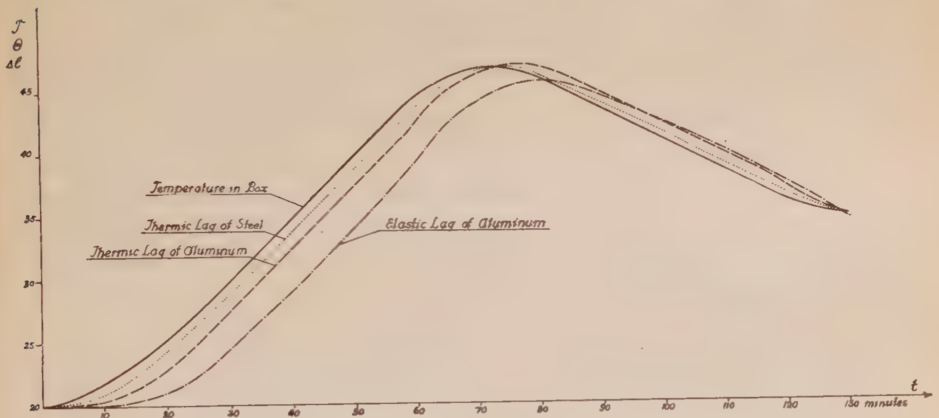

FIG. 13.—THERMIC LAG OF ALUMINUM AND STEEL IN A MAGNETIC SYSTEM.

to show how peculiarly a magnetic system will act under the influence of various outside temperature gradients even if it has been completely compensated for extremely slow variations in temperature.

For such a system, that is, a system in which all parts have the same temperature at any time, $\frac{ds}{d\theta} = 0$ or $s_\theta = s_0$ or

$$\frac{MH(1 - \mu\theta) - g[m_{st}y_{st}(1 + \gamma\theta) - m_{al}y_{al}(1 + \beta\theta)]}{MZ(1 - \mu\theta) - g[m_{al}x_{al}(1 + \beta\theta) - m_{st}x_{st}(1 + \gamma\theta)]} = \frac{MH - mga}{MZ - mgd} \qquad [36]$$

The dimensions y_{st}, y_{al}, x_{st}, x_{al} of such a system have been computed by the writer. This system has consequently the temperature correction 0 for any temperature θ.

Assume that this system is subjected to a temperature gradient, as shown in Fig. 13, the temperature first increasing rather rapidly and then dropping slowly, as illustrated. The result is shown in Fig. 15. We obtain first a positive temperature correction and then a negative

correction if we have a system with excessive thermic and elastic lag. The computation of this result has been a rather cumbersome procedure. The mathematical steps followed in obtaining it will be outlined in the next chapter.

The first task is to determine the actual temperature of the aluminum and of the steel at any time t if the temperature of the air is T. If the air temperature were constant, the temperature θ would approach it, following a simple exponential function as given by the Cooling Law of Newton, which states that

$$T - \theta = (T - \theta_0) \cdot e^{-\frac{t}{\lambda}} \qquad [37]$$

λ in this equation is a coefficient of retardation. We can write for any metal

$$\lambda = c\frac{C\sigma V}{KhS} \qquad [38]$$

where c is a constant, V the volume, S the exposed surface. The ratio $\frac{K}{C\sigma}$ is commonly known as the "absolute conductivity of temperature." K is the absolute conductivity of heat, σ the density and C the specific heat; h is a "coefficient of conductivity of boundary surfaces" and depends on the condition of the metal surface, whether it is rough, or polished, etc. Substituting the correct values in formula 38, as furnished by physical tables for aluminum and steel, we find that the ratio $\frac{\lambda_{al}}{\lambda_{st}} = \frac{5}{2}$, approximately.

The temperature T, however, is not constant, but changes. For such a case, the solution of equation 37 becomes difficult. If the rate of change of T is $\frac{dT}{dt}$, we may approximately let

$$T - \theta = \lambda \cdot \frac{dT}{dt} \qquad [39]$$

We thus obtain the actual temperatures θ for aluminum and steel which are illustrated in Fig. 13.

With these values θ_{st} and θ_{al}, we then plot the deformations Δl_{st} and Δl_{al}, as given by the relations

$$\left.\begin{aligned} \Delta l_{y,al} &= y_{al}\beta\theta_{al} \text{ and } \Delta l_{y,st} = y_{st}\gamma\theta_{st} \\ \Delta l_{x,al} &= x_{al}\beta\theta_{al} \text{ and } \Delta l_{x,st} = x_{st}\gamma\theta_{st} \end{aligned}\right\} \qquad [40]$$

We will have to assume that the deformations do not follow the temperatures immediately but are subjected to elastic lag. In connection with this the question comes up whether we have to consider elastic hysteresis also.

This question may be answered by considering the curves obtained with magnetometers if the temperature varies. The general type is such

as given in Fig. 14b. The reading first drops slowly as the temperature rises, then more rapidly when the temperature attains its maximum. With the temperature decreasing, the reading stays first down and then goes up slowly, to increase more rapidly at the end of the cycle. The reading is thus a "non-conversible" function of the temperature. The curve points clearly to elastic and thermic lag. This will be without hysteresis if the curve is like that in Fig. 14b. In Fig. 14a, however, there occurs hysteresis, because the readings are different at the beginning and the end of the temperature cycle. Such curves are extremely rare. If there is a difference of the two positions of equilibrium, it can be traced down in most cases to an abrupt change of the base position of the instrument or similar causes not exactly related to elastic hysteresis. We seem to be justified, therefore, in disregarding hysteresis for our problem.

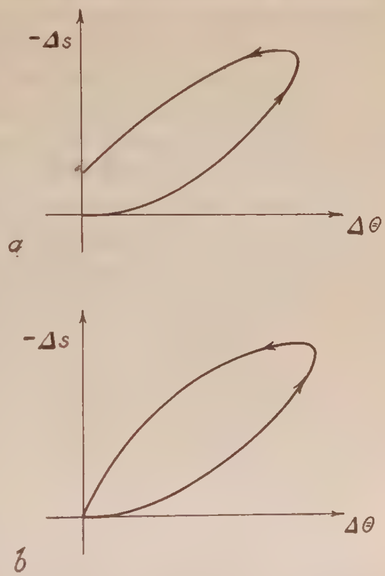

FIG. 14.—READING TEMPERATURE CURVES: *a*, WITH THERMIC LAG AND HYSTERESIS; *b*, WITH THERMIC LAG ONLY.

For the variation of the deformation Δl as time, we assume that the following relation holds, which is again approximate only, but sufficient for these purposes:

$$\Delta l - \Delta\Lambda = u\cdot\frac{d(\Delta l)}{dt} + \frac{\Delta l}{v}\cdot e^{\frac{t_{max.}-t}{R}} \qquad [41]$$

Where u and v are constants, R a "time of relaxation" (assumed as 30 min.) $t_{max.}$ is the time (after the beginning of the curve) of the maximum temperature; Δl is the deformation of the metal as given by equation 40, and $\Delta l - \Delta\Lambda$ is the difference of the actual expansion and the deformation as determined by those equations.

Using formula 41, $\Delta\Lambda$ curves have been constructed for aluminum and steel; one is shown in Fig. 13. The difference of the lags for aluminum and steel has been purposely somewhat exaggerated in order to show effects more clearly. In reality, the effects resultant from these assumptions, illustrated in Fig. 15, are not quite as great in practice, but their general character is usually the same.

Hence the results have been computed according to the following formula:

$$s_\theta' = 2f\left[\frac{MH(1-\mu\theta_{st}) - g\{m_{st}(y_{st}+\Delta\Lambda_{y,st}) - m_{al}(y_{al}+\Delta\Lambda_{y,al})\}}{MZ(1-\mu\theta_{st}) - g\{m_{al}(x_{al}+\Delta\Lambda_{x,al}) - m_{st}(x_{st}+\Delta\Lambda_{x,st})\}}\right] \qquad [42]$$

It is very interesting to note, from Fig. 15, that a positive and negative temperature correction may result for a completely compensated magnetic system. The curves have the character of sine curves; their amplitude is approximately proportional to the temperature gradient; their period is for the positive correction equal to the time elapsed from zero to a short period after the maximum of the outside temperature, while the period of the negative correction is about equal to the time elapsed between shortly after the maximum and the end. No attempt has been made, however, to define these relations more exactly and to use them for the elimination of the effects of varying temperature gradients from field observations. First, more accurate procedures have been suggested by Venske[14] which require the analysis of the variation of temperature into various trains of waves. Secondly, the consideration of the tempera-

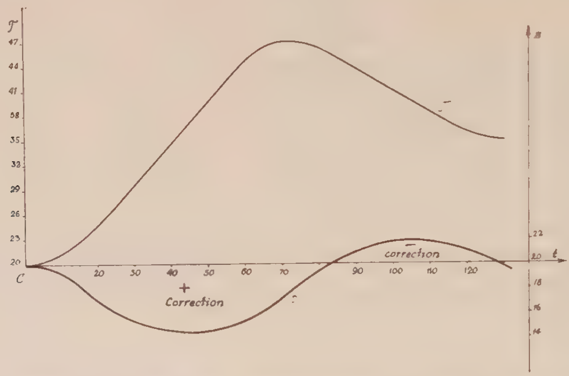

FIG. 15.—NATURE OF READINGS OBTAINED WITH A SYSTEM COMPENSATED AGAINST INFINITELY SLOW TEMPERATURE CHANGES, DUE TO THERMIC AND ELASTIC LAG (EXAGGERATED CONDITIONS).

ture gradient, while advisable in observatory work, is almost impossible in field observations with the magnetometer. As we have seen, from the equations given before, a continuous record of the temperature variations must be available. The temperature read at the time of observation is not sufficient for this analysis. The thermic history of the instrument must be known. This requirement, however, would necessitate attaching a thermograph to the packing box, which would be highly impracticable. For the time being, the observer has to be content with the fact that he can determine the temperature correction for his instrument only approximately. He can aim to reduce this error by protecting the instrument from rapid temperature changes. Especially, direct sunshine should be prevented from striking the instrument; the mistake of allowing this contact is frequently made. Instead of carrying an extra piece of equipment like a thermograph, it would

[14] O. Venske: *Op. cit.*

be better still to have some arrangement whereby the temperature of the inside of the instrument would be kept constant, such as an alternating current resistance heater, the current being furnished by dry cells and induction coil. Another possibility would be a magnetic instead of a mechanical compensation. Aside from requiring fundamental changes in the instrument, it calls for the constant use of an auxiliary magnet, the moment of which can not be kept sufficiently constant in the field and thus increases the error of the base correction. The use of steels with a small coefficient μ will probably be most advantageous of all, as it requires little mechanical compensation and consequently involves a small time lag. The best results relative to the avoiding of time lag might possibly be obtained by using a magnetic system made of one metal only, a solution which at present does not seem to be attainable for technical reasons.

6D. *Influence of Temperature on Effect of Auxiliary Magnets*

An increase in temperature gives rise to an expansion of the deflection rod, as well as to a drop of the moment of the auxiliary magnet. From formula 22, it is seen that $\frac{dF}{dr} = \frac{-3M_a}{r^4}$. If the north pole of the auxiliary magnet is in the north, the readings increase when the deflection rod expands due to the rise of temperature. If the north pole is in the south, the readings become smaller. The change of the readings due to this influence is extremely small and may be neglected. For a deflection of 40 scale divisions, for instance, produced by a magnet of 162 c.g.s. in a distance of 30 cm., the difference in reading due to an increase of temperature by 10° C. is only $\mp 0.3\gamma$.

The second effect of temperature on the magnet is much greater, yet in most cases is negligible. From the equation in paragraph A, page 286, we recall that

$$S'\epsilon = F = \frac{1}{r^3}M_a(1 - \mu\theta);$$

hence,
$$\frac{dF}{d\theta} = -F\mu. \qquad [43]$$

If the north pole of the auxiliary magnet is in the north, the readings increase when the temperature rises; if the north pole is in the south, they decrease. For the magnet and distance referred to, we obtain for an increase of 10° C. a change of the reading by $\pm 3\gamma$. This influence is small for conditions assumed but must be taken into account if larger magnets are used close to the instrument for compensating great anomalies. Taking, for instance, one of the largest magnets in use, with an approximate moment of 2000 c.g.s. in a distance of 20 cm., it figures out that a change of reading of about 12 gammas will be produced for every change of 1° C. in temperature.

7. *Influence of Shape of Knife-edge*

The shape of the knife-edge is of much greater importance for the behavior of the instrument than is generally assumed. It is necessary for the derivation of the theory of its influence to consider (1) the shape of the knife-edge in a cross-section parallel to the plane of oscillation of the system, (2) in a cross-section through the longitudinal axis of the knife-edge.

Due to the procedure of grinding and polishing the quartz, the shape of the edge will generally be the surface of a cylinder. Its cross-section, therefore, is a circle; in a few instances, a parabola. If the shape of the edge is circular, the distance of the center of gravity from the axis of rotation does not remain the same if the system is inclined. If the inclination is η, as before, and R the radius of curvature of the edge, the horizontal projection a of the distance of the center of gravity will be increased for readings above 20 by $R \cdot \sin \eta$ and for readings less than 20 it will be decreased by the same amount. Consequently the readings will be too small if they are more than 20 and too great if they are less than 20 in comparison to the true readings that should be obtained if the axis of rotation were a line. This influence is much greater than generally assumed. The vertical distance d of the center of gravity will also be changed when the system rolls off on the cylindrical surface. The amount by which it is changed is $R(1 - \cos \eta)$. This change is very small; there is hardly a noticeable difference between the scale value obtained if the surface of the knife-edge is a cylinder and the scale value corresponding to a straight line as axis of revolution.

The numerical evaluation of these formulas yields a surprising result —that the manufacturers of knife-edges, with their present machinery, can probably not produce a quartz edge that would give readings that would check the readings obtained if the axis of revolution were a straight line within less than about one-half scale division at the end of the scale. There will probably always be a difference of this "true" reading and the actual reading obtained with our present edges. Such a difference obviously brings about a difference in the scale value for different parts of the scale. This is not contradictory to the statement already made, that there is hardly any influence of the shape of the knife-edge upon d. The influence is appreciable on a, but as we determine the scale value empirically by dividing the deflecting magnetic field by the number of deflected scale divisions, it follows that the influence on a brings about, indirectly, a change of the scale value. To detect such a change, by experiment, it is not advised to determine the scale value on different parts of the scale, because this procedure is not accurate enough. It is better to use the auxiliary magnet in one position, move it up step by step and thus increase the readings gradually. The deflecting field is

then plotted against the readings obtained; the resultant curve gives an idea of the scale value in any portion of the scale, as $\epsilon = \frac{dF}{ds}$.

As stated before, there is some doubt as to whether it will be possible to manufacture a knife-edge that does not show a variation of scale value as deflection. For such a knife-edge, the radius of curvature should be smaller than seems to be attainable with our present mechanical devices.

Let s be a reading obtained if the axis of revolution were a perfect line and s' the actual reading obtained because the knife-edge has a cylindrical surface. We can then compute what the radius of curvature of the edge-surface must be in order not to make the difference $s' - s$ excessively great. We write as before,

$$s = 2f\left(\frac{MH - mga}{MZ - mgd}\right) \text{ and}$$

$$s' = 2f\left(\frac{MH - mg(a \pm R \sin \eta)}{MZ - mgd}\right) \qquad [44]$$

Assuming that the influence of the shape of the knife-edge on d is negligible, we have

$$s' - s = \frac{mgR \sin \eta}{M\epsilon} \qquad [45]$$

$$\text{or} \quad R = (s' - s)\frac{M\epsilon}{mg \sin 1°43'20''} \qquad [46]$$

The permissible difference $s' - s$ depends naturally on the deflection η; hence, the radius R must be computed for the reading on the end of the scale (+60 or −20) or a deflection of 40 scale divisions, which corresponds to an inclination of the system of 1°43′20″.

The factor of $(s' - s)$ in formula 46 is about 8.10^{-5} cm. for the customary magnetic system. If we expect to determine the anomaly in the "horizontal intensity" (the Schmidt balance measures only *differences in H*) with an accuracy of about 2.5 per cent., the difference $(s' - s)$ is one scale division, or the radius of the cylindrical knife-edge must be at least 0.001 mm. This accuracy is by no means great. If we expect the same accuracy at the end of the scale as in the middle—that is, about one-tenth of a scale division middle error—the radius of curvature of the knife-edge would have to be about 8.10^{-5} mm. There is some doubt whether such precision will be attainable in the process of grinding the edge. If it should be attainable, the problem arises how to keep this radius constant over the entire length of the quartz wedge (about 2.3 cm.). At any rate, it is always advisable to test the constancy of the scale value of a new instrument, over the range of the entire scale, as described. It would be possible to compensate this effect of the curvature of the knife-edge by an according arrangement of the optical system.

The disadvantage of this procedure would be that the compensation holds only for one knife-edge and becomes ineffective when it is necessary to replace the knife-edge. The discussed effect may be somewhat reduced by using magnets with greater magnetic moments in the system or by decreasing its sensitivity.

So far, we have discussed only the effect of a circular cross-section of the knife-edge. For a parabolic cross-section, similar formulas may be derived, but it is disadvantageous to use systems with knife-edges of such a shape, as the change of scale value with the magnitude of the deflection will be much greater.

It may also happen that the cross-section of the knife-edge is not regular. Very dangerous is a cross-section where the radii of curvature of the outline are greater on one side than they are on the other. Such a knife-edge will produce unsymmetrical deflections under the influence of a symmetrical (positive and negative) field such as is used in scale-value determinations. In other words, $\frac{s_1 + s_2}{2}$ will not check s_0. The only remedy for too great differences is replacement of the knife-edge. (For a circular cross-section, the readings will always be symmetrical; for a parabolic cross-section only if s_0 lies in the axis of symmetry of the parabola.)

The cross-section of the knife-edge in the plane of oscillation of the system, which we have considered so far, furnishes only an explanation for the change of the scale value in different parts of the scale. For the explanation of the difference of subsequent readings, we have to take into account: (1) the variation of this cross-section in the longitudinal direction of the quartz wedge; (2) the cross-section at right angles to it through the longitudinal axis, and particularly the variation of the vertical distance of the edge from the flat top of the wedge along this axis.

If the vertical distance mentioned in the second instance is the same for the whole length of the edge, it may happen that the deepest points of subsequent cross-sections, as referred to in the first case, are not in one straight line. In other words, if we should take a quartz wedge, lay it on its top so that the edge points up, the line where the edge touches the bearing is not straight, but wavy. If the arresting mechanism has some play sideways—that is, in the longitudinal direction of the knife-edge—different parts of this wavy edge come in contact with the surface of the bearing. Consequently, the horizontal distance of center of gravity and touching point of system and bearing does not remain constant. Extremely slight undulations in the edge are actually effective, and deviations of this wavy line from a straight line by a *fraction of a wave length of light* produce noticeable differences in readings. It is surprising that the manufacturers are able to turn out knife-edges that produce middle errors as small as they are actually observed. Of course, extreme

care must be devoted to the arresting mechanism. The effect of the undulations in the knife-edge can be almost entirely suppressed by a perfectly working arresting mechanism.

If this mechanism has some lateral play, we must expect that different parts of the edge come in contact with the bearing that may produce a lateral difference of the distance of center of gravity and turning point. We can compute how much lateral displacement of the turning point is permissible to produce a certain middle error of the readings. From the principal equation 11 it follows that

$$\frac{da}{ds} = -\frac{M\epsilon}{mg} \tag{47}$$

Even if we allow the middle error to be as great as one scale division, it follows from formula 47 that changes of the distance a of axis of rotation and center of gravity by 25.10^{-6} mm. are sufficient to produce such differences. This amount is one-twentieth of the wave length of yellow light. For one-tenth of a scale division this amount is even 10 times smaller. It is interesting to note that the computation of the difference in a made previously[15] for the example and for the vertical balance gave 50.10^{-6} mm.; this is in accord with the result obtained now, because $\epsilon_H = \frac{1}{2} \epsilon_Z$. The experience shows that the manufacturers turn out instruments which fulfill the requirement that the instrument should have a middle error of 0.1 scale division for $\epsilon = 15\gamma$. If we had derived the theory of this instrument first and had deduced the practical possibilities from its results, probably we would have assumed that it would be totally impossible to manufacture an instrument with a middle error of subsequent readings of virtually 1 to 2 gammas. On the other hand, the results demonstrate not only what amount of care is necessary but also what high degree of precision may be obtained with modern machinery and skilful mechanicians.

Equation 47 is also of interest from a viewpoint not related to the shape of knife-edge. In the transportation of the instrument it often happens that, due to shocks etc., tensions in the magnetic system are released or the masses of the system are displaced very slightly one with respect to the other. The resultant displacement of the center of gravity may only be as much as 25.10^{-6} mm. and the "base correction" will be one scale division, which is nothing unusual. The fact that for the vertical variometer the displacement necessary for a "base correction" of one scale division is 50.10^{-6} mm. shows that *it is necessary to handle the horizontal variometer in transportation with twice as much care as the vertical variometer.*

This obviously is due to the fact that the absolute sensitivity of the horizontal magnetometer is twice as great as that of the vertical magneto-

[15] C. Heiland and P. Duckert: *Op. cit.*, 312.

meter, while their relative sensitivities $\frac{H}{\epsilon}$ and $\frac{Z}{\epsilon}$ are about the same for higher magnetic latitudes.

It can be derived from the theory that the instrument is much more sensitive against horizontal shocks than against vertical ones. Generally speaking, the readings are much more affected by physical influences which change a than by those which change d. This is seen from another differentiation of the principal equation 11, which gives

$$\frac{dd}{ds} = \frac{2f}{mgs^2}(MH - mga) = \frac{2fM\epsilon}{mgs} \tag{48}$$

For an extreme deflection of 40 scale divisions, we obtain from formula 48 that a change in d by 83.10^{-5} mm. will produce a difference of one scale division. In other words, the vertical change of the center of gravity must be about 30 times as great as the horizontal change in order to produce the same difference in the reading (at the end of the scale). Consequently, we may conclude that the balance is at least that much more sensitive against horizontal shocks than against vertical ones.

Returning to the influence of the knife-edge, we have considered before the fact that the axis of revolution is not a straight line in the longitudinal direction of the edge. There is also the possibility that the radius of curvature of the cross-section changes in that direction even if the axis is a straight line. This would produce an increased middle error if the arresting mechanism has a play in the longitudinal direction of the knife-edge. The error produced by this change in radius of curvature may be expected to be small and will depend on the deflection.

If the knife-edge is chipped and the arresting mechanism has some play, either in the longitudinal or transversal direction, different parts of the irregular surface of the edge will come in contact with the bearings in subsequent readings. This will be equivalent to a change in the distance of center of gravity and axis of rotation; in other words, a will be changed. For this change we have derived the equations. The influence of such chips in the knife-edge is considerable and may often cause middle errors of several scale divisions; that is, make the instrument useless. Therefore very great care is advised in releasing the system. This should always be done very slowly and while the system is being released the observer should always observe its behavior in the telescope. If a knife-edge is slightly chipped, it is not recommended that the knife-edge be replaced; move the bearings until they will touch undamaged portions of the edge. Replacing the edge is like making a new magnetic system. For another knife-edge, the temperature coefficient, the latitude adjustment and the scale value will be different. The replacement of the knife-edge is a delicate affair because of the necessary complete

readjustment of the system and should only be done by experts, in the factory or a geophysical laboratory. Moving the bearings, however, will not only change slightly the latitude adjustment, but will sometimes alter considerably the scale value, depending on the depth of the chips in the knife-edge. Therefore, the scale value must be redetermined after the bearings have been moved.

This change in scale value is the chief effect of changes in the vertical distance of the edge from the top of the quartz wedge; in other words, we proceed now to the discussion of the influence of the shape of the edge in the longitudinal cross-section (item 2, page 298). If the system has a scale value of 15γ and there is only a chip of 0.01 mm. the scale value will then be 19.5γ. In general, the changes in d necessary for a change of ϵ is given by the equation

$$\frac{dd}{d\epsilon} = -\frac{2fM}{mg} \qquad [49]$$

so that a change in d by $2/1000$ mm. will produce a change in the scale value of 1γ.

This change in d will, obviously, not only produce a change in ϵ, but also in the reading itself. In addition, it must be expected that a chip in the knife-edge will produce a change in a, so that a change in reading is also produced on that account, as stated. If the arresting mechanism has a play in the longitudinal direction of the edge, it is likely that different portions of the wedge with different vertical distances of the edge from the top come in contact with the bearings. This, in other words, explains also the existence of the middle error of subsequent readings.

This section will have demonstrated sufficiently the necessity of extreme care with the knife-edge.

8. *Optical Influences*

There are three optical influences which require a brief discussion: (1) a deviation of the optical axis of the telescope from the vertical; (2) a deviation of the plane of the mirror attached to the system from the magnetic axis of the system; (3) an eccentric position of the graduated scale, in reference to the optical axis of the telescope, in the direction of oscillation of the magnetic system. If these optical factors remain constant for the instrument during a set of readings taken at different localities, there is no correction required. If, however, the top of the instrument is taken off to clean the inside or to make other adjustments, it is necessary to repeat measurements at the base station.

These three factors together may produce a reading that is different from 20 if the magnetic axis of the system is exactly vertical. It is of some importance to know which reading corresponds to this vertical

position, as we have to figure the correction for vertical intensity (page 281) from this reading. On the other hand, it is very difficult to determine this reading. An approximate determination is possible by adjusting the position of the system to 20 with a horizontal small magnet placed horizontally on the side of the instrument or with the lateral screw; then a strong vertical magnet is inserted in the deflection tube so as to direct the magnetic axis of the system as nearly vertical as possible. The resultant reading is the figure to be used in formula 24. The difference of this reading and 20 may also be removed by a lateral adjustment of the graduated scale.

Relation of Theories of Vertical and Horizontal Balances

In going over the theory which has been set forth in this article the reader will have noticed that many points have been discussed in great detail which were only very briefly dealt with in the theory of the vertical field balance published several years ago.[16] The reason for this is that considerable experience has been gathered from the reports received by many users of the more than 200 vertical balances now employed in this country, as well as frequent work done on these instruments. These data could readily be applied also to the theory of the horizontal field balance. Users of the vertical balance in turn will therefore be able to modify readily this theory and to apply it to their instruments. In order to enable them to do so, the general formulas for the vertical balance have been included in this article (equations 5 and 7); the theory of the influence of false orientation, magnets, temperature, knife-edge can thus readily be derived for the horizontal balance by merely substituting the formulas for the vertical balance.

Accuracy of Measurements

We have discussed a number of errors produced by various influences; also, the magnitude of these errors. It has been found that these errors, as would be expected, are a function of the magnitude of these influences. The question comes up: What is the error of measurements if all these influences are kept down as much as is possible with the present arrangement of the instrument without hindering the speed of observations too much? In other words, the question is *not:* What is the middle error of the result on one station? It can be readily determined that this middle error is less than 1 gamma for four to six readings. The problem is to determine the accuracy of the whole survey; that is, the error of an individual station. The best criterion for determination of this error is the difference in the horizontal intensity obtained when the station is repeated.

[16] C. A. Heiland and P. Duckert: *Op. cit.*

This accuracy depends, naturally, on the accuracy desired for the particular purpose, and thus on the care of the observer. In other words, when surveying a magnetite deposit where the anomalies are several thousand gammas, the error may be readily increased to about 50 gammas without affecting the interpretation. Then it will be advantageous to disregard the correction for variation and temperature, and it might not even be necessary to level the instrument accurately. When surveying a salt dome, however, we will find that we have to check all these influences as precisely as possible.

The maximum error, and thus also the average error, obtained for a magnetic station depends altogether on the particular situation or the accuracy desired.

Hence the only criterion, which may be said to define correctly the dependability of the instrument, is the error obtained under the highest requirements for accuracy. This error depends naturally upon the care which the observer wants to devote to the checking of various detrimental influences. It would be false to determine this error by summing up the minimum errors produced by the various influences set forth above. This would not be in accordance with the theory of probability.

The only and best criterion for the accuracy is the error that is obtained when a station is repeated. Actual experiments with the horizontal magnetometer have shown that it is possible to obtain a difference of not more than about 5 gammas between the first and another reading taken at the same station within an interval of several months; that is, if the observer is careful, observes the diurnal variation and works at an average speed of about 15 to 20 stations a day in good terrain about 1 to 3 miles apart. To state the maximum error obtained is impossible, as mentioned, as it depends on the accuracy, speed and care desired.

Rieber[17] has come to the conclusion that it is impossible, under average working conditions, to obtain closer checks than within 24 gammas with the type of magnetometers herein described. This estimate is not in agreement with the actual results obtained. In addition to the reasons stated on page 276, this discrepancy is due to the fact that Rieber assumes for these Schmidt balances a method of observation which is not being applied on these instruments (attachment of a magnet in the deflection tube, and change of its position until it compensates the anomaly and a zero reading is obtained). This principle is applied in the Thomson-Thalén magnetometer and the Oertling field balance but is very detrimental to the accuracy of the results. The discrepancy between Rieber's estimates and the actually obtained errors would be greater yet had he taken into consideration the main sources of error—temperature and displacement of center of gravity.

[17] F. Rieber: *Op. cit.*

Under average working conditions, the difference of two set-ups on the same station may be kept down to within 8 gammas. As an example, we may state results obtained under very adverse conditions, in Alaska, where, on account of the vicinity of the magnetic pole, the amplitude and irregularity of the diurnal variations introduces greater errors. E. E. Maillot has kindly informed me that he repeated, for instance, eight stations after a period of six weeks and obtained differences which varied from 1 to 8 gammas. The speed in making such observations was about 40 stations a day, with the vertical magnetometer.

These accuracies are altogether sufficient for even the most precise geologic work. It is hardly probable that anything will be gained by merely increasing the sensitivity of magnetometers.

Rieber has the merit of having built a very sensitive micromagnetometer. This instrument has been used so far only for the determination of angles, where a great accuracy always may be obtained, because the influence of temperature and of many other factors affecting intensity measurements (change of magnetic moments, etc.) cancel out. It is well known that the declination may be readily determined with an accuracy of 0.001° by using a fiber for the suspension of a magnetic needle and mirror and telescope for reading. The rotary inclination furnishes an accuracy of 0.1″ of an arc, or 0.002°. Besides, these instruments are simple.

Unfortunately, we cannot make very much use of this fact, because the geologic interpretation of disturbances of the magnetic angles is difficult. We need magnetic intensities. Modifying Rieber's magnetometer for the determination of intensities as suggested by him, the problem of constancy of current and temperature arises. In order to obtain the same accuracy with Rieber's micromagnetometer as is now obtainable with the field balance, much additional equipment would be required, which would make it impossible to obtain the same speed with it as can be obtained with the field balances.

Progress in magnetometer construction may be obtained in the future only by a design on which difficulties of temperature and center of gravity are completely removed, which may be operated at a reasonable speed and which is convenient in transportation.

Appendix I. Determination of Constants and Corrections, and Adjustment of Instrument

A. Latitude Adjustment

When ordering a magnetometer, the customer should inform the factory in which magnetic latitude he expects to use the instrument, so that the manufacturers may arrange the steel bars so that the system is approximately balanced in the area of survey. The steel bars must

never be taken loose by the observer, because this changes everything, temperature correction, latitude adjustment and scale value. A skilful worker may be in a position to replace the steel bars so as to have again the proper latitude adjustment and scale value, but he is likely to miss the temperature compensation altogether or produce at the most a system that may be compensated against temperature but has an enormous time lag. The only necessity to take off the bars may arise if a knife-edge has to be replaced, but that operation should also be left to the factory or a geophysical laboratory. The observer should always remember that the less he changes a system, the better off he will be. There is really no need to move the steel bars in ordinary use of the instrument because the adjustment of the lateral screws makes possible the use of the instrument at least 20° in latitude north and 20° in latitude south of the latitude for which the instrument has been adjusted. Upon receipt of the instrument the observer should go to a point in his area of survey which is supposed to be fairly free of magnetic disturbances and should adjust the lateral screws until the reflected scale is in the field of vision and the reading is from 10 to 30. If the reading is 20 on this station, the instrument carries a normal increase or decrease of the horizontal intensity of 40 scale divisions, or 600γ either way, if the (normal) scale value is 15γ; that is, on the North American continent, about 1° north and 1° south in latitude from the original point. If within this range the reflected scale should disappear on account of too great local anomalies, auxiliary magnets should be used in the manner previously described.

If the observer wishes to use his instrument in North America as well as in South America, in localities that are more than about 20° in latitude apart, it is better to have two systems, one permanently adjusted for North America, the other for South America. If the lateral screws are turned in or out excessively and their position differs greatly from their position when the instrument arrived, it is advisable to test the temperature coefficient and scale value after this adjustment. Although it should be superfluous to emphasize it, it must be remembered that all adjustments and measurements in general should be made far enough away from trolley lines, direct current motor or generator plants, direct current power lines, streets, etc. to avoid their influence. The observer himself must have no iron objects upon him. The presence of a few ferruginous objects at a station where such adjustments are made is not dangerous unless they are too close to the instrument and unless their location is changed during the experiment.

B. Scale-value Determination

It is well to test whether the instrument shows an excessive variation of the scale value with the deflection by measuring and plotting an $H - s$ curve (see page 296). If the scale value should remain fairly

constant, so that an average value for ϵ can be used, the determination of this average scale value is made as follows. A magnet of known moment is placed at a known distance under the instrument horizontally. For this, the deflecting rod should preferably be used. First the reading is made without the magnet, or with the magnet vertical, if far enough from the instrument. Then a reading is made when the north pole of the magnet is in the north and a second reading when the north pole is in the south. This operation is repeated by turning the magnet about the axis of its holder two or three times. Measurements should be made at four or five distances from the deflecting magnet. For the computation of the results, formula 21 is applied. For the scale value determination it is advisable to use, if necessary, a second magnet in the same horizontal plane as the instrument to adjust the undisturbed reading to about 20. The scale value should be about 15γ, for the average geological purposes. It is useless to make the instrument too sensitive, as the errors (friction on bearings, shape of knife-edge, temperature correction) are increased in proportion. The sensitivity may be changed by turning the vertical screw underneath the wedge, but the screw hardly needs a change after the scale has been once adjusted for approximately 15γ. Fig. 16 illustrates how the scale value depends on the turns of the vertical screw.

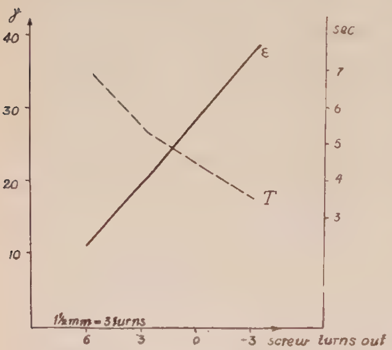

FIG. 16.—RELATION OF SCALE VALUE AND PERIOD TO POSITION OF VERTICAL SCREW.

Although all magnets are seasoned, there is always a drop of their magnetic moment in use. With the vertical balance, this causes the scale value to increase slowly. The opposite happens to a horizontal balance; the scale value decreases, but in proportion not as much as the scale value of the vertical balance (see formulas 8 and 12 for scale values). If the instrument is to be used for great anomalies and if the movement of the vertical screw does not make the instrument unsensitive enough, another screw may be inserted of heavier material (gold, for instance). The scale value should be determined about once every month or every other month. The determination should be made at about the same temperature.

In case no magnets are available, the procedure described on page 274 may be used. Also, the observation of the period of oscillation of the magnetic system gives a good approximate idea of the scale value (see Fig. 16). This figure shows also that it would be possible to make the instrument theoretically infinitely sensitive, but, as emphasized before, there is no gain by such "supersensitive" instruments, unless the sources

of error (temperature, knife-edge, etc.) are also decreased by special arrangements. The normal scale value of 15γ permits of determining the horizontal intensity on one station with an optical accuracy of $\pm 1.5\gamma$, which is just as accurate as the best determination of the diurnal magnetic observations.

C. Moments of Auxiliary Magnets

These moments should be determined about every half year in a geophysical laboratory. Magnets which have just been received from the factory need standardization about every two months for the first half year. It is always advisable to leave one magnet at home and use the two others in the field. If an observer has two instruments, one for the observation of the diurnal variation, the other for field work, it is advisable to leave one complete set of magnets at home. The advantage of this procedure is that the magnets left permanently at one place are better protected against changes than those used in the field and affected by shocks, etc. In other words, the magnets left at home may be used as standards from which the moments of the field magnet may be determined as often as desired.

For this purpose, the deflecting rod should be used. The standard magnet with the moment M_{st} is placed at an arbitrary distance and the deflection (double field) $(s_1 - s_2)_{st}$ produced. The unknown magnet with the moment M_x is placed at the same distance after M_{st} has been removed and the deflection $(s_1 - s_2)x$ is produced. Then it is possible to derive the moment of the unknown magnet from that of the standard without knowing their distance (only if k is used, the distance should be known) using the formula

$$M_x = M_{st} \frac{(s_1 - s_2)_x}{(s_1 - s_2)_{st}} \cdot \frac{k_{st}}{k_x} \qquad [50]$$

k_{st} and k_x are the correction factors to be derived from formula 21. From formula 50 it is seen that $\frac{k_{st}}{k_x} = 1$ if magnets of the same length are used for the comparison. The most accurate results are obtained therefore by comparing magnets of the same dimensions. The deflection $s_1 - s_2$ should be observed at least twice at the same distance of the magnet and measurements should be made in at least four or five distances.

Instead of using magnets for all the operations for which they are applicable, coils supplied with electric current (Helmholtz coils) could be used. As far as the author knows, only one or two operating companies employ these coils; all others use magnets. The reason is as follows:

In the field use of a magnetometer, accidents may happen which require an immediate standardization of the instrument. A magnet is

always available, whereas a Helmholtz coil must be left at home on account of its great dimensions as well as because there is no provision made on the magnetometers for the attachment of such coils. The main objection against magnets seems to be their loss of magnetic moment and their change with temperature. The actual experience shows, however, that it is possible to keep good magnets constant within 0.5 per cent. during more than one-half year's field work. Besides, this change can always be checked by comparing the field magnet with the magnet left at home. The sensitivity of magnets against temperature changes is not a great handicap provided they are used far enough away from the instrument and provided that scale-value determinations are made always at the same or similar temperatures.

Virtually the same objections as made against magnets could be raised against Helmholtz coils. If dry cells are used to supply the current, their output changes with time. It would thus be necessary for accurate work to use a potentiometer and standard cell with the Helmholtz coil, which in turn makes the equipment more cumbersome. If the coil is wound with copper wire, the resistivity changes with temperature. The change in the field of a permanent magnet equals $\Delta F = -F\mu$ and the change in the magnetic field of a solenoid is $\Delta F = F \cdot \frac{-\alpha}{1 + \alpha}$, which equals approximately $-F\alpha$ (α being the temperature coefficient of resistance). Hence it follows that the relative sensitivity of a magnet against temperature as compared to that of a Helmholtz coil is proportional to the ratio of $\frac{\mu}{\alpha}$ which means that a magnet is about 10 times less sensitive against temperature change than a copper Helmholtz coil. It thus seems that for the time being a Helmholtz coil may be useful for the standardization of a magnetometer only under permanent laboratory conditions.

The error of magnetometer readings produced by changes of magnets such as recently described by Rieber is not in accordance with the experience. First of all, the procedure of observation of a magnetometer upon which such estimates are based is not applied to the Schmidt balance. In other words, the deflecting magnets are not used permanently and their distance is not changed, as assumed by Rieber, until zero reading is obtained. In reality, their use is restricted as much as possible to standardization only. Of course, there occur changes in the magnetic moment of the deflected system, but experience shows that they are not only much smaller than changes due to displacements of the center of gravity, but also occur as a more or less linear variation. In other words, such changes may be eliminated to a very great extent by the interpolation of the base correction.

D. Determination of Central Reading, and Vertical-intensity Correction

Directions should be followed as given on page 302. The correction is computed from formula 24. To facilitate the computation of this correction, it is recommended that 20 be subtracted from all readings beforehand.

E. Determination of Influence of Temperature

From the detailed discussion of this influence (pages 285–95) it will be seen that it is very difficult to determine it accurately. It is possible to determine an average correction for the instrument by observing a number of readings s_0, s_1, s_2, s_3 . . . at the temperatures θ_0, θ_1, θ_2, θ_3, etc. Then the temperature correction is the average of the ratios:

$$-\frac{s_1 - s_0}{\theta_1 - \theta_0};\ -\frac{s_2 - s_0}{\theta_2 - \theta_0};\ -\frac{s_3 - s_0}{\theta_3 - \theta_0}\ \text{etc, or}$$

$$-\frac{s_1 - s_0}{\theta_1 - \theta_0};\ -\frac{s_2 - s_1}{\theta_2 - \theta_1};\ -\frac{s_3 - s_2}{\theta_3 - \theta_2}\ \text{etc.}$$

A least-square adjustment may be applied to compute the best average of these determinations.

The correction which has been derived in such a manner is approximate only and its application is frequently not very satisfactory. As stated, the correction depends on the gradient of temperature. Its magnitude may also be different on different parts of the scale. It is therefore recommended that four empirical charts be used for temperature corrections, which represent the variation of the reading under the following conditions:

1. Start at 10° C., heat up to 30° C., cool down to 10° C.
   Temperature gradient: 1° C. per 5-min. interval.
2. Start at 10° C., heat up to 30° C., cool down to 10° C.
   Temperature gradient: 1° per 2-min. interval.
3. Start at 20° C., heat up to 40° C., cool down to 20° C.
   Temperature gradient: 1° C. per 5-min. interval.
4. Start at 20° C., heat up to 40° C., cool down to 20° C.
   Temperature gradient: 1° C. per 2-min. interval.

In the field, too great temperature gradients should be avoided: caution should be taken that the gradients covered by the foregoing experiments are not exceeded. The observer keeps record, as close as that is possible, of the temperatures recorded by the instruments, plots a temperature-time curve for the whole day and applies the correction as derived for the particular gradient (up or down) and for the particular temperature from the charts; if necessary, using interpolation. The determination of the reading-temperature curve is done best by using a

metallic heating box. This box is heated by alternating current of 110 volt going through a double (or noninductive) winding to avoid the effect of surges or demagnetization of the magnetic system. The magnetic variation must be taken into consideration during these experiments by observing a second instrument which is kept at constant temperature if possible. The observations on both instruments are made with magnetic systems permanently released.

The reading-temperature diagrams should be checked about every other month. The correction is best expressed in gammas. If it is stated in scale divisions, it must be multiplied with the ratio of old and new scale value if the latter is changed.

F. Determination of Base Correction

If a station has been repeated and s_1 is the first corrected reading and s_2 the second corrected reading, the base correction is: $-\frac{(s_2 - s_1)}{n} \cdot p$ for the station having number p after the first reading, if n is the number of stations that were occupied between the two subsequent readings on the base station. This formula applies only, however, if the difference $(s_2 - s_1)$ is small (about one scale division) and if the assumption is justified that the change in the base reading took place continuously. If the record of the observer indicates that an abrupt change (due to a shock, etc.) is likely, the entire difference $s_2 - s_1$ must be applied as correction to all stations occupied after that event. It is recommended to check in on the base station three times a day—morning, noon and night.

G. Determination of Auxiliary-magnet Correction

See directions given on page 280.

H. Determination of Middle Error of Subsequent Readings on One Station

Numerous subsequent readings are made, interrupted by arresting and releasing. To find the middle error, a least-square adjustment of the obtained values may be made. The difference between maximum and minimum value should not exceed two-tenths of a scale division. If it does, there is either a strong magnetic variation due to natural or artificial causes, or the observer is not free of iron, or the magnetic system has been released too slowly so that the points on the arresting mechanism pushed the magnetic system over to the side, or there are impurities on the knife-edge, or the knife-edge or the bearings are damaged.

I. Determination of Variation Correction

The diurnal variation curve of a magnetic observatory is used, if close enough, and replotted after the records have been corrected for

temperature. The same is done if the observer uses his own photographic recording outfit.[18] If a stationary instrument for visual observations (like the field instrument) is used, it is read every 10 or 20 min., the readings are corrected for temperature and the variation curve is plotted. The resulting amplitudes in all three cases must be multipled by the scale values of the instrument. This amplitude (in gammas) must be subtracted as variation correction from the readings of the field instrument for the time when the readings were taken.

K. Determination of Latitude Correction

As the horizontal intensity increases towards the equator, all stations south of the base station must be provided with a negative correction (opposite for the south hemisphere). There may also be a correction for stations east or west of the base station if the isodynamic H-lines on the official magnetic maps do not go exactly east-west. The distance of two subsequent lines in the area under survey is measured in the north-south and in the east-west direction and the difference in their intensity divided by this distance. The result is the correction per mile distance from the base station.

Appendix II. Application of Instrument

The following is a scheme which may be followed in making the measurements in the field:

1. Select a station that is far enough away from all objects containing iron or steel or electrical current, such as fences, oil tanks, pipelines, derricks, boilers, all kinds of mechanical and electrical plants, d.c. power lines, wells, bridges, railroads, trolley lines, etc.
2. Remove the instrument from the box so that it acquires the temperature of the air.
3. Remove all ferruginous objects.
4. Set up and level tripod. Orient tripod head with compass in the magnetic meridian, fasten with screw. Read and note position of index.
5. Set up instrument on tripod, *N* toward north, and relevel with tube levels.
6. Turn lateral mirror until scale appears equally lighted, focusing by moving eyepiece.
7. Read and note temperature.
8. Make a number of readings, interrupted by slowly arresting and releasing. At least three readings are necessary, their difference not to exceed two-tenths of a scale division.
9. If necessary, screw in auxiliary magnet.

[18] C. A. Heiland: Directions for the Use of the Vertical and Horizontal Field Balance. Askania-Werke A. G., Houston, Tex.

10. Read and note time.
11. Read and note temperature.
12. Dismantle station. Avoid horizontal jolts when putting instrument back in box.

Appendix III. Computation of Results

The computations are most conveniently made in a book, of which one side of each page is arranged for entering the field data, while the other side takes the computations. A standard form commonly in use is shown herewith.

Left-hand Pages of Book

No.	Location	Date	Time	Tripod	Readings s	Magnet	Temperature
2	One mile north of base	July 20 1926	11 a. m. 58 −12.02	42°	3.8 3.9 4.0	$M = 100$ $r = 30$ N in S	32°.0

Right-hand Pages of Book

s-20	(s-20) $\times \epsilon$	Temp. Corr.	Magnet Corr.	Var. Corr.	Latit. Corr.	Vert. Int. Corr.	Base Corr.	Corr. Read	Minus Base = result	Remarks
−16.1	−242	−12	−370	−279	−8	−6	−12	−889	-619γ	Base = -270γ

The final results may be represented graphically in three different ways.

1. By lines of equal anomaly of the horizontal intensity or ΔH-curves. Their interval cannot be chosen smaller than the middle error of the result; 5 gammas is probably a fair average for the uncertainty of the results under ordinary circumstances. It is therefore not advised to make the intervals of the isodynamic lines less than 15γ.

2. By profile curves at right angles to the strike of the magnetic formation. However, the total value of ΔH must not be plotted, but its projection ΔH_p upon a direction at right angles to the strike. If α is the strike of the formation from astronomic north over east, then the following equations may be applied for the computation of ΔH_p.

(a) $\alpha < 90°$

Declination (δ) east.

$\Delta H_p = \Delta H \cdot \sin (\alpha - \delta)$

Declination west

$\Delta H_p = \Delta H \cdot \sin (\alpha + \delta)$

(b) $\alpha > 90°$

Declination east.
$\Delta H_p = \Delta H \cdot \sin (\alpha + \delta)$
Declination west.
$\Delta H_p = \Delta H \cdot \sin (\alpha - \delta)$

3. By disturbance vectors, if the vertical intensity has also been observed, which is usually the case. These vectors are plotted for the vertical plane at right angles to the strike of the magnetic formation, The magnitude and direction of the vector is most conveniently found by geometric construction. ΔH_p is plotted northward if positive and southward if negative. ΔZ is plotted downward if positive and upward if negative. If φ is the inclination of the vector (positive from the horizontal north direction downward),

$$\text{tang } \varphi = \frac{\Delta Z}{\Delta H_p} \text{ and the vector} = \sqrt{\Delta H_p^2 + \Delta Z^2}$$

DISCUSSION

D. C. Barton, Houston, Tex. (written discussion).—In connection with the comments on the history of the modern use of the magnetometer in this country, it may be interesting to record that apparently the first Schmidt-Lloyd variometer brought to this country was imported in March, 1924, at the writer's instigation, by Mr. DeGolyer of the Rycade Oil Corpn. and Amerada Petroleum Corpn. Beginning with a survey of the Blue Ridge salt dome in late March or early May, 1924, the use of the magnetometer was tested out under the writer's direction on the salt domes of the Gulf Coast, on faults in eastern Kaufman and southeastern Hunt counties, and at Thrall, and in the winter of 1924–25 was tested out under Sidney Power's direction on various types of structure in Oklahoma, Texas, and the Panhandle area. A test survey was made of the Nocona granite ridge in May, 1925, which showed definitely that the magnetic method had considerable power in mapping certain granite ridges. Torsion wire magnetometers made by the now defunct G. P. G. of Freiburg were imported during the summer of 1925 by the Rycade Oil Corpn. and were put in use by the Rycade Oil Corpn. and the Amerada Petroleum Corpn. Our extensive tests of the salt domes indicated that although the presence of the salt dome was indicated in the results of the magnetic survey, the indication was too unreliable for practical use in competition with the torsion balance or seismic methods.

C. A. Heiland (written discussion).—I am glad indeed Dr. Barton made these contributions to the history of the magnetometer as applied to the location of oil deposits for hitherto I have thought that the use of the magnetometer in search for granite ridges started at a somewhat later date. When I came to this country as the representative of the manufacturers of these instruments in 1925, I suggested to officials of the Humble Oil and Refining Co. to try the instruments on oil-bearing structures underlaid by granite ridges. The discovery which, according to Dr. Barton, had been previously made by the Rycade Corpn. that granite ridges could be located with the magnetometer was evidently kept secret by that company, because in the autumn of 1925 all the companies with whom I talked about the possibilities of magnetic prospecting for granite ridges were very dubious about it. At the end

of that year, the Humble Oil Co. and two or three other purchasers of magnetometers were successful on granite ridges. Some of these released this information, which soon became widely known, the manufacturers of the instruments of course being keenly interested in spreading it. The beginning of the year 1926 thus marks very distinctly the start of an epoch of general use of the magnetometer. While in 1924 only two of the modern magnetometers were used in this country and in 1925 not more than five, the end of the year 1926 found about 60 magnetometers sold and in use.

It often happens in science that facts are rediscovered, which may have been found years, decades, and even centuries before. Frequently, the original discoverer was not certain about his results and hesitated to publish his discovery; or he wanted to utilize his invention himself and kept it secret; or his contemporaries paid no attention to him, believing his discovery to be meaningless or of no practical value, perhaps only because he did not give an altogether correct description of it.

Therefore it cannot surprise us that we have to go back about twenty years to find the scientist to whom credit must be given for first calling attention to the fact that oil fields are frequently associated with magnetic anomalies. George F. Becker published a paper in 1909 entitled "Relations Between Local Magnetic Disturbances and the Genesis of Petroleum." However, he interpreted his discovery by the probable genesis of the oil from iron carbides. This evidently was the reason why his discovery was completely disregarded and soon forgotten. We know today that his observation lacked only the right explanation, but was altogether correct. As far as we can say at present, the close relation between magnetic anomalies and oil deposits is due to the following facts: (1) that oil fields generally lie in zones of relative uplift; (2) this uplift is either produced by the deposition of the oil-bearing strata on preexisting ridges of igneous material or by later processes of folding which also elevated igneous material, or both; (3) the igneous rocks thus associated with the oil-bearing formations carry magnetite which produces the magnetic anomalies.

On the basis of this reasoning, the magnetometer is now being applied in this country on as great a scale as any other geophysical method. The successes which have been reported—unfortunately very few in number, as most companies anxiously keep their results secret—are certainly very encouraging. They might be more encouraging if the magnetometer did not involve two decided disadvantages for most operators: the interpretation of the results is too hard, and the handling too easy.

A Background for the Application of Geomagnetics to Exploration

By Noel H. Stearn,* St. Louis, Mo.

(Boston Meeting, August, 1928)

When the Age of Machinery was suddenly thrust upon civilization about the beginning of the 19th century, an unprecedented demand for mineral resources sprang up. This demand brought about the rapid development of the known ore deposits and the discovery of such other deposits as could be encountered by energetic and canny prospectors. Thus in the countries now reasonably accessible nearly all orebodies have been located except those which are totally concealed even from the eyes and "senses" of experienced and expert prospectors.

The high rate of exhaustion of known ore deposits and the necessity of looking farther and farther ahead for raw materials as a matter of insurance on the stupendous capital outlays involved in mining, development, and conversion operations, have resulted in exceptional activity in three different fields of effort. The first is purely geographic. Prospectors are now penetrating the few hitherto inaccessible regions in the hope of stumbling upon exposed ore deposits or locating visible indications of their presence. The second is chemical. Industrial chemists and metallurgists are seeking to facilitate the exploitation of lower grade ores or to find methods of recovering valuable metals from common substances.

In the third field of effort lie the activities of the economic geologist and the mining engineer, who endeavor to locate concealed ore deposits which would remain hidden from the prospector no matter how expert or conscientious he might be. Obviously the task which such a man must face is both exacting and indefinite, and the tools which he has heretofore been permitted to bring to it are, for the most part, abstract—a knowledge of certain fundamental principles and processes involved in the origin, occurrence, structure, and distribution of deposits of natural resources and of the rocks which form the environments of those deposits; and a process of reasoning by analogy supplemented by the use of the diamond drill, or such artificial exposure as the test pit, trench, shaft, or tunnel.

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His primary efforts are usually bent on acquiring as accurately as possible a knowledge of the distribution of the rocks. Over very extensive areas, however, he meets the extreme difficulty of being unable to see the rocks, except at rare intervals, since they have been concealed by an overburden of glacial material, of alluvium, of soil, or of other rocks.

To meet this contingency geophysical methods have been developed, based on measurements of variations in certain physical properties of the rocks or ore deposits; *i. e.*, density, elasticity, electrical conductivity and magnetic permeability. To carry out each of these methods various instruments have been designed, and all of the methods, as well as nearly all of the instruments, have special and unique fields of applicability, although there is some overlap.

The foregoing approach to the subject of geophysics is designed to emphasize the viewpoint, which is occasionally stated but apparently with little conviction by geophysicists, that the whole field of applied geophysics constitutes merely another tool for the economic geologist and mining engineer, and in any other hands that tool is as inadequate and may be as harmful as a sculptor's chisel in the hands of an adolescent.

No one of the various geophysical methods is simple, but one in particular is characterized by relative simplicity in principle, in the instruments involved, in the field procedure, and in the interpretation of results. The use of this method involves the application of the principles of magnetism to field geology, and this combination of two already established sciences may be fittingly termed geomagnetics.

The Physical Phase of Geomagnetics

Magnetic Properties of Magnets

Although the ultimate causes of the phenomena exhibited by magnets are still hypothetical, the phenomena themselves have been well known for many centuries. For practical geomagnetic work it is sufficient to know merely the empirical laws governing the common attributes of magnets—attraction, induction, polarity, and the magnetic field. These seem to have been so thoroughly established as to require nothing beyond mere statement.

Magnets.—The name magnet was originally applied to specimens of magnetic iron ore or magnetite (Fe_3O_4) found in Magnesia, Asia Minor, which possessed the power of attracting iron. The name has since been extended to include pieces of iron, steel, or certain alloys, so treated that they exert an attraction similar to that of natural magnets.

Induction.—The treatment by which artificial magnets are produced makes use of the process or property of "induction" by which the

unmagnetized specimens when subjected to the influence of a magnetic field, either by contact or contiguity with another magnet, or by location within a solenoid, become polarized magnets. The stronger the magnetization developed in a material by a given magnetizing force, the greater is said to be the susceptibility of that material to magnetization.

Polarity.—Polarity is the name given to that property by which a magnet possesses loci of maximum attractive force, these loci being referred to as poles. Each magnet possesses two poles with different characteristics, so that if two magnets are brought near each other certain poles will repel and other poles will attract each other. The empirical law is that like poles repel and unlike poles attract each other, and the similarity of poles is established by the fact that if the magnets are suspended by their centers of gravity and left free to move, one of their poles will invariably seek to point northward. This is known as the north-seeking or north pole of the magnet while the other is called the south pole.

Field of Force.—The magnetic field is the space surrounding a magnet, through which its influence is detectable. Its observable limit, therefore, depends only on the sensitivity of the detecting device. The magnetic field possesses the properties of direction and intensity and therefore is measurable in vector quantities. The direction is measured relative to the magnetic axis, which is a straight line drawn between the two poles of a magnet. The intensity of the field varies inversely as the square of the distance from a pole for a bar magnet, and as the first power of the distance for a sheet magnet. The magnetic field is customarily represented diagrammatically by lines of force which indicate the direction by their trend and the intensity by their spacing.

Lines of Force.—These are lines along which an isolated north pole would travel if free to move in a magnetic field. By definition, they are such that the tangent to a line of force at any point gives the direction of the resultant force at that point. If lines of force are drawn throughout the magnetic field so that the number of lines per unit area is numerically equal to the strength of the field in any given area, the lines turn out to be continuous curves.

The conception of a group of lines of force filling a magnetic field is merely a mental mechanism for visualizing the relations of magnets to each other and to the surrounding space, but it is important because it leads naturally to an examination of the media in which the poles are situated.

Magnetic Properties of Matter

Magnetic Classification.—In 1845, Faraday demonstrated that every substance is affected by a powerful magnet with varying degrees of intensity. The effect on some substances is repellent. These he called

diamagnetic. On the majority of the substances which he tested, the effect is attractive, and these he called paramagnetic. Modern work on magnetism has led to a threefold classification brought about by subdividing the paramagnetic substances into two groups. Certain substances, notably iron, nickel, cobalt, and to a much lesser degree manganese, have very pronounced magnetic properties. These are now placed in a separate group called ferromagnetic.

Paramagnetic and diamagnetic reactions are relatively feeble. It is important to note that experiments indicate that a paramagnetic substance will act as though diamagnetic if it is surrounded by a medium more paramagnetic than itself.

TABLE 1.—*Specific Susceptibilities of the Elements*

Where S is the Specific Susceptibility Relative to Unit Volume

Paramagnetic	$S \times 10^6$	Diamagnetic	$S \times 10^6$
Praesodymium	163.17	Helium	−0.002
Erbium	130.36	Chlorine	−0.007
Cerium	106.06	Argon	−0.01
Manganese	80.00	Cæsium	−0.188
Palladium	66.00	Silicon	−0.29
Uranium	60.96	Indium	−0.57
Platinum	26.00	Germanium	−0.62
Chromium	26.00	Copper	−0.80
Vanadium	13.20	Sulfur	−0.85
Rhodium	13.00	Zinc	−1.00
Ruthenium	11.01	Lead	−1.12
Niobium	10.80	Bromine	−1.26
Tantalum	9.40	Selenium	−1.30
Barium	6.90	Gallium	−1.34
Titanium	6.65	Arsenic	−1.40
Molybdenum	5.13	Silver	−1.5
Iridium	4.90	Phosphorus	−1.60
Tungsten	4.76	Boron	−1.66
Aluminum	1.80	Iodine	−1.73
Calcium	1.67	Beryllium	−1.87
Magnesium	1.44	Tellurium	−2.10
Osmium	1.35	Mercury	−2.50
Thorium	0.89	Thallium	−2.73
Strontium	0.86	Gold	−3.10
Potassium	0.52	Antimony	−4.70
Sodium	0.50	Carbon	−8.00
Zirconium	0.39	Bismuth	−14.00
Tin	0.35	Cadmium	−15.23
Lithium	0.23		
Oxygen (O_2)	0.146		
Rubidium	0.126		
Hydrogen (H_2)	0.008		
Nitrogen (N_2)	0.001		

In Table 1 the various paramagnetic and diamagnetic elements for which the figures are available are arranged according to the indices of their magnetic susceptibility.[1]

Permeability.—The mental mechanism involving the conception of lines of force is very useful in giving a visual grasp of the reactions of various substances in a field of force. Every substance may be regarded as possessing a certain power of conducting lines of force and of offering resistance to them. These two reactions, resistance and conductivity, may be summed up as permeability. The magnetic permeability of diamagnetic substances is less than that of a vacuum while that of paramagnetic substances is slightly more. Ferromagnetic substances show extreme permeability.

Properties of Magnetic Fields

Symmetry.—The magnetic field of an ordinary magnet under ideal conditions possesses a high degree of space symmetry with respect to the magnetic axis, which is demonstrated geometrically by the lines of force. Moreover, since the lines of force show intensity as well as direction in a magnetic field there is a constant relationship between the space symmetry and the strength of the field.

The shape of the field can be calculated mathematically by using the inverse square law (Force $= \frac{m_1 m_2}{r^2}$ when m_1 and m_2 denote pole strengths measured in any arbitrary unit and r denotes distance.) It can also be plotted instrumentally by introducing a short magnet into the field in various places and moving it in the direction in which it points until it has described a series of closed curves representing the lines of force of the magnetic field in that particular plane in which it has been moved.

The ideal conditions which permit the highest degree of symmetry involve a uniform medium filling the space occupied by the magnetic field, and the absence of magnetic attraction due to any other magnet. The symmetrical shape of a magnetic field of force, as described by the convention of lines of force, is dependent only on the shape and strength of the magnet; and from these the shape of the field can be predicted. *Any variation from the predicted shape must be due to a combination of*

[1] Data from Smithsonian Physical Tables, Ed. 5. Smithsonian Miscellaneous Collection (1910) **58**, No. 1, and Landolt-Bornstein-Roth-Scheel: Physikalisch-Chemisch Tabellen 5., Auflage II. Translating from the latter: "If a homogeneous body occurs in a magnetic field of strength H and if the intensity of magnetization of the material is I, then $S = 1/H$ where S is its magnetic susceptibility relative to the unit volume and $x = S/d$ where x is its specific susceptibility relative to the unit mass. The susceptibility of a vacuum is taken as 0. Between the susceptibility S and the permeability P, as well as the strength of the magnetic field H, the intensity of magnetization I, and the magnetic induction B, there is this relationship: $S = 1/H$; $B = H - 4\pi I$; $P = B/H = 1 + 4\pi S$.

more than one field, or the presence of a non-uniform medium within the space occupied by the field, or both.

Distortion.—If a second magnet is brought into the vicinity of the first, the magnetic fields of the two magnets will combine so that the force on any unit pole free to move in the field will be the resultant of the four forces emanating from the poles of the two magnets acting on that pole. The shape of the resulting field, as traced by lines of force, will therefore differ from that of a single magnet in that it will have a lower degree of symmetry. The resultant field can be calculated mathematically, given the shape of each magnet, the pole strengths of each magnet, the distance between the two magnets, and the space relationship between the magnetic axes. It is simpler, however, to plot the field by introducing into it a small compass and tracing out the lines of force. This instrumental determination of the magnetic field will reveal by its shape the presence of two magnets.

Just as every substance has some response to magnetic attraction, every substance has a corresponding influence on a magnetic field, differing in power according to the intensity with which it responds to magnetic attraction. The lines of force which have been conceived to define a magnetic field find one substance more penetrable than another. If two substances of differing permeability occupy a magnetic field there will ensue an uneven distribution of the lines of force, since there will be a crowding through the more permeable substance. This crowding necessarily alters the shape of the magnetic field, especially near the borders of the two substances, and the symmetry is decidedly decreased or destroyed.

Mathematical determination of such a field would be possible but extremely involved. The instrumental plotting, however, would involve no more difficulty than that of plotting a symmetrical field. The position and direction of the lines of force would show where the different media lay in the magnetic field.

Summary.—It is perhaps wise to call especial attention to the fact that in the conception of geomagnetic work here introduced, the important consideration does not involve magnets themselves but deals primarily with magnetic fields. The possibility of detecting combined magnetic fields and the presence of media of differing magnetic permeability has been proved by simple laboratory experiments performed yearly by high school students of elementary physics. If it is possible to plot a geometric representation of the magnetic field surrounding a set of heterogeneous magnetic factors, it should also be possible to deduce something about the magnetic factors from a plotted magnetic field. Experimental physics has worked from the source of magnetic energy to its results. Geomagnetics seeks to work from the results to the source, using the earth's magnetic field as its laboratory.

Terrestrial Phase of Geomagnetics

Since the year 1600, when Gilbert's treatise "De Magnete" first came out, the earth itself has been recognized as a magnet, having the properties displayed by an ordinary magnet—the powers of attraction and induction, the property of polarity, and the field of force. Gauss calculated the field of force of the earth to be equivalent to that of eight sextillion, four hundred sixty-four quintillion (8464 by 10^{18}) unit bar magnets measuring 14 by 1¼ in., each weighing 1 lb., and all located around the earth's center. Overbeck's conception of the same idea is that the earth's field is equivalent to that of a soft iron sphere magnetized to saturation, having a radius of 101 statute miles, or one twenty-sixth that of the earth, and located around its center.

Although there has been some inconclusive speculation regarding the cause of this earth magnetism (*i. e.*, whether it is due to a metallic earth-core or to a system of deep-seated, surface, subsurface, or supersurface electrical currents or to any combination of such causes), nevertheless the one outstanding feature, which fortunately is independent of such speculation, is also the feature of real significance in the application of geomagnetics. All modern investigations, according to Bauer, lead to the conclusion that there exists *a dominant deep-seated* magnetic force, whose field of attraction suffers modification to various degrees by various means. The ideal and undistorted symmetry of that field must be understood in order to understand departures from that ideal symmetry.

As a first approximation, the earth's field may be represented as that due to a short magnet placed at the earth's center. Because the earth acts as a spherical magnet, a bar magnet within the earth which would produce the magnetic effects observed at the surface would have its poles practically coincident. Kraft and Biot found that the nearer to each other they assumed the poles of a fictitious bar magnet at the center of the earth, the closer was the agreement between their computed results and the observed facts, so that the "equivalent magnetic poles" of a spherical magnet are practically the same distance from all points on the surface. This has a direct bearing on the geometry of the earth's field.

The earth's field can be measured only at the approximate surface. If the magnetic force which gives rise to that field were coextensive with the earth itself, the measurements would be taken at the surface of a magnet and the shape of the field would depend only on the shape of the magnet, because the medium through which the lines of force delineating the field would pass would be only air. However, on account of the fact that the earth's magnetic field is dominantly due to a *deep-seated magnetic force*, the lines of force which define the field must pass through the outer crust of the earth and into the air. The outer crust of the earth consists of heterogeneous materials which would be expected to have differing

permeabilities, and so the lines of force should consequently be warped from their symmetrical paths in coming to the surface of the earth.

Two types of variation are found modifying the earth's magnetic field. In one type the factor of time is dominant, and variations of this type may be regarded as cyclic or irregular *fluctuations*. These may be and probably are due to the influence of electrical phenomena. In the second type the place factor is dominant, and variations of this type are the modifications in the symmetry of the earth's field with which the application of geomagnetics has to deal. The symmetry variations may be regarded as *anomalies*. The symmetry anomalies must obviously be isolated from the time fluctuations in order to appear in their true significance.

Clearly, then, an accurate knowledge of the cross-section of the earth's magnetic field obtainable at the surface of the earth would involve a record at every instant of time and at every point on the earth of the magnitude and direction of the earth's field. It has long been recognized that great importance attaches to such a knowledge of the earth's field, to the navigator for practical purposes, and to the theoretical investigator who attempts to describe and explain the magnetic state of the earth, and the science of terrestrial magnetism has been developed in response to that recognition. Worldwide magnetic surveys have been conducted with the aim of collecting such data. A detailed description of the processes, instruments, and computations by which these data have been assembled may be found in the United States Declination Tables for 1902, or in any advanced textbook on magnetics.

For convenience, the field at any one point is always represented by the three components which lend themselves most readily to accurate instrumental determination. These are (1) the declination, (2) the dip or inclination, and (3) the horizontal component of intensity. Obviously, when these three elements are known the field can be completely determined for a given location at a given time.

Since the points of intersection of the projection of the ends of the earth's magnetic axis with the earth's surface (commonly referred to as the magnetic poles), do not coincide with the geographic poles, it follows that the magnetic meridians are not parallel to the geographic meridians. The angle between the two meridians at a given point is known as the angle of declination. The angle made by the lines of force of the earth's field with a horizontal plane at any point is known as the dip or inclination. These two elements give the direction in space of the magnetic field. It remains necessary to determine its intensity, or any component of it, the most convenient of which is the horizontal component, H. From this the vertical component, V, may be calculated by the formula: V/H equals the tangent of the angle of inclination; and the total intensity, I, may be derived from the formula:

$$I^2 = H^2 + V^2$$

In the surveys of terrestrial magnetism heretofore conducted, these three elements of the earth's magnetic field have been determined at various points separated one from another by distances measured in tens and hundreds of miles. The determinations of each element have been plotted on a map with lines drawn through all points of equal value. Such lines on maps showing magnetic declination are called isogonal lines. Such lines on maps showing magnetic inclination are called isoclinal lines. Such lines on maps showing magnetic intensity are called isodynamic lines. From these maps can be derived a picture of the approximate symmetry of the earth's field.

At certain points observatories have been established where records are kept of the time fluctuations in the earth's field. Since these fluctuations must be either isolated from or evaluated with relation to the symmetry anomalies, it is necessary to understand their effect on the earth's field. Some of these are cyclic, some quasiperiodic, and some entirely irregular.

Time Variations

Secular Variation.—The cyclic change with the longest periodicity is known as the secular variation. Records of the magnetic elements do not go far enough back to permit an accurate computation of the secular change, but its periodicity is of the order of magnitude of 960 years.

The magnetic system, as noted by Lord Kelvin, is slowly rotating from east to west, making a revolution in about 960 years, so that in 960 years the magnetic system lags behind the earth by one rotation. The rate of change of the secular variation differs in different localities. It is measured and plotted on maps similar to those of the other magnetic elements. It is significant that the rate of change is of the order of magnitude of two minutes a year for the declination. The other elements go through corresponding cycles with a corresponding order of magnitude. The secular variation may be considered to have a negligible influence on the symmetry anomalies.

Annual Variation.—A change in the declination has been noted whose periodic time is one year. This occurs simultaneously in opposite directions in the northern and southern hemispheres. At London the maximum easterly deviation occurs in August, and the westerly in February. The amplitude of the deviation in London is about 2.25 min. At Toronto it is about 1.1 min. and for an average of North American stations it amounts to about 1 min. This too may be considered negligible in the general application of geomagnetics.

Diurnal Variation.—Another change in the magnetic elements occurs daily. Along the 49th parallel the average maximum easterly deflection

is 3.2 min. at about 8 o'clock in the morning, while the average maximum westerly deflection is 3.6 min. at about 1:30 in the afternoon. The diurnal range of the variation in declination increases with an approach to the magnetic pole and decreases toward the magnetic equator. The formula for this change in diurnal range with magnetic latitude is $d = 2.58' \sec^2\phi$, where d is the diurnal range and ϕ is the magnetic latitude as found from the formula $\tan \phi = \frac{1}{2} \tan I$, in which I is the inclination.

The order of magnitude for the average diurnal range of variation in declination is from 5 to 9 min. for the region of the United States. The horizontal intensity varies on ordinary days not more than 0.25 per cent. The vertical intensity varies about 0.1 per cent. on ordinary days and 0.5 per cent. on extreme days. This fluctuation is sufficient to mask or obliterate some symmetry anomalies which might have significance. Thus it is necessary, in the practical application of geomagnetics to some field problems, to take steps toward eliminating the effects of this fluctuation. This elimination is usually accomplished by certain methods of field procedure.

Sun-spot Variations.—The period of 11 years during which the frequency of the occurrence of the sun spots goes through a cycle coincides with a period of change in the magnitude of the daily variations of the magnetic elements. This change in magnitude involves a variation of about 6 min. of declination. Any field procedure which takes care of daily variation automatically takes care of this variation. However, it is necessary, in years of high sun-spot frequency, to correct for diurnal variation in certain instances where it would not be necessary in years of low sun-spot frequency.

Minor Periodic Fluctuations.—Chief among these may be mentioned the variation depending on the position of the moon relative to the sun and the earth. The range of this variation is so minute that many years of extremely precise work have been required to detect it.

Another fluctuation has been called pulsatory. By using very slight magnets with small moments of inertia, it has been found that the magnetic system is subject to constant vibrations of very small amplitude. These are known as pulsations.

Irregular Variations.—Magnetic storms frequently accompany volcanic eruptions, earthquakes, exceptional sun-spot activity, and auroral displays. They also occur unaccompanied by any such startling phenomena. They are irregular disturbances of the earth's field; they may last from a few minutes to several days, but they are usually of short duration. Occasionally, they cause variations of considerable magnitude, one having been recorded as causing a declination deviation of 5° within a period of 14 min. The ordinary magnetic storm, however, does not cause deviations greater than 0.5°.

A tabulation of observations taken for 18,000 consecutive days at the Kew Magnetic Observatory in England gives an idea of the frequency and magnitude of these irregular disturbances (Table 2).

TABLE 2.—*Observations of Magnetic Disturbances at Kew, England*

	PER CENT.
Undisturbed days	12
Days having a change of less than 10′ in declination	66
Days having a change of 10′ to 30′ in declination	20
Days having a change of 30′ to 60′ in declination	1.6
Days having a change of over 60′ in declination	0.4

From an examination of this table it becomes obvious that a magnetic storm of the magnitude of the one mentioned above is a decided rarity; nevertheless it is necessary to guard against the influence of these storms.

It is perhaps noticeable that no attempt has been made to present explanations for these various fluctuations, partly because the explanations are still in the hypothetical stage and partly because they have no direct bearing on geomagnetic work. The presentation of these time variations in the magnetic field is introduced merely to permit the establishment of an estimate of their influence on the symmetry modifications or the anomalies with which geomagnetic work primarily has to do.

It is possibly a source of some mental confusion to try to picture the interaction of magnetic fluctuations with anomalies. Perhaps the following analogy, though not a very close one, will help to clarify the picture. Suppose a snag reaches nearly to the surface of a stream. No matter which way the current flows, no matter how swiftly, there will remain a riffle on the stream surface to localize the position of the snag. Current riffles may develop with increased velocity but they can be distinguished from the snag riffle up to a certain point of magnitude. However, if the stream suddenly becomes swollen the snag becomes hopelessly submerged. The snag riffle represents a symmetry anomaly in the earth's field, the changes in current represent time fluctuations, and the torrential swelling represents a magnetic storm.

Symmetry Variations.—To the terrestrial magneticist the symmetry modifications caused by the differential permeability of the earth's crust are known as local disturbances and the tendency has been to minimize them as much as possible in plotting the results of terrestrial magnetic surveys. Hazard says:[2] "While the lines on the world magnetic charts appear as smooth curves, the isogonic charts of land areas which have been surveyed in detail, such as the 1920 chart of the United States, are characterized by crooked lines. *Even then they do not fully represent the irregular distribution* of declination over the country. In general it is

[2] D. L. Hazard: Earth's Magnetism. U. S. Coast and Geodetic Survey *Spec. Pub.* No. 117 (1925) 34.

found that when observations are made at any additional stations in any region, additional irregularities are developed. These are usually referred to as local disturbances, as contrasted with departure from regular distribution which are regional or continental in extent."

Although the plotting of the results of terrestrial magnetic surveys smooths out the local disturbances to a considerable extent, their possible significance was long ago recognized by such terrestrial magneticists as A. W. Rucker, whose extensive work in the British Isles brought him to the conclusion that the curiously anomalous behavior of his magnetic instruments from place to place was due to local disturbing forces centered in the rocks, and Dr. Rijckevorsal, whose survey of Holland led him to this statement: "Little even as we know about the geology of the Netherlands, the magnetic maps must bring everyone to the conviction that in some cases, in many perhaps, there must be a direct relation between geology and terrestrial magnetism, and that many of the magnetic features must be in some way determined by the geological structure of the underground."

Summary.—The combined picture afforded by physical and terrestrial magnetics of the primary working basis of geomagnetics involves only the very simple conception, then, of a locus of magnetic force situated about the center of the earth, surrounded by a magnetic field of force similar to that of an ordinary spherical magnet, a cross-section of which is obtainable at the earth's surface. The lines of force by which this field is defined suffer warping from their normal symmetrical relationship by reason of the differential permeability of the media through which they must pass; *i. e.*, the rocks which constitute the earth's outer shell. In substantiation of this picture we have already adduced evidence that such warping occurs wherever terrestrial magnetic surveys have been made, and the possibility that those warpings have a geological significance has been suspected and suggested. It remains to discuss whether, why, how, and how far, it is possible to measure these warpings in such a way as to interpret their suspected geological significance.

Geological Phase of Geomagnetics

The observable portion of the earth is composed of rock formations the influence of which upon the magnetic field may be considered to be dependent on their mineral composition, shape, attitude, and the accident of their arrangement. These factors of influence divide themselves into two groups, mineralogical, and geometrical.

Mineralogical Factor

Faraday's work seems sufficient to justify the statement that all minerals display a certain reaction to magnetic forces and can be classified

as diamagnetic, paramagnetic, or ferromagnetic, according to the type and intensity of the reaction which they display. Unfortunately, little quantitative work has been done toward ascertaining the relative intensity of reactions of the common earth minerals to magnetic attraction. Certain general qualitative facts are known, such as that compounds of iron are paramagnetic, as are most compounds of nickel, cobalt, and manganese, while such minerals as calcite, zircon, wulfenite, halite, and quartz are diamagnetic. But no useful scale of the magnetic properties of rock minerals has yet been formulated. Table 3 shows certain indices of values for the magnetic susceptibility of minerals for which comparable data are available, but this table obviously contains very few of the important rock-making minerals, nor are the figures precise because they do not take into account the analogy between the axial

TABLE 3.—*Magnetic Susceptibilities of Minerals*

Ferromagnetic	$S \times 10^6$	Diamagnetic	$S \times 10^6$
Iron (Fe)	80,000.00	Epsomite ($MgSO_4.7H_2O$)	0.63
Magnetite (Fe_3O_4)	32,122.00	Niter ($NaNO_3$)	0.70
		Water (H_2O)	0.72
Paramagnetic		Niter (KNO_3)	0.72
		Covellite (CuS)	0.74
Hematite (Xtal) (Fe_2O_3)	426.00	Marble ($CaCO$)	0.75
Manganosite (MnO)	349.44	Chalcocite (Cu_2S)	0.78
Hausmannite (Mn_3O_4)	318.07	Copper (Cu)	0.80
Alabandite (MnS)	177.28	Halite ($NaCl$)	0.82
Pyrolusite (MnO_2)	131.22	Sulfur (S)	0.85
Pyrite (FeS_2)	120.00	Sassolite ($B(OH)_3$)	0.89
Hematite (Amor) (Fe_2O_3)	107.12	Sylvite (HCl)	0.91
Melanterite ($FeSO_4.7H_2O$)	80.00	Kalinite ($Al_2K_2(SO_4)^4$-	
Bieberite ($COSO_4.7H_2O$)	68.00	$24H_2O$	1.00
Limonite ($Fe_2O_3.NH_2O$)	57.00	Calcite ($CaCO_3$)	1.00
Platinum (Pt)	26.00	Berzelianite (Cu_2Se)	1.01
Morenosite ($NiSO_4.7H_2O$)	18.00	Anhydrite ($CaSO_4$)	1.12
Chalcanthite ($CuSO_4.5H_2O$)	14.30	Villiaumite (NaF)	1.12
Cuprite (Cu_2O)	4.38	Quartz (SiO_2)	1.20
Rutile (TiO_2)	0.28	Lead (Pb)	1.30
Brookite (TiO_2)	0.262	Cotunite ($PbCl_2$)	1.31
Octahedrite (TiO_2)	0.257	Silver (Ag)	1.50
Air	0.024	Bromyrite ($AgBr$)	1.53
		Cerargyrite ($AgCl$)	1.55
		Iodyrite (AgI)	1.66
		Arsenic (As)	1.70
		Diamond (C)	1.80
		Zincite (ZnO)	1.85
		Fluorite (CaF_2)	2.00
		Graphite (C)	8.00
		Bismuth (Bi)	14.00

variations of the magnetic properties in crystals and those of the optical properties. But the table will serve to give an idea of the relative range of variation to be expected.

Ferromagnetic Minerals

The ferromagnetic minerals clearly may be held responsible for the most readily detectable modifications of the earth's field. These are very few in number: native iron and native nickel (with certain rare combinations of the two such as awaruite, josephinite, souesite, kamacite, tacnite, and plessite), native platinum, pyrrhotite, franklinite, chromite, ilmenite, and magnetite. Naturally the feasibility of geomagnetic work based on the detection of variations caused by these few minerals must depend on their distribution, origin, and habit of occurrence.

Native Iron.—Native iron has a meteoric as well as a terrestrial origin, for it occurs in almost all meteorites of all three types, making up nearly the entire mass of the metallic meteorites. Meteoric iron is always alloyed with nickel in amounts usually varying from 5 to 10 per cent. Some of these alloys have been given mineral names such as kamacite, tacnite, and plessite. These are ferromagnetic minerals also.

The occurrence of native iron of terrestrial origin is widespread, though quantitatively unimportant. It has been reported from basaltic rocks from Antrim, Ireland; Cassel, Germany; and Ovifak, Disco Island, Greenland, where large masses of iron weighing as much as 20 tons have been weathered out as boulders and where lenticular masses of iron are still embedded in the rock, associated with magnetic pyrites and graphite. It has been reported from trap rocks in New Jersey, and from the dolerite of Dry River, New Hampshire, where the grains of native iron are enveloped by magnetite, suggesting its secondary origin; and it has been found in auriferous gravels from Brazil, British Columbia, and Berezovsk in the Urals, where traces of platinum occur in the iron.

Native Nickel.—Nickel is always found alloyed with iron, but certain alloys approach native nickel in composition. Some have been given mineral names such as awaruite, josephinite, and souesite. Placer deposits from various localities (notably the George River, New Zealand; Josephine and Jackson Counties, Oregon; the Elvo River, Piedmont; the Fraser River, British Columbia; the Smith River, California; and the Yukon River, Alaska) have yielded these nickel-iron alloys and in all cases except one they have been definitely traced to neighboring serpentine rocks or peridotites.

Native Platinum.—Some varieties of platinum are ferromagnetic, especially the kind called iron-platinum. Native platinum and its highly paramagnetic associates, osmium, iridium, ruthenium, and palladium, have their origin in peridotites and occur in placer deposits

in various places scattered over the world. Notable occurrences are found in the Pinto River, Columbia; in Nijni Tagilsk, Goroblagodat and Bisersk, Russia; in Borneo, New Zealand, New South Wales; in Trinity County, California; Fort Orford, Oregon, and Tulomeen, British Columbia.

Pyrrhotite (Fe_7S_8 to $Fe_{11}S_{12}$).—As a minor accessory of igneous rocks, pyrrhotite or magnetic pyrite occurs frequently in the ferromagnesian varieties such as diorites, gabbros, diabases, and basalts. It commonly appears in norites and peridotites either disseminated or segregated by magmatic differentiation, and occasionally it is found in nepheline rocks.

Amphibolites and crystalline schists, as well as contact metamorphic rocks, contain pyrrhotite, the latter in considerable quantities in places, but the largest masses occur in metalliferous veins.

Since it is an accessory mineral, its geographic distribution is as widespread as that of the containing rocks. The following localities are mentioned as having notable occurrences of pyrrhotite: Kingsberg, Norway; Bodenmais, Bavaria; Sudbury, Canada; Stafford and Ely, Vermont; Ducktown, Tennessee; Gap Mine, Pennsylvania; Falun, Sweden; Andreasberg, Germany; Schneeberg, Saxony; Leoben and Lavantal, Carinthia; Minas Geraes, Brazil; Cornwall, England; Standish, Maine; Diana, New York; Elizabethtown, Ontario.

Franklinite (Fe, Zn, Mn)O.(Fe, Mn)$_2$O$_3$.—Franklinite is a very rare mineral occurring in contact metamorphic rocks near Franklin, New Jersey, and Eibach and Altenberg, Germany. It is mentioned only because it is one of the few ferromagnetic minerals.

Chromite ($FeO.Cr_2O_3$).—Chromite is a primary mineral in some basic igneous rocks, particularly peridotites and the derivative serpentines. It occurs disseminated in the rocks, or segregated into irregular masses by magmatic differentiation.

Chromite is known to occur also in some crystalline schists, and rarely in dolomites. It has been found in veins. It is frequently found in the black sands of placer deposits.

The wide geographical distribution of notable occurrences of chromite is shown by this list of localities from which it has been reported: Silesia, New Zealand, Rhodesia, New Caledonia, Asiatic Turkey, Texas, Pennsylvania, Maryland, California, North Carolina, Oregon, Washington, Wyoming, New Jersey, Vermont, Styria, Shetland Islands, Norway, France, Bohemia, and the Ural Mountains.

Ilmenite ($FeTiO_3$).—Ilmenite is a common though usually sparsely distributed, constituent of igneous rocks where it occurs either as an accessory mineral or in veins and segregated masses near the borders of the parent rock, supposedly concentrated by local differentiation or fractional crystallization. It is also common in small amounts in many anamorphic rocks. The black sands of placers usually contain some ilmenite.

France, the Ilmen Mountains, Cornwall, Tyrol, Switzerland, Norway, Quebec, New York, Connecticut, Massachusetts, and Arkansas, are localities from which notable occurrences of ilmenite have been reported.

Magnetite (Fe_3O_4).—Of the ferromagnetic minerals listed here, magnetite and ilmenite are the only ones that can be called common earth minerals; but these, especially magnetite, occur in appreciable amounts in almost all types of rocks except shales and limestones.

Magnetite is the only common mineral that has a marked influence on the earth's magnetic field. Pierre Wess[3] has made a quantitative study of the effect of two magnetite specimens on weak fields like that of the earth. In one specimen the magnetism induced by a field of 0.6 Gauss was 85 times that of the inducing field. The induced magnetism in the second specimen was 285 times that of the inducing field. A magnetite content of 1 per cent. in a rock would cause nearly 100 per cent. variation in the earth's field at its surface even if the smaller of the values quoted were considered representative. The magnetite content of rocks, therefore, is a very important consideration.

If the magnetite and ilmenite in the earth's crust were evenly distributed, there would be about 2.951 per cent. in all rocks, and the resultant warping of the earth's magnetic field would be so general as to be undetectable. Fortunately, however, the magnetite-ilmenite content of rocks has a wide range of variation, as shown by Tables 4 and 5.

TABLE 4.—*Magnetite-Ilmenite Content of Rocks*

Rock	Magnetite, Per Cent.	Ilmenite, Per Cent.	Combined, Per Cent.
Average granite	1.72	0.31	2.03
Average basalt	5.80	0.73	6.53
Average igneous rock (65 per cent. granite; 35 per cent. basalt)	3.15	1.45	4.60
Average shale			
Average sandstone	0.58	0.25	0.83
Average limestone			
Average sedimentary rock (82 per cent. shale; 12 per cent. sandstone; 6 per cent. limestone)	0.07	0.02	0.09
Average rock (95 per cent. igneous; 5 per cent. sedimentary)	2.95	0.001	2.951

It is apparent from these tables that not only is there a wide range of variation in the content of ferromagnetic minerals in the various rock groups, but also within each group there are distinct differences; so that the chances of two contiguous masses of igneous rock exerting the same influence on the earth's magnetic field are very slight.

[3] L'Éclairage Electrique (1896) **7**, 487, quoted by W. O. Hotchkiss: Magnetic Methods for Exploration and Geologic Work. *Trans.* (1923) **69**, 36.

TABLE 5.—*Magnetite-Ilmenite Content of Igneous Rocks*

	Magnetite, Per Cent.			Ilmenite, Per Cent.			Combined, Per Cent.		
	Min.	Max.	Av.	Min.	Max.	Av.	Min.	Max.	Av.
Quartz porphyries	0.0	1.4	0.82	0.0	0.5	0.3	0.0	1.6	0.94
Rhyolites	0.2	1.9	1.00	0.0	0.8	0.45	0.2	2.2	1.10
Granites	0.2	1.9	0.90	0.6	0.8	0.7	0.2	2.5	1.15
Trachyte-syenites	0.0	4.6	2.04	0.0	1.2	0.70	0.2	5.8	2.74
Eruptive nephelites	0.0	4.9	1.51	0.0	3.7	1.24	0.0	8.3	2.75
Abyssal nephelites	0.0	6.6	2.71	0.0	2.0	0.85	0.0	8.4	3.56
Pyroxenites	0.9	8.4	3.51	0.0	1.5	0.40	2.1	8.4	3.91
Gabbros	0.9	3.9	2.40	0.0	6.4	1.76	1.9	8.7	4.16
Monzonite-latites	1.4	5.6	3.58	0.9	2.6	1.60	2.3	7.3	5.18
Leucite rocks	0.0	7.4	3.27	0.5	4.1	1.94	0.5	8.9	5.2
Dacite-quartz-diorites	1.6	8.0	3.48	0.0	3.4	1.94	2.1	11.4	5.4
Andesites	2.6	5.8	4.50	0.5	1.8	1.16	4.1	7.8	5.6
Diorites	1.2	7.4	3.45	1.1	5.4	2.44	2.3	9.7	5.8
Peridotites	1.6	7.2	4.60	0.0	7.1	1.31	1.6	13.1	5.9
Analcite rocks	1.9	9.0	5.54	0.0	2.5	1.05	2.0	10.8	6.5
Basalts	2.3	8.6	4.76	1.1	3.1	1.91	3.4	11.7	6.67
Diabases	2.3	6.3	4.35	1.2	4.3	2.70	4.7	8.8	7.05
Basaltic rocks[a]	2.8	7.0	4.80	0.0	5.1	2.80	3.4	12.1	7.60
Femic syenites	4.2	6.0	5.24	1.4	3.7	2.74	6.5	9.7	7.98
Basic and titaniferous rocks	3.5	49.9	27.99	1.1	59.0	35.11	25.11	75.3	53.10

[a] Leucite, nephelite, or melilite, basalts.

Table 5 also shows that magnetite and ilmenite occur in all of the general igneous rock groups, intrusive and extrusive, though not in all igneous rocks. The minerals may be sparsely disseminated or segregated into rock masses of which they form the principal constituents. Certain of these rock masses have the ferromagnetic constituents so concentrated that they are mined as ores.

In rocks formed by katamorphic, anamorphic, or metasomatic processes, magnetite is also an extremely variable, though a common, constituent.

Because magnetite is relatively very refractory to the processes of weathering, the formations derived from the breaking down of magnetitic rocks are likely to contain the original magnetite—probably sorted into lenses because of its high specific gravity (sp. gr. 5.168 to 5.180)—as well as secondary magnetite derived from the partial decomposition and oxidation of ferruginous minerals. Among sedimentary rocks, sandstones are the only ones which as a general group contain enough magnetite to be recorded in average analyses, but it is known that shales and limestones may contain magnetitic beds and lenses. And among the unconsolidated sediments, black sands,[4] composed for the most part of mag-

[4] For example, those of New Zealand.

netite, and placer deposits in which magnetite almost invariably occurs, are of economic importance.

Since anamorphic processes tend to develop magnetite in ferruginous rocks, and since shales and limestones are often ferruginous, it is to be expected that their anamorphic equivalents would tend to be magnetitic. This development of magnetite in ferruginous rocks subjected to dynamic or contact metamorphism is perhaps most strikingly illustrated by the actinolite-magnetite-quartz rocks, or the "amphibole-magnetite rocks" which occur in the Lake Superior region, seemingly wherever later intrusive rocks are found in or near the iron-bearing formations, wherein the iron was presumably originally in carbonate form.

It is also possible for magnetite to develop from limonite and hematite under anamorphic conditions. Products of extreme metamorphism, such as gneisses[5] and schists, contain much magnetite. In some cases these have had their origin determined as sedimentary, in other cases they appear to have been derived from igneous rocks, but in any case the magnetite is frequently appreciable. Within the rock groups of gneisses and schists the content of ferromagnetic minerals varies extremely.

Occurrences of magnetite supposed to have originated as metasomatic replacements have been reported from various localities. At the Cornwall mine in Pennsylvania the ore is reported to be a replacement of Silurian limestone resting upon a trap dike as an impervious basement.[6]

Texada Island, British Columbia, furnishes an example of the occurrence of magnetite the origin of which is attributed to metasomatic replacement accompanying the contact metamorphism surrounding a diorite intrusive.[7]

In the Iron Springs district of Southern Utah, andesite laccoliths in Paleozoic and Mesozoic sediments have given rise to pneumatolytic after effects, one phase of which involved the deposition of magnetite as dike-like masses in the andesite, as fissure fillings and replacements in the adjacent limestone, and as cements in breccias of andesite, limestone, and quartzite. At intrusive contacts there frequently occurs a concentration of magnetite, irregularly distributed, to be sure, and not often quantitatively important enough to form an ore, but still strong enough to have an appreciable influence on the earth's magnetic field.

This review of the occurrence of the ferromagnetic minerals is designed to emphasize an important point. If to the so-called "local disturbances" encountered in terrestrial magnetism there is to be attributed

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[5] For example, those of the eastern metamorphic region of the United States, notably at Cranberry, N. C.

[6] C. R. Van Hise: Treatise on Metamorphism, 1197. U. S. Geol. Survey *Monograph* 47.

[7] C. O. Swanson: The Genesis of the Texada Island Magnetite Deposits, 106A. Can. Dept. of Mines, *Summary Report* (1924) *Part A*.

geological significance, some basis must be adduced besides mere empirical observation for making such an attribution. Obviously, *the ferromagnetic minerals alone, few as they are, have a sufficiently widespread geological and geographical distribution, and have at the same time a sufficiently varied local distribution, to cause marked variations in the earth's field.* A review of the occurrence of these minerals furnishes justification for the conclusion that the geomagnetic method should be able to locate orebodies of magnetite and pyrrhotite, dikes, sills, veins, intrusions, extrusions, contacts, placer deposits, meteors, and ferruginous sediments. As a matter of fact, this list is far from exhaustive of the geological phenomena traceable by the geomagnetic method, and there are known examples of the location of every phenomenon mentioned, with the possible exception of meteors.

Diamagnetic and Paramagnetic Formations

Aside from and supplementing the influence of the few ferromagnetic minerals whose magnetic properties are readily perceptible, there is the influence of the vastly more numerous and quantitatively more important diamagnetic and paramagnetic minerals. The magnetic properties of these minerals are so unpronounced in laboratory specimens that they have been called nonmagnetic. However, their permeabilities do differ to some extent.

When rock masses of the magnitude of lime formations, for instance, composed chiefly of one mineral, are contiguous with other rock masses composed of minerals of different permeability, it is natural to suppose that this difference in permeability would be impressed upon a weak magnetic field like that of the earth by the very mass of the formations. The detection of such distortion in the shape of the field would necessarily require a more delicate technique than would the detection of the warpings caused by an appreciable ferromagnetic mineral content. But it is known that so-called nonmagnetitic formations differ in magnetic permeability sufficiently to bring about changes in the earth's field which can be detected by delicate measurements. For instance, the location of salt domes by magnetic measurements in Germany and Texas has been carried out on just this principle.

A consideration of the mineralogical factor, then, leads to the conclusion that the general distribution of rock masses should be reflected by anomalies in the earth's magnetic field, having a wide range of variation in intensity, so that some rock masses should be more readily traced than others. This is a very general consideration, however, leaving out of account the shape, attitude, and accident of arrangement of the rock masses. Thus, although it seems reasonable to suppose that magnetic anomalies should yield some clue to the areal geology of a region, and so directly or indirectly to the natural resources of that region, what can

be read from them regarding structure involves a consideration of the geometrical factor involved.

Geometric Factor

Concentrations of Ferromagnetic Minerals

Actually very little is known concerning the distribution of ferromagnetic minerals within the various rock masses. In igneous rocks the degree of concentration of the magnetite is known to vary to some extent within masses generally considered to be homogeneous, while segregations of magnetite at contacts are common. These take the form of dikes, fissure fillings, replacements, tabular veins, and irregular masses.

A quantitative study of the distribution of the ferromagnetic minerals magnetite and ilmenite, in one of the Keweenawan lava flows of Wisconsin, led Buckstaff[8] to the conclusion that the two together show a marked and systematic variation in amount with depth in the flow, and that the magnitude and regularity of this variation may well be one cause for the linear magnetic disturbances which occur parallel to the flows. In this case, at least, the ferromagnetic minerals of an igneous rock show a concentration at definite horizons, or roughly parallel to definite planes, so that should these planes be distorted or disrupted, a consequent change should occur in the magnetic anomalies caused by them.

In sedimentary rocks the concentration of ferromagnetic minerals into definite horizons is more marked. In the present beach sands, some lenses contain much magnetite and some none. Some stream bars have magnetite, ilmenite, chromite, nickel-iron or platinum, and some have none. In sandstone formations, magnetitic beds are intercalated between beds free from magnetite. In shales and impure limestones, magnetitic lenses may occur. In schists and gneisses, the tendency to banding brings about a still more marked tabular distribution of the ferromagnetic minerals. Concentrations of ferromagnetic minerals, then, in general may be said to occur in irregular masses and in tabular or lenticular forms, the effects of which upon the earth's magnetic field are distinguishable one from the other.

The same generalization may be carried over to concentrations of paramagnetic and diamagnetic minerals. A salt dome is a diamagnetic concentration of distinguishable shape. A limestone bed associated with ferruginous shales would act as a diamagnetic concentration of tabular form. A paramagnetic substance, it is wise to recall, has a diamagnetic effect on the earth's field when it is associated with a substance more permeable than itself. Thus we find in northern Michigan diabase dikes acting as lenticular diamagnetic concentrations.

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[8] S. Buckstaff: A Study of the Distribution of Magnetite in One of the Keweenawan Lava Flows. Unpublished thesis, University of Wisconsin, 1923.

Attitude of Tabular Concentrations

A mathematical analysis of the effect on the earth's field of tabular forms (for example, magnetic iron formations) in different attitudes, has been worked out by H. L. Smyth.[9] Obviously, the attitude of tabular forms with relation to the earth's field would have a bearing on the degree of influence they would exert toward distorting it. Certain ferromagnetic masses are sufficiently concentrated so that even in the earth's weak field induced polarity occurs in them. In the northern hemisphere, under ordinary conditions, induction in a tabular mass produces a south polarity along the outcropping edge. There are certain very limited attitudes, however, which such a mass can assume, in which the relationship to the earth's magnetic field is such that the induced polarity in the outcropping edge will be north polarity. For example, if a magnetic bed strikes east and west (magnetic) and dips against the earth's field at an angle very slightly greater than that of the inclination of the lines of force, its outcrop will show north polarity.

Consequent upon the fact that a change in attitude can bring about a change in polarity, it follows that there must be a certain attitude where the polarity would be theoretically neutral. Such a case has been actually reported by Broderick.[10] In the Duluth gabbro, long east-west bands of titaniferous magnetite ore failed to cause distortion in the earth's magnetic field, measurable by the methods then used. Their dip, as closely as could be determined, brought the lenses into a position normal to the direction of the earth's field.

These considerations are very significant in demonstrating that the same geological phenomenon can produce magnetic anomalies of widely different characteristics, and of widely different degrees of distortion, dependent simply on attitude.

In order to make a practicable generalization concerning the relations between the attitude of rocks and their disturbing influence it is necessary to refer their attitude to the direction of the earth's field, rather than to a horizontal plane, as is the current convention. This is necessary on account of the variation between the relationship of the earth's field and horizontality. For instance, in the upper Peninsula of Michigan, where the normal dip of the earth's field is about 75° north, the critical dip of a tabular ferromagnetic body at which induced polarity would be expected to be neutral is about 15° south. In Texas, however, where the normal magnetic inclination may be about 60° north, this critical dip would be expected to be about 30° south. Where the inclination of the earth's field is 70° north, dips between the approximate and estimated limits of

[9] H. L. Smyth. U. S. Geol. Survey *Monograph* 36, Part II, Chap. 2.

[10] T. M. Broderick: Some Features of Magnetic Surveys of the Magnetite Deposits of the Duluth Gabbro. *Econ. Geol.* (1918) **13,** 35.

5° to 20° south in a tabular ferromagnetic body would be expected to give rise to north polarity. Elsewhere the limiting dips would have to be determined experimentally. The actual occurrence of induced north polarity along the outcropping edges of such polarized bodies is rare.

Furthermore, in the so-called crust of the earth even these polarized bodies are rare. Qualitatively, however, the changes in influence ascribed to polarized tabular bodies can be carried over to those bodies whose influence on the earth's field is due to differential permeability rather than polarity. If a tabular paramagnetic or diamagnetic body presents itself broadside to the lines of force, its influence will be much less concentrated and therefore less localized than if it presents itself edgewise. For an analogy, consider that it is easier to see flatwise through a pane of glass than edgewise.

In general, it may be said that a more sharply defined distortion of the earth's magnetic field is made by steeply dipping formations—or, referring the attitude of the formations to the direction of the earth's field, those whose dip approaches parallelism with that of the normal lines of force. Obviously, then, the attitude of tabular formations influences the shape of any magnetic anomaly they may cause, and so it should be possible to draw inferences regarding the attitudes of formations from a study of the shapes of the magnetic anomalies.

Accident of Arrangement

Topography

The topographic conditions encountered in the process of measuring anomalies in the earth's magnetic field offer another geometrical factor which can neither be overlooked nor adequately evaluated. Of course, measurements must be taken at the land surface, and, obviously, the ideal condition for obtaining a cross-section of the earth's magnetic field from which to make geological inferences would be a horizontal surface coincident with the surface of the country rock; but this condition seldom, if ever, exists. Two different phases of the topographic factor contribute variations from this ideal condition.

Bed-rock Topography.—First, topographic variations in the country rock may have an influence on measurements of magnetic anomalies. This can be readily understood by visualizing the relationship between the earth's surface and the lines of force of the earth's magnetic field under the influence of a magnetite mass, if that mass occurred in the top of a peak, the bottom of a ravine, the side of a hill, the face of a cliff, or the midst of a level. Clearly although the total shape of the anomaly would differ only in so far as the permeability of the surrounding rock differed from that of air, the actual shape of the measured anomaly would be markedly different for the cases mentioned—as different as

the difference between the planes in which the cross-section of the anomaly was measured, just as the cross-section of a cylinder differs in shape with the direction of the plane of the section. But knowing the direction of the plane of the section of a cylinder and the concomitant shape of the cross-section, one can reconstruct the shape for any other direction. Similarly, knowing the topographic features surrounding an anomaly, one can at least qualitatively visualize the shape of that anomaly referred to a horizontal plane. And there always remains the fundamental fact that there is an anomaly directly connected with a geological feature.

Overburden.—The second phase of the topographic factor is that introduced by variations in the depth and shape of the overburden. Here again visualization of a series of cases will be an aid toward understanding. Picture the relationship between the earth's surface (the plane of the section) and lines of force distorted by a ferromagnetic concentration if over that magnetic mass is superimposed glacial drift in the form of a hill, a flat plain, a valley, or a slope. Obviously the recorded shape of the distortion would be different in each case, but the analogy of the cross-section of the cylinder still holds; and there still remains a direct relationship between the anomaly and a geologic feature, so that the actual location of the source of the anomaly is possible within narrow but varying areal limits. As the thickness of the overburden increases, a point is reached where the influence of that particular distorting feature can no longer be detected. This thickness, of course, depends on the intensity and shape of the distorting influence and the sensitivity of the measuring instrument, and a quantitative estimate of the distance through which an anomaly can be observed is made very difficult by the fact that the simple laws of energy radiation have been observed to be inapplicable. For example, the intensity of a magnetic anomaly over Spur Mount Michigan[11] has been found to vary inversely as about the 0.3 power of the vertical distance above the top of the mountain.

A variation in the thickness of the overburden which may lie between the earth's surface and the source rock of a major magnetic anomaly may cause a change in the vertical relationship between the anomaly as recorded at the surface and the rocks that caused it. This is exemplified by the buried igneous hills in the Panhandle of Texas. These are assumed to represent an extension of the Wichita mountain range of Oklahoma which has been submerged under a thick section of later sediments. In the Wichita mountains themselves the magnetic anomalies have a direct vertical relationship to the igneous rocks which cause them, but as these rocks plunge deeper and deeper beneath the sediments toward

[11] T. B. Brooks: Iron-bearing Rocks. Geol. Survey of Michigan.

the west the magnetic anomalies shift farther and farther north of a vertical projection on the surface, so that the horizontal distance between the anomaly and the vertical projection of its source becomes of the order of magnitude of ¾ mile when the igneous rocks are buried approximately 4000 ft. This shift is due to the inclination of the lines of force of the earth's magnetic field and its amount would vary, therefore, in different latitudes. At the magnetic poles, for instance, where the lines of force are vertical, no such shift would be expected.

One other complication may be introduced by variations in the permeability of the overburden itself, which can be conceived to superimpose minor anomalies on the major anomaly. Here it seems fitting to introduce the final and general conception that the shape of the field at any one point is the result of the total effect of *all* the rocks through which it passes. This introduces the possibility of further complications into the interpretation of anomalies. The absence of an anomaly may mean that a deep-seated paramagnetic reaction has been compensated by an overlying diamagnetic reaction. A slight diamagnetic anomaly may be caused by a deep salt dome or a shallow lens. Once more the warning against simplicity of interpretation unguided by geological data is obvious. This warning gains point in a comparison of certain areas in Texas. In some regions, notably the eastern Panhandle, magnetic anomalies have been found when properly interpreted to reflect subsurface structural features with surprising fidelity, but when the work was carried southwestward the anomalies seemed to show less and less structural significance. This does not mean that they have no geological significance but merely that the geological significance in which the operators were interested has become masked by another geological feature, in this case probably the encroachment of the younger geological section with its concentrations of ferromagnetic minerals and its intercalated igneous extrusives.

The geometrical consideration, then, in spite of certain complicating features, introduces the idea that the ferromagnetic, paramagnetic, and diamagnetic concentrations in the earth's crust take certain definite shapes and attitudes which may be reflected in the magnetic anomalies which they cause. Thus it is possible that veins, dikes, contacts, and the strikes and dips of magnetic beds can be traced, while faults may be located by the sudden discontinuance or offsetting of a linear magnetic anomaly. It would seem, therefore, from the geological set-up, that geomagnetics can be looked to for a clue to features of structural as well as areal geology. This has been borne out in actual field results.

Furthermore, specific geological considerations introduce a number of other possible variable factors, such as the degree and localization of the activity of metamorphic processes as reflected by the secondary development of magnetite. If the observations of changes in the earth's mag-

netic field be made in close conjunction with such geological observations as it may be possible to make, certain of these variables become known quantities and can be fitted into the correlation between anomalies and causes in such a way as to permit the drawing of conclusions regarding the variables that remain unknown. Thus the conclusions to be drawn from geomagnetic methods may not be confined to areal or structural features, but in special cases may be extended to features of metamorphic history or subsurface topography.

Summary of Phases

As a clarifying, unifying, skeletal background, then, for the application of geomagnetics to exploration, we can maintain a visualization of a deep-seated magnetic source within the earth surrounded by a magnetic field of which the ideal symmetry can be and has been established. Within this field lies the earth's crust, composed of different substances grouped into rock formations. The differential permeability of these substances causes distortion from the ideal symmetry of the earth's magnetic field. This differential permeability is definitely related to geological features. Thus the distortion in the earth's magnetic field can be definitely related to geological features. Obviously this bald statement reduces the subject to a degree of simplicity almost absurd.

Application

However, it is safe to say that every symmetry variation in the earth's field has a definite geological significance and so every anomaly recorded by a magnetic instrument is susceptible to geological interpretation. But there remains that impressive array of geological factors which may influence the shapes shown by the anomalies. For example, variations in these shapes may be due to any one or any combination of the following:

1. Variations in mineral composition of the formations:
   a. Due to primary distribution.
   b. Due to secondary development.
2. Variations in the attitude of the formations.
3. Variations in the distribution of the formations:
   a. Due to structural causes.
   b. Due to erosion.
4. Variations in the shape of the formations.
5. Variations in topography:
   a. Bed-rock topography.
   b. Topography of the overburden.
6. Variations in the overburden:
   a. In depth.
   b. In mineral composition.

Clearly, then, unless the influence of certain of these variables is known, only general interpretations can be drawn. Such general inferences as areal distribution and the broader structural features, which, of course, are of prime importance, can always be drawn. But the desirability of recording every observation of geological significance during the progress of a geomagnetic survey is apparent, for if definite conclusions regarding the actual part played by some of these variables can be ascertained, those conclusions can be fitted together in such a way as to indicate the influence of the unknown variables.

In specific cases it is possible, by analyzing the shapes shown by the anomalies, to determine strikes, directions of dip, marker horizons, faults, folds, local intrusives, dikes, contacts, changes in bed-rock topography, metamorphic reactions, and other special features. It is merely a matter of solving an equation for unknown quantities.

But to carry the idea of the effect of differential permeability upon the earth's field to its logical conclusion, we must realize that wherever there is a difference in substance in the earth's field there is a distortion—the smaller the mass and the difference in permeability of the two substances, the smaller the distortion. Thus it is conceivable to have minor distortions superimposed upon or masked by major distortions. A good example of a case where this feature becomes of real importance in exploration is in the "shoestring" regions of eastern Kansas, where the distortions caused by the developed "shoestring oilpools" appear as faint linear trends upon much greater anomalies, probably due to regional structure.

Clearly no rule of thumb methods for attacking geomagnetic problems or interpreting geomagnetic results can be set down, for each field problem is a unit and the method of procedure will depend on the character of the exploration problem and the results sought for. But there are two common factors in every problem—area, and the earth's field. The area to be studied may vary from an acre to a state, and the earth's field may vary from a fraction of a per cent. to percentages measured in thousands. The desired end geologically is to know all the facts bearing on the distribution of the rocks in the area. The desired end geomagnetically is to take enough observations of the earth's field to be able to plot its variations within the area and correlate them with the geological data.

These ends require a systematic attack. The usual procedure is to run traverse lines at intervals varying from 50 ft. to 10 miles along which observations are taken at intervals varying from 12½ ft. to 2 miles, depending on the scale of operation. For example, in tracing out a buried mountain range, as has been done in Texas, readings taken at 1-mile intervals along traverse lines several miles apart suffice to detect the major variations in the earth's field caused by the buried range. But in tracing out minor drag folds, as has been done in the Crystal Falls iron-

bearing district of Michigan, it is desirable to take readings at intervals of about 25 feet.

The traverse system of taking geomagnetic data has the advantage of inducing a systematic survey of the area, which often brings to light geological data which might not otherwise be found. This fact has been proved by the discovery of outcrops during the process of a geomagnetic survey of areas previously mapped geologically, which in themselves would have materially altered the previous geological survey.

Too much emphasis cannot be placed upon the advantage of keeping, as far as possible, a continual correlation between the geomagnetic and the geologic data. Thus it is best, when attacking a new area, to start, if possible, where exposures are most numerous and the geology is most clearly ascertainable without the aid of geomagnetic data.

Instruments

The relationship of differential permeability to distortion in the earth's field brings out the fact that the magnitude of these anomalies may range from the infinitesimal (such as would accompany the difference between two crystals) and the enormous (such as would accompany a deposit of lodestone). Field instruments have been devised which measure anomalies through only a portion of that range. Different instruments are adapted to measure different portions. Some of the instruments which have been adapted to or devised especially for geomagnetic use are the ordinary compass, the dial compass, the Swedish dip needle, the ordinary dip needle, the portable magnetic theodolite, the Thomson-Thalén magnetometer, the Thalén-Tiberg magnetometer, the Schmidt field balance, the Haalck universal variometer, the Schering-Wild earth inductor, the Hotchkiss Superdip, and others. New improvements and new types of instruments are being worked on continually.

Of the instruments now in use, the ordinary dip needle, by virtue of its superior simplicity of construction, facility and speed of manipulation, and definiteness of interpretation, seems best adapted for the measurement of the more pronounced portion of the range of anomalies. It has been extensively and successfully used in the Lake Superior region, and its applicability seems far more widespread than its present use.

The Schmidt field balance, which is the instrument commonly meant when the term "magnetometer" is used, has, by reason of its extensive use in the last half decade, become more or less the standard field instrument for measuring relatively minute anomalies.

During two years of intensive development, the Hotchkiss Superdip has been brought to a state where in direct field comparison with the work of the magnetometer it has shown superiority in speed, practical sensitivity, and directness of interpretation.

These instruments, with the micromagnetometer described by Rieber, have attained the greatest degree of sensitivity concomitant with practical field work. In fact, of these the Hotchkiss Superdip has an adjustable sensitivity theoretically capable of attaining infinity; and the higher the sensitivity, the more anomalies will be detected. However, there is a point where the practicability of any further sensitivity ceases to exist. As the instruments are made more sensitive they respond to anomalies of the order of magnitude of the time fluctuations. These, then, must be corrected out, but the very correction is susceptible of error, and a sensitivity that responds to anomalies of the order of magnitude of this error has passed beyond practical limits, since such an anomaly cannot be distinguished from instrumental or computational error. A case in point occurs over certain orebodies of the Tri-state zinc district, where certain of the anomalies, which must be caused by the differential permeability of the chert-zinc masses as compared with the massive limestone country rock, seem to have an order of magnitude below that of field manipulative error, so that in some cases the anomalies are completely masked.

An extensive body of empirical data is being built up by the practical application of the geomagnetic method of exploration through the use of various instruments, but since much of this work is still held as confidential, the impossibility of assembling it for a back sight for future guidance is obvious. Heretofore it has been impossible to predict much regarding the possible results to be expected in new applications of the method. Fortunately, the method is sufficiently economical in application that it usually pays to try it out in a problematical case. But some logical basis for a prediction of its applicability has been the grail in quest of which most of those interested in geomagnetics have sent many thoughts. An approach to such a basis of prediction bids fair to lie in tabulating either the specific or relative permeabilities of all common rocks, and work along this line is now actively progressing. Another desirable advantage would lie in the devising of a simple apparatus whereby the permeabilities of rock specimens could be obtained quickly. This, too, is now receiving attention.

DISCUSSION

L. B. Slichter, Madison, Wis. (written discussion).—Mr. Stearn's interesting paper suggests a few comments concerning the need which he voices in common with other writers on the subject, for more and better data on the fundamental magnetic constants of rocks and rock-forming minerals. Greater reliability in these data and the consequent possibility for a more penetrating analysis of field results, when needed, is a logical expectation in the further progress of geomagnetics. Despite the relatively large inaccuracies permissible or unavoidable in many geophysical interpretations, our present information on magnetic susceptibilities seems quite inadequate.

Naturally the most disturbing discrepancies are those pertaining to the few strongly magnetic minerals. For example, writers differ greatly in the values assigned to such a common mineral as magnetite. In Stearn's paper, the susceptibility of magnetite is listed as 0.032 (reference wanting), but elsewhere in the paper he refers to Weiss' experiments as indicating that in specimens of magnetite the "induced magnetism" was 85 to 285 times the inducing field. While the meaning of the term "induced magnetism" is ambiguous, technically speaking, apparently a permeability of 85 to 285, corresponding to a susceptibility of 7 to 23, is thereby implied. The vast difference between these values and the one of 0.032 in his table seems worthy of question.

Probably the truth of the matter is that both values are in error. It seems unfortunate to interpret Weiss' work in the way quoted, or to suggest from his experiments the possibility of a 100 per cent. change in the magnetic intensity due to only 1 per cent. magnetite content. Weiss' results are really less startling. They were not obtained at a field strength equal to the earth's (0.6 Gauss) but at higher fields, and only a single specimen was observed down to a field of 0.9 Gauss; or about 50 per cent. greater than the earth's. Some of his data, however, allow extrapolation down to the earth's field. His values for the susceptibility, extrapolated to a field of 0.6 Gauss, would be only about one for three magnetite samples, and about 20 for a fourth. This last value seems inexplicably high.

The writer recently published some results[12] on the susceptibility of powdered magnetite measured at a field strength of 0.6 Gauss. The values of 0.3 to 0.8 (depending on concentration) which were obtained have recently been substantiated by further measurements with new apparatus on solid drill-core specimens from the vicinity of St. Peters, Pa. Two specimens showed susceptibilities of 0.4 and 0.5 (corrected for the percentage of magnetite in them, which was about 55 per cent.). From the curve (Fig. 6 in the article cited), we read a susceptibility of 0.44 for a 55 per cent. magnetite. This close agreement is considered good evidence of the approximate validity of the above values.

TABLE 6.—*Magnetic Susceptibilities of Major Magnetic Minerals Listed by Various Writers*

	Stutzer, Gross and Borneman			Stearns			Slichter			Weiss			Welo and Baudisch		
	k	Specimen	Field Strength	k	Specimen	Field Strength	k	Specimen	Field Strength	k	Specimen	Field Strength	k	Specimen	Field Strength
Magnetite....	0.097 (Approx.)	P	?	0.032			0.3 to 0.8 0.44 @ 55 per cent. Fe_3O_4	P S	0.6 0.6	1. to 20 (extrapolated to 0.6 Gauss)	S	0.9 to 20	0.23 to 0.63	P	10
Pyrrhotite....	0.007	P	220				0.028	P	0.6						
Ilmenite.....	0.03	P	220				0.044	P	0.6						
Specular hematite...	0.003	P	220	0.00043			0.004	P	0.6						
Franklinite...	0.0035	P	220												

k denotes magnetic susceptibility.
P denotes pulverized specimen.
S denotes solid specimen.

[12] L. B. Slichter: Certain Aspects of Magnetic Surveying. See page 238.

For comparison, a brief table of published susceptibilities on some of the important magnetic minerals is appended (Table 6). It is clear that many of these values need to be brought into much closer accord.

The importance of a reasonable degree of trustworthiness in magnetic data is illustrated by the following actual problem. Some magnetite ore had been pierced by several drill holes at a depth of about 120 ft., but it was doubtful whether sufficient tonnage was present to constitute a mine. By a field survey of the intensity of magnetic disturbance produced by this ore, combined with susceptibility measurements on core samples, it was indicated that no tonnage of the required magnitude could exist. Such a conclusion would have been impossible if the value of the susceptibility of magnetite really is uncertain within the hundredfold range suggested by Stearn's figures.

The Dip Needle as a Geological Instrument

By Noel H. Stearn,* St. Louis, Mo.

(Boston Meeting, August, 1928)

Of the many instruments devised for the measurement of magnetic anomalies, the ordinary dip needle, by virtue of its superior simplicity of construction, facility of manipulation, and definiteness of interpretation, seems to merit serious attention as the instrument most readily applicable to the detection of magnetic anomalies of the higher order of magnitude. In spite of the fact that it has been used for many years for the discovery of iron ore in Europe and in the Lake Superior region of America, its capabilities as a geological instrument have not yet been thoroughly plumbed. The Wisconsin Geological Survey, under Dr. W. O. Hotchkiss, E. F. Bean, and H. R. Aldrich, has been most thorough in developing the technique of the instrument, and the results it has obtained have aroused a general interest in a more widespread use of geomagnetics as an exploration method and the dip needle as a geological instrument. The simplicity of the instrument is an especial advantage, as it can be used by anyone, regardless of his technical training; but to insure useful results not only is a knowledge of the background of geomagnetics necessary but also an understanding of the principles of the instrument itself.

Principles of the Dip Needle

The major feature of the dip needle is an elongated magnet suspended with fixed bearings in such a way that it can move in a single plane. If the point of suspension were exactly at the center of gravity of the magnet, the instrument would be identical with the "dip circle" or "dipping needle" of terrestrial magneticists, and as such could be used to determine variations in the inclination of the earth's magnetic field when observed in the plane of the magnetic meridian. The dip needle, however, differs from the "dip circle" or "dipping needle" in having an adjustable counterweight affixed to one end of the magnet. This counterweight serves to balance the magnet at an angle to the inclination of the earth's field when the instrument is oriented in the plane of magnetic

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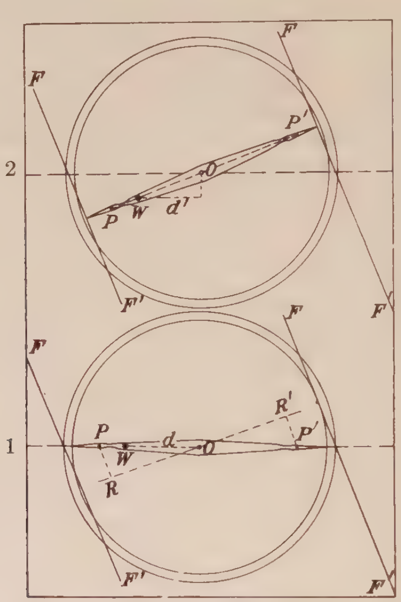
meridian, and therefore to introduce the action of two forces, the resultant of which determines the position of rest of the needle. One of these is the force of gravity, whose direction and intensity may be assumed as relatively constant from place to place; the other is the force of the earth's magnetic field, whose direction and intensity vary from place to place, thus varying the position of rest of the needle.

Figs. 1 and 2 represent the interaction of these two forces. Obviously, although the force of gravity may be considered constant in direction and intensity, its effect on the needle varies because it is applied as a turning



FIGS. 1 AND 2.—DIAGRAMS SHOWING THE RELATIONS BETWEEN THE DIP NEEDLE AND THE FORCES ACTING UPON IT.

moment. This turning moment is the product of the weight and the effective distance from the fulcrum (*wd*, Fig. 1) and clearly it is at a maximum when the needle is horizontal, because the effective distance decreases with a departure from horizontality (Wd^1, Fig. 2). This decrease is proportional to the cosine of the angle of departure from horizontality.

The turning moment of gravity is opposed to that of the magnetic force which is acting on the needle. This force is the product of the strength of the earth's field I and the magnetic moment of the needle, which is in turn the product of the pole strength of the needle s and the length of the magnetic axis L. Thus the magnetic force M is equal to SLI, and acts as a turning moment of which the maximum value occurs when the needle is normal to the inclination of the earth's field, for in that position the force M acts at the maximum effective distance from

the fulcrum. A departure from that position involves a shortening of the effective distance in proportion to the cosine of the angle of departure.

It is clear from the principles involved that this instrument does not measure quantitatively any of the three components of the earth's field but that it responds to variations in both intensity and inclination. Thus its function is to measure various degrees of distortion from any assumed condition of normality.

Construction of the Dip Needle

In spite of the long period of time during which the dip needle has been used, its mechanical construction is still in the developmental

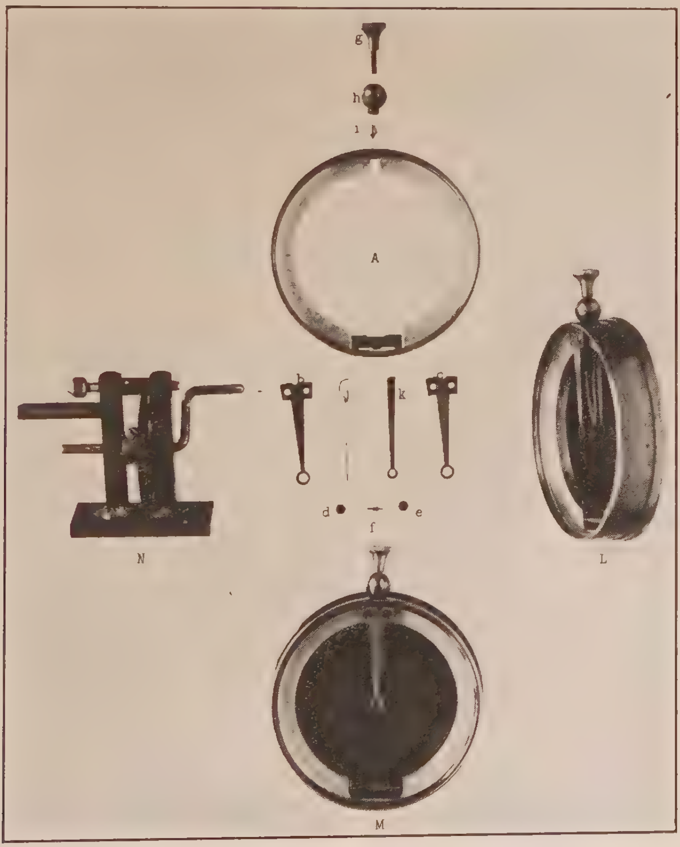


Fig. 3.—Dip needle, dismantled and assembled, and dip lathe.
A = The case
$b-f$ = Needle-mounting assembly
$g-k$ and L = Release assembly
M = Assembled dip needle
N = Dip lathe

stage. Fig. 3 shows the model made by W. and L. E. Gurley, which seems best adapted to the precision of technique that is necessary in

order to use it as a geological instrument. Their later model, unfortunately, is not as satisfactory in field use as the one here pictured.

There are three major constructional features: (1) the case; (2) the needle mounting; (3) the device for stopping and releasing the needle.

The case (*A*, Fig. 3) is a circular brass drum about 3⅞ in. dia. and ⅞ in. thick. It is graduated with a scale reading 0° at horizontality. The horizontal position is established by a level bubble mounted at the bottom of the case. There is about 1° variation in the position of the instrument between the points of disappearance of the bubble. The case is fitted with two glass plates which are sealed in place with wax, and two brass covers held in place by friction.

The needle-mounting assembly is indicated on Fig. 3 by the letters *b* to *f*. Of these, *b* and *c* are rigid brass arms, which are screwed to the case and in which are mounted the jewels *d* and *e*—concave agates in which rest the ends of the pivot *f*. Obviously this bearing introduces more friction than a needle-point bearing, since there must be more or less rolling and sliding movement while the needle is swinging. The pivot is screwed through the needle at its center of symmetry. The needle is built of two or more thin steel strips to insure saturation when magnetized. At one end is mounted a counterweight, either in the form of a brass rivet in a tapered needle, or a movable rider on a straight-sided needle.

The release assembly is indicated on Fig. 3 by the letters *g* to *k*, of which *j* and *k* are two brass spring clamps through the ends of which the pivot passes. They can be seen in position in *L*. When at rest they press firmly against the two sides of the needle. In order to release them it is necessary to turn the thumbscrew *g*, which pushes a wedge *i* against shoulders shaped in the clamps, and spreads them apart, thus releasing the needle.

The completely assembled dip needle is shown by *M* (Fig. 3). Its weight is 13 oz. With the leather case in which it is carried in the field, it weighs 22 oz. The dimensions of the leather case are 5 by 4¾ by 1¼ in. Thus the maximum space requirement for the instrument is 29¾ cubic inches.

Manipulation of the Needle

Because the dip needle is an instrument that registers *relative* changes, all care must be taken to see that the only relative changes it registers are those in the earth's magnetic field. Every gesture in manipulating the instrument should be routinized as far as possible. Every item of personal equipment should be kept in the same relative position.

The readings are taken with the instrument oriented so that the needle is swinging in the plane of the magnetic meridian. This orien-

tation requires a series of gestures, which differ in minor details with different manipulators. The following procedure has been found to be the quickest and most easy to duplicate:

1. Get a firm footing facing approximately west.

2. Hold the instrument so that the needle swings in a horizontal plane and release the needle.

3. Determine the direction of the magnetic meridian; orient the instrument to that direction and clamp the needle.

4. Raise the instrument to a position directly in front of the face so that the needle will swing in a vertical plane.

5. Steady it and release the needle.

6. Read the needle on the swing, always referring the upward swing to the same graduation, preferably the 0° position.

With this method of procedure, a reading can be taken in less than 20 sec., after routine facility has been acquired. For the expert manipulator it is safe to say that a maximum of 30 sec. is required to take and record a reading under ordinary field conditions.

Sensitivity of the Dip Needle

The sensitivity of the dip needle to changes in the earth's magnetic field depends on a number of factors, chief of which are the magnetization of the needle, the amount of friction in the bearings, and the adjustment of the normal position of rest relative to the direction of the earth's magnetic field.

A needle magnetized to saturation is most sensitive because it gives the maximum magnetic moment to act against the gravitational force.

In a mounting that gives the smallest and smoothest surface of contact between the pivot and the jewels, the damping effect of friction on the sensitivity is reduced to a minimum.

The setting of the normal position of rest involves a consideration of the interaction of the magnetic force and the gravitational force. The measurement of different degrees of variation in the earth's field brings about a series of changes in the position of the needle. These changes are accompanied by progressive variations in the values of the forces acting on the needle.

When the inclination of the earth's field is vertical, the positions of maximum value for the gravitational and magnetic forces coincide at horizontality, and the progressive variations that ensue from a departure of the needle from horizontality are parallel. However, when the earth's field has any angle of inclination other than that of 90°, these positions of maximum value do not coincide, and the progressive variations in the gravitational and magnetic forces that ensue from a departure of the needle from its normal position of rest are not parallel.

For example, in Fig. 2, the needle PP' is in the position of rest at which the maximum magnetic force is exerted upon it, since it is normal to the inclination of the earth's field FF', of which the angle is represented as 70°. But the gravitational force is not at its maximum, and any departure from this position of rest caused by a positive variation in the intensity of the earth's field would involve a decrease in the proportion of the magnetic force acting on the needle and a corresponding increase in the proportion of the gravitational force for the first 20° of departure.

The problem, then, is to find the position at which to set the needle in a given region where the normal inclination is known, so that for a certain change in the earth's field there will ensue the greatest deflection of the needle from that normal position of rest. This will be the position of maximum sensitivity.

The instrument registers variations in both intensity and inclination, and so its working position of maximum sensitivity must involve a compromise between the positions of maximum sensitivity for each of those magnetic components.

Formulas of Maximum Sensitivity

The positions of maximum sensitivity to intensity changes and changes in the inclination of the earth's magnetic field have been computed mathematically[1] and the resulting formulas are as follows:

1. For maximum sensitivity to intensity changes $x = -\left(\frac{90 - a}{2}\right)$ where x is the angle (measured from horizontality) at which the dip needle should be set for its normal position and a is the angle of inclination of the earth's field; the minus sign indicates that the north end of the needle should be above horizontal. Thus for any constant inclination of the earth's field the position of maximum sensitivity to intensity changes is halfway between the position at right angles to the field and the horizontal position, or half the complement of the angle of inclination. When the earth's field has an inclination of 70°, for instance, the position of maximum sensitivity to intensity changes is $-10°$, or with the north end of the needle 10° above the 0° reading.

2. For maximum sensitivity to variations in the angle of inclination the formula is $x = +\frac{a}{2}$ where x is the angle of departure from horizontality that the dip needle should register as its normal position of rest and a is the angle of inclination; the plus sign indicates that the north end of the needle should be below horizontal. Thus, if the intensity of a given field

[1] The former by R. R. Purucker and E. J. Peters, whose results were checked by Dr. Warren Weaver, Associate Professor of Mathematics at the University of Wisconsin; the latter by Dr. Weaver.

remain constant, the position of maximum sensitivity to variations in the inclination is a point below the 0° reading equal to half the angle of inclination; *i. e.*, for a 70° field, the normal should be +35°.

Because these two positions of maximum sensitivity do not coincide, the actual working normal of the instrument must lie between them, and its position between them must be determined by the relative importance of the variations in intensity and inclination. If they were expected to be of equal importance the working normal would be halfway between the position of maximum sensitivity for intensity changes and that for inclination changes. But this is not the case.

3. Hotchkiss has shown that these two components are actually not of equal importance but that the relative importance of the intensity component is about five times that of the inclination component. The working normal can be found from the formula $x = -\left(\frac{90 - a}{2}\right) - \frac{1}{2} \tan^{-1} \frac{1}{k}$ in which the first two expressions give the position of maximum sensitivity to intensity changes and the last expression gives the compromise from this position which should be made when the intensity factor is k times as important as the inclination factor. Thus if the ratio of 5 to 1 be considered, the formula would signify that the instrument normal should be varied from the position of maximum sensitivity to intensity changes by an amount equal to half the angle whose tangent is $\frac{1}{5}$ or by about 5.5°. Thus for a field whose inclination is 70° the most favorable working normal for the dip needle is about −4.5°, or with the north end of the needle 4.5° above horizontal.

Instrument Behavior

Heretofore the position of normality for the dip needle has been arbitrarily set. From purely empirical observation it became apparent that for a field whose normal inclination is about 70° a dip needle normal of about 12° (*i. e.*, with the north end of the needle about 12° below horizontal) would work satisfactorily. Set thus, the instrument has its normal established on the basis of equality of importance of the two magnetic components. Hotchkiss[2] has computed the behavior of an instrument of which the normal is established thus arbitrarily:

"For a needle which has a normal of 12° below horizontal in the earth's field at Madison (intensity = 0.75 Gauss, inclination = 76°) the counterweight was weighed and found to be 22 mg. The distance a (effective arm) was measured and found to be 1.9 cm. With these constants known, it was possible to compute the moment of the counterweight for any position, and hence the moment of the force M needed to balance it. Conversely it was possible to compute a counterweight for this needle which would give any desired reading for any field selected.

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[2] Mineral Land Classification. Wisconsin Geol. Survey, *Bull.* **44,** 100.

"From these data a series of curves (Fig. 4) was drawn. These show the positions this particular needle would assume for all variations of the intensity from 0 to 1.4 Gausses, and for angles of inclination varying by 5° intervals from 65° to 90°. The ranges of angle and intensity were chosen to include all intensities and all degrees of inclination ordinarily found near magnetic formations. The curves in *A* show the dip-needle readings when the needle is counterbalanced so as to have a reading of 12° below horizontal in the normal field at Madison. . . .

"These curves give a complete and definite picture of the behavior of the ordinary dip needle under the conditions usually met with in the field. From *A* it appears that a positive (downward) dip of 20° may be due to any inclination of the field from 65° to 90° but cannot indicate an intensity greater than 0.91 or less than 0.69 (Gauss) within that range of inclinations.

"In the space included between the dips of plus 2° and minus 24°, the change in the dip-needle reading is about 12° for a 10 per cent. change in the intensity, and an equal amount for a 20° inclination of the earth's field. Thus 10 per cent. change in intensity may be said to be the equivalent of 20° change in inclination so far as effect upon the dip needle is concerned. The intensity changes found in areas of local attraction vary 10 times this amount and the total change in inclination ordinarily found is probably less than twice 20°. Thus for ordinary fields the changes in *intensity* may be said to have at least five times the effect of changes in *inclination* upon the dip-needle readings.

"From the curves it is apparent that the dip needle is most sensitive to intensity changes near the center where the curve approaches a horizontal direction. Toward the ends of the curve the change in inclination has a relatively greater effect."

Quantitative Sensitivity

The actual quantitative sensitivity of the dip needle can be determined in three ways: by computation, by laboratory experimentation, and by field measurement. From all three of these methods figures are available.

TABLE 1.—*Sensitivities of Dip Needle*

Instrument Number	Gauss Per 1° Variation	Per Cent. Variation per 1° Dip Variation
15	0.0016	0.21
36	0.0021	0.28
1	0.0022	0.29
33	0.0024	0.32
10	0.0029	0.39
5	0.0030	0.40
27	0.0030	0.40
39	0.0032	0.42
32	0.0035	0.46
21	0.0041	0.53
26	0.0042	0.55
41	0.0046	0.61
37	0.0056	0.72
29	0.0075	0.99

It is obvious from the computed curves (Fig. 4) that in the ordinary working quadrant of the dip needle a change of from 1° to 2° is the response to a 1 per cent. change in the earth's field.

In the laboratory of Mason, Slichter, and Hay, at Madison, Wis., dip needles were placed in an artificial field oriented with its axis parallel to the earth's field. A controlled current then was permitted to vary the earth's field a known amount.

Table 1 shows the sensitivities of 14 different instruments which had been conditioned for field use.[3]

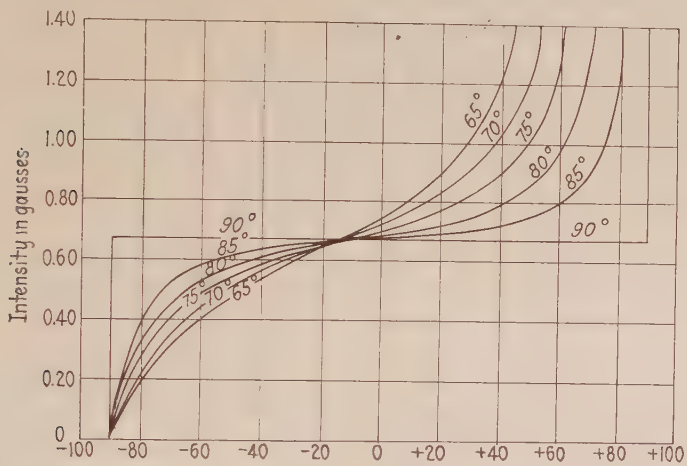

Fig. 4.—Positions assumed by a dip needle with normal of +12, in magnetic fields of various intensities and inclinations. (After Hotchkiss.) The normal was set in a field of 0.75 Gauss with an inclination of 76°.

A variation of 1° of the dip needle was produced by a variation of intensity ranging from 0.25 to 1 per cent., depending on the instrument. The average, however, was slightly less than 0.5 per cent.

Field data secured in northern Wisconsin by W. O. Hotchkiss and E. M. Terry[4] show one case where a 5 per cent. increase in the intensity produced an 8° variation in the dip-needle reading. Thus 1° variation of the dip needle corresponds to 0.625 per cent. variation in the intensity of the earth's field within the working range of 8°. An increase of 52 per cent. in the intensity produced a 66° variation in the dip-needle reading, or an average of 1° dip-needle change for 0.788 per cent. change in the field intensity over a working range of 66°. Clearly, the smaller the variation, the greater the relative sensitivity.

Thus the results of computation, experimentation, and observation check closely enough to give rise to the generalization that for the

[3] Instruments from the Wisconsin Geological Survey prepared by J. M. Hansell.

[4] Mineral Land Classification. Wisconsin Geol. Survey, *Bull.* **44,** 134.

northern United States, and regions magnetically comparable, a 1 per cent. variation in the intensity of the earth's field produces a deflection of the dip needle of from 1° to 2°.

Theoretically, then, 1 per cent. concentration of magnetite buried under 100 ft. of cover would produce a dip-needle deflection of from 1° to 2° if it is assumed that the energy radiation law involved is that the force varies inversely as the first power of the distance. The laws of energy radiation, however, are based on assumptions that the energy radiates from a point or a line, neither of which conditions can be said to obtain in magnetic formations. But the actual change in intensity over a magnetic body was measured by T. B. Brooks.[5] The curve of change plotted from his observations points to a more rapid rate of diminution near the source of magnetic force than at a distance from it. Neither of the ordinary laws of radiation are discernible in the curve. Its regularity and shape suggest that it is exponential. When plotted on logarithmic paper, it shows that its equation is $y = \frac{1}{x^{0.3}}$. That is, the force varies inversely as the 0.3 power of the distance. In the observed case, a force which at its surface was 370 per cent. of the earth's field had decreased to 140 per cent. of the earth's field at a distance of 94 ft. above its surface.

The dip-needle deflection does not increase in direct proportion to the percentage variation in the earth's field; so that it does not follow that a dip needle that is said to register 1° deflection in response to 0.5 per cent. variation will also register 80° deflection in response to 40 per cent. variation. The relative sensitivity decreases with an increase in the percentage variation in the earth's field according to the cosine factor of proportionality. But, obviously, the instrument is relatively most sensitive to the smallest variations in the earth's field.

Sources of Error in Use of Dip Needle

There are many sources of error in the use of the dip needle, all of which should be evaluated, and borne in mind, but only a few of which are really quantitatively important. The sources of error can be divided into three groups: those introduced by the actual manipulation of the needle, those introduced by mechanical defects, and those introduced by certain magnetic factors.

Manipulative Sources of Error

Orienting the Instrument.—Since the dip needle is held in the hands, its orientation must necessarily be approximate, and importance attaches to the consideration of just what effect this approximation may have

[5] T. B. Brooks: Iron Bearing Rocks, Geol. Survey of Michigan (1873) 235.

upon the readings of the needle. Ideally the instrument should be *level, vertical*, and *exactly in the plane of the magnetic meridian.* The level bubble is so constructed that a variation of about 1° in the dip-needle reading occurs between the two points of disappearance of the bubble. Thus a fraction less than 0.5° of variation in the reading can at most be ascribed to failure to center the level bubble. Obviously, this fraction will usually be less than 0.25°, an amount very difficult to detect.

The approximation to verticality in the plane of the magnetic meridian is not so precise; therefore a quantitative investigation was conducted to ascertain how much effect failure to orient the needle with precision would have on the readings. In these experiments the various initial positions of rest were obtained (1) by altering the gravitational component acting on the needle (*i. e.*, shifting the instrument normal in a constant field) and (2) by varying the magnetic component acting on the needle (*i. e.*, changing the field with a constant instrument normal). The results showed that it is necessary to vary the instrument about 20° from verticality and from 15° to 20° from parallelism with the plane of the magnetic meridian in order to vary the reading by 1°. The approximation achieved in the field, even in very rapid work, probably comes well within 5° of parallelism with the magnetic meridian and 2° of verticality, and the resulting deviation even from those limits would be extremely difficult to detect.

Taking the Readings.—The practice of reading the needle on the swing introduces a source of error due to the relative change in the amount of the forces acting on the needle. The difference between the reading taken on the swing and that taken in the position of rest is very slight. For a deflection of 7° read from 0° the difference would be about 0.25° This difference decreases with lower readings and increases with higher ones. By the time it reaches a detectable magnitude the deflections are so great as to render the difference proportionally negligible.

The fact that the graduated scale is not in the same plane with the needle gives rise to the possibility of error due to parallax. This can amount to as much as 0.5° in careless reading, but can be minimized by carefulness.

In the field use of the needle the variations due to manipulative error will be negligible if one consideration is continually borne in mind; *i. e.*, the manipulator should reestablish for each reading exactly similar conditions, as nearly as possible, so that the only variations recorded by the instrument may be variations in the magnetic field that he is examining.

Mechanical Sources of Error

Certain mechanical factors introduce sources of error of much importance:

1. Defects in the jewels, such as pitted, fractured, or irregular surfaces.
2. Defects in the pivot points, such as rust flakes, pits, irregularities (for example, an oval cross-section at the surface of contact with the jewel), blunted ends.
3. Dust particles or moisture between the contact surfaces of the jewels and pivot points.
4. Imperfections in the adjustment of the release clamps, which cause them to snap the needle.
5. Gradual demagnetization of the needle.

The general symptoms of mechanical imperfection shown by an instrument are:

1. Failure to duplicate readings at a given station.
2. Irregularity in the periodicity of the swing.
3. Jerkiness at the time of release.
4. Comparative sluggishness of movement.

Repair of Dip Needle

These mechanical imperfections occur often enough to make it necessary to carry a repair kit in the field. An adequate repair kit contains the following items:

1. A lathe for grinding pivots and polishing jewels. (Devised by W. O. Hotchkiss for the Wisconsin Geol. Survey. See N, Fig. 3.)
2. A fine oilstone or hone.
3. Fine crocus paper for polishing.
4. A small screwdriver.
5. A small pair of pliers.
6. Beeswax for sealing the instrument.
7. A small roll of cotton.
8. A can of oil.
9. A can of gasoline.
10. Spare jewels.
11. Spare pivots.
12. A spare release assembly.
13. Spare glasses.
14. A high-power hand lens.

As soon as any of the mechanical symptoms are observed the dip needle should be overhauled. This involves the following general procedure.

1. Remove one glass plate from the instrument, using gasoline to dissolve the wax which seals it in place.
2. Adjust the release arms so that they exert a constant and equal pressure on each side of the needle.
3. Unscrew one jewel, release the clamp on the needle, and lift it from the other jewel.

4. Unscrew the pivot from the needle after noting their relationship.
5. Repoint the pivot by grinding it in the lathe. The lathe is designed for one man. If an assistant is available it is better to remove the set screw which holds the large cogwheel to the crankshaft, and rotate it back and forth with a leather thong for a belt. This obviates the possibility of grinding a point with an ellipsoidal cross-section.
6. Polish the points as perfectly as possible.
7. Place a small wooden peg in the lathe and polish the jewels.
8. Remount the needle on the pivot as it was before.
9. Examine pivot points and jewels with extreme care to insure freedom from dust.
10. Reset the needle and screw in the jewel until the needle swings freely but has little play normal to the plane in which it swings.
11. Retest the action of the release.
12. Test and, if necessary, reset the normal position of rest by altering the counterweight.
13. Be sure that the inside of the case is free from dust.
14. Reseal the glass cover.

The process of polishing produces a certain amount of static electricity, which will influence the normal to some extent, so that the day following the repair of an instrument it usually shows a normal slightly different from the one established during the process of repair.

The shape of the pivot has a direct bearing on the sensitivity of the instrument. Obviously, a pivot so shaped that it gives a minimum of surface contact with the jewel will produce the least friction. But a pivot with an extremely delicate point is too readily blunted; and so a compromise must be established in favor of workability. Heretofore this compromise has been made by the mechanician doing the repairing. The result has been satisfactory. Nevertheless, a quantitative investigation of the matter seems worth while.

The gradual loss of magnetism in the needle may produce variation over long periods of time, but the change during the period of one unit investigation is negligible. However, accidental demagnetization may occur, which would have a decided effect on the readings.

Magnetic Sources of Error

Aside from the manipulative and mechanical sources of error, which can be actually observed by a careful manipulator, there are other sources introduced by factors unconnected either with the instrument or its user.

The most important of these is the diurnal magnetic variation. In the latitude of the Lake Superior region the intensity of the earth's field varies on ordinary days about 0.27 per cent. and on extreme days about 1.12 per cent. Table 1 shows the percentage of variation in intensity

required to cause a deflection of 1° in 14 dip needles which had been conditioned to work in the Lake Superior region. On ordinary days one of them will undergo a deflection of over 1°; at least three will undergo deflections of approximately 1°; and eleven will undergo deflections of approximately 0.5° or over. On unusual days, all of them will vary over 1° as a result of diurnal variation. This change, however, is very gradual and the effect of it does not enter materially into a series of readings taken at close intervals of time.

The daily shift in the dip-needle normal, which is a common observed phenomenon, may be ascribed partly to the diurnal magnetic variation and partly to the effect of temperature variation on the mechanics of the instrument. An increase in temperature causes a decrease in the instrument readings. Although this has not been quantitatively determined, its effect is slight enough to be negligible in field practice.

Magnetic storms of comparatively rare occurrence cause inconsistencies in dip-needle readings which are not likely to be misleading.

Static electrification of the glass face of the instrument is a common occurrence, especially on cold days. This produces marked inconsistencies in dip-needle readings which can readily be eliminated by breathing on the glass.

The dip needle, then, is an instrument designed to measure the relative degrees of distortion in the earth's magnetic field caused by the differential permeability of the formations forming the earth's crust. Its sensitivity is sufficient to detect the variations caused by the ordinary concentrations of ferromagnetic minerals found in the earth's crust, but there exist certain mechanical and theoretical limits to its sensitivity.[6]

Problems Solved by Aid of the Dip Needle

The dip needle has been used to assist in the solution of many varied problems. T. M. Broderick has located inclusions of iron formation in an intrusive gabbro. H. R. Aldrich has located faults in iron formation, mapped areal distribution of lavas and intrusives, and traced copper-bearing lodes. A. E. Walker has located iron orebodies in Missouri. W. O. Hotchkiss has traced igneous contacts, dikes, and ore lodes. These are only a very few examples.

The following case[7] is a single example of the applicability of the dip needle as a geological instrument to one type of geological problem. In one of the iron districts in Upper Michigan there exists a complicated structural situation which has puzzled geologists for a long time, although

[6] The new Hotchkiss Super-Dip has overcome these limits, so that it is capable of attaining theoretically infinite sensitivity.

[7] Work done by the writer in 1924 assisted by Dr. C. O. Swanson, under the direction of L. P. Barrett.

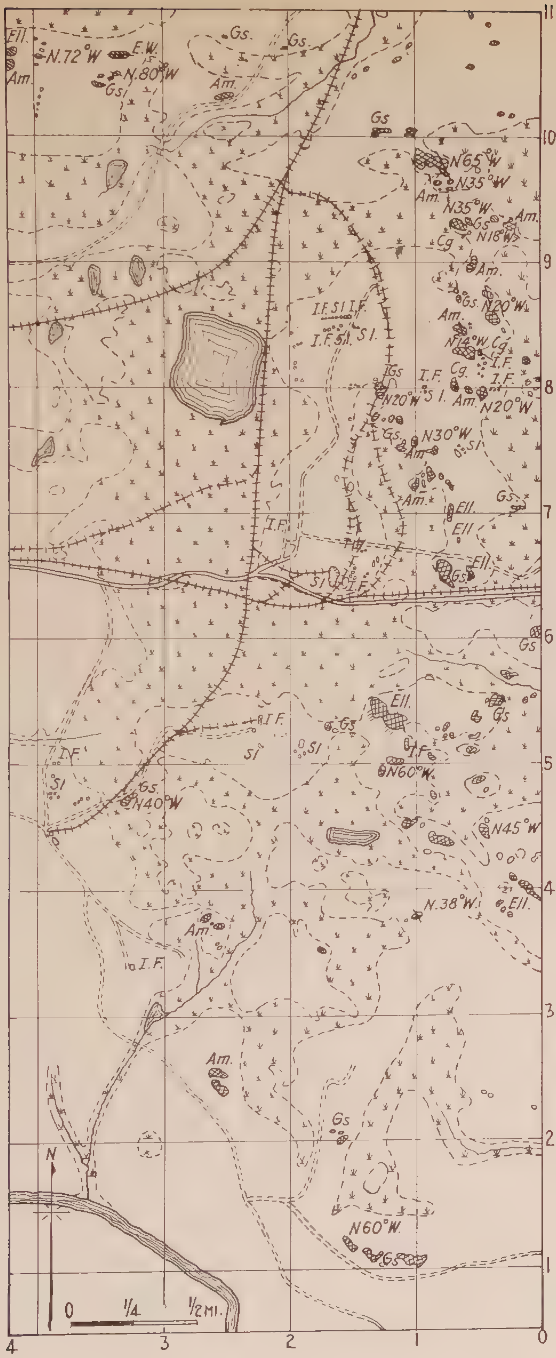

FIG. 5.—DETAILED GEOLOGICAL MAP OF AREA "C," UPPER PENINSULA OF MICHIGAN.

it has been subjected to considerable study on account of the presence of iron formation. In fact, five mines have been operated in the region.

One obstacle encountered in the study of the area was the difficulty of determining the geologic section on account of the lack of sufficient exposure of a marker horizon. The relative resistance to erosion of the rocks is such that the exposures are mostly of volcanic rocks which were not considered interdistinguishable. The difficulty of obtaining any but the most general of conclusions regarding the area from an ordinary geological survey, even though it be very detailed, is made clear by an examination of Fig. 5, which is a detailed fact map recording all the available information in the area. During the progress of the magnetic survey, it became apparent that at least two horizons produced distinct magnetic anomalies. From these horizons it was possible to work out the geologic section for the areas as follows:

The lowest formation of the sequence is composed of a series of volcanic agglomerates and tuffs, which is overlaid by a horizon of ellipsoidal flows. These two phases produce about the same reaction in the dip needle. They are succeeded by a black highly magnetic tuff with interbedded flows. Just above this occurs a horizon of slate and iron formation which was observed definitely to be intercalated between volcanic rocks. This iron formation in places is slightly more and in other places slightly less magnetic than the underlying volcanic horizon, so that the location of the magnetic maximum shifts from one to the other horizon. The iron formation is characterized by granular jasper. It is immediately overlaid by a conglomerate with a tuffogene matrix, containing pebbles of iron formation and volcanic rocks. It seems to represent a hiatus in volcanic activity and so would be expected to be lenticular in shape, so that it possibly disappears along the strike. Next is a succession of ellipsoidal greenstone flows followed by a horizon of amygdaloidal flows of a rich vivid green color, with amygdules containing a blackish green serpentine. The vividness of this horizon makes it easily recognizable.

The foregoing sequence constitutes the section of volcanic rocks in the area. They are considered as Middle Huronian in age. Over the outer volcanic rocks there occurs a gray, siliceous, fissile slate, containing easily discernible octahedra of magnetite. This horizon is very magnetic. It grades upward into a ferruginous slate or slaty iron formation whose upper horizon is characterized by the presence of dark red limpid chert in lenses and thin even bands. The iron formation is also supposed to be middle Huronian. Over the iron formation occurs a gray to greenish micaceous slate and graywacke slate which causes little deflection in the dip needle. This is supposed to overlie the Middle Huronian iron formation unconformably.

FIG. 6.—CONTOUR MAGNOGRAPH OF AREA "C."

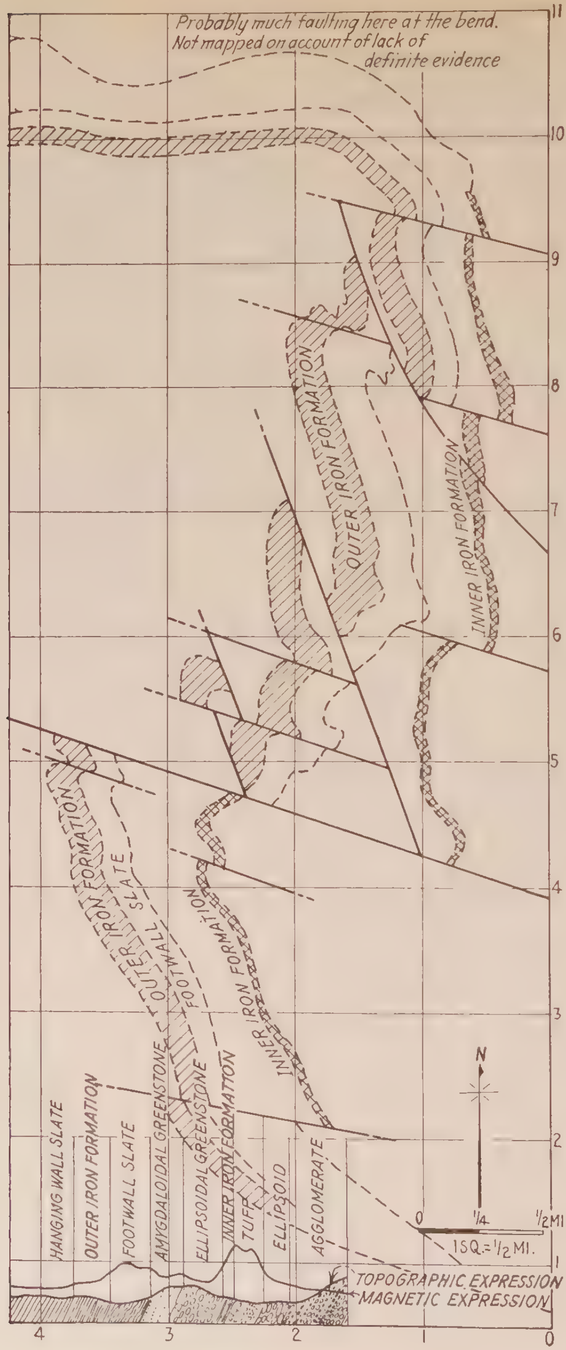

FIG. 7.—GEOLOGICAL MAP OF AREA "C."

The contour magnograph of the magnetic data (Fig. 6) suggests very strikingly certain structural relationships between the magnetic horizons. With these suggested relationships as a key and the stratigraphic sequence as a check, it was possible to review all the observed geological facts such as strike observations, fracture systems, schistosity observations, topographic indications including cliffs, scarps, swamp distributions, and fit them into a structural pattern so that none were discrepant. Fig. 7 shows the resulting map, together with the stratigraphic section and the magnetic and topographic expressions of the rocks which compose it. Although this map was made without reference to underground data from the mines in the area on account of the inaccessibility of the data at the time of the survey, it was later discovered that the workings of one of the mines show faulting corresponding in strike, location, and stratigraphic displacement to the faulting shown by the map.

To present a general idea regarding the cost of making a dip-needle survey, it is worth while to mention that all of the work shown on these maps, including the mapping of the surface features, the geological data, and the magnetic observations, was done in less than a month by two geologists assisted by two compassmen and a cook. The equipment involved a capital outlay of less than $300 in addition to an ordinary camp outfit, and included five dip needles, two dial compasses, and a repair outfit.

DISCUSSION

G. D. Barron, New York, N. Y.—One of the cases cited by Mr. Stearn was for a company with which I was connected. The only thing I wish to point out is that in that geological survey the dip needle was used as an accessory instrument, covering 280,000 acres at a total cost of $20,000, which included field expenses—the full expenses of the survey and the complete report, and everything else. So that the dip needle has certain advantages of cost where it is used, where its use will give the interpretation that we need.

Magnetometric Investigation of Gold Placer Deposits near Golden, Colo.

By C. A. Heiland,* Golden, Colo., and William H. Courtier,† Denver, Colo.

(Boston Meeting, August, 1928)

The investigations described were made on a portion of Clear Creek basin near Golden, Colo. (A portion of the area under survey is shown in Fig. 1. The photograph was taken in the vicinity of station No. 2 of Line II.) Three principal purposes were in the minds of the investigators:

1. To repeat with modern, more sensitive and more reliable instruments magnetometric placer investigations previously made.

2. To disprove the recently advanced theory that deposits of the glacial period have no pronounced magnetic effects.

3. To develop in detail the theory of the magnetic effects produced by placer deposits.

Six years ago, the first magnetic observations were published, which proved that it is possible to locate gold-bearing channels with relatively crude instruments.[1] Two years ago, K. C. Laylander published results obtained with similar instruments.[2] The measurements of the latter were discussed by the senior author of this paper.[3] At that time, however, the material available for such a discussion was somewhat scarce and not made with the most modern magnetic instruments, although very brilliant results were obtained. Therefore, the senior author felt that magnetic investigations should be made on very clear geologic conditions with the most modern instruments.

Comparison of Magnetometers

The junior author undertook this work in the spring of 1928. The magnetic field balances, for vertical and horizontal intensities, designed by Schmidt were used (Figs. 2 and 3). The scale value of these instruments was approximately 25γ and 15γ, respectively. The middle

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† Assistant Geologist, The Midwest Refining Co.

[1] A. Gibson: Magnetometric Determinations Applied to Placer Mining. *Engng. & Min. Jnl.* (1922) **114,** 1064.

[2] K. C. Laylander: Magnetometric Surveying as an Aid in Exploring Placer Ground. *Eng. & Min. Jnl.* (1926) **121,** 325.

[3] C. A. Heiland: Prospecting with the Magnetometer. *Engng. & Min. Jnl.* (1926) **122,** 59.

Fig. 1.—Looking east in Clear Creek basin from a point near Station No. 2, Line II.

error was about $\pm 10\gamma$ for the vertical and about $\pm 8\gamma$ for the horizontal balance.

There happened to be a Thomson-Thalén magnetometer temporarily in the possession of our department. The scale value of such an instrument is not constant but depends on the distance of the auxiliary magnet. For small intervals of distance, however, the reading-intensity curve is fairly straight. The instrument at the school had a scale value of about 27γ for $\frac{1}{100}$ mm. at a distance of 33 to 34 mm. For greater readings the scale value is somewhat greater, because the readings increase as the distance of the auxiliary magnet from the system decreases.

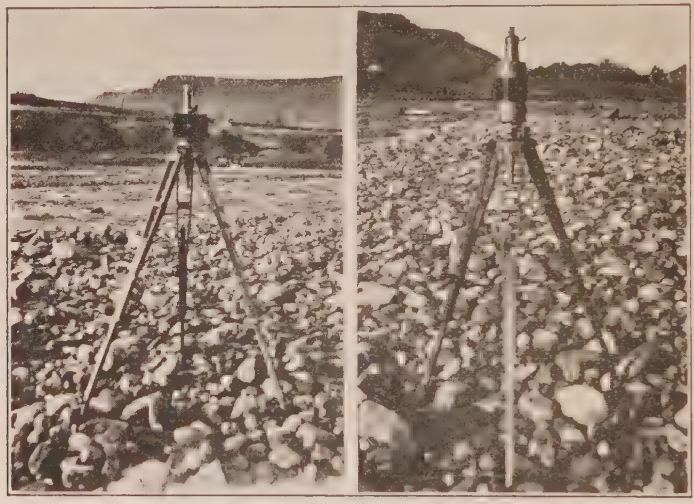

FIG. 2.—THE VERTICAL MAGNETOMETER.

FIG. 3.—THE HORIZONTAL MAGNETOMETER.

K. C. Laylander expressed his magnetometer curves at Quesnel River in scale divisions of the Thomson-Thalén magnetometer, so that we could obtain an approximate idea of what magnetic disturbances to expect. His maximum disturbance above the strongest concentration was 22 scale divisions (hundredths of a millimeter). Assuming for his magnetometer the same scale value as that of the magnetometer tested at the school (although his absolute readings were greater), we obtain a disturbance of 660γ above the strongest concentration. This disturbance seemed extremely great; yet it may be even greater on account of the greater readings.

We were surprised that the anomalies which we obtained on the concentrations of Clear Creek with our instruments were so much smaller. The maximum amplitudes above the concentration *A* (see Figs. 6-11) are: in line I, 65γ; in line II, 155γ; in line III (above *B*), 165γ; in line IV, 165γ; in line V, 235γ; in line VI, 195γ. The average

anomaly, therefore, is 165γ on the strongest concentrations; that is, about 3.3 times smaller than that observed by Laylander.

These figures suggested a possible difference of the amplitudes recorded by the Schmidt balance and other magnetometer observations previously observed by the senior author.

In order to obtain more data on this problem, observations were made with both the Thomson-Thalén and the Schmidt balance on a number of stations on line III. Results are given in Table 1.

TABLE 1.—*Observations with Two Different Magnetometers*

Station	1	2	3	4	5	6	7	8	9	10
Schmidt, gammas	155	115	95	120	140	135	125	135	145	125
Thomson-Thalén, scale div.	35	30	11	25	18	41	31	36	27	13

From these values it may be seen that the absolute values of the anomalies measured with the Thomson-Thalén are 10 times larger at Clear Creek than those measured with the Schmidt balance but that the general trend of the curve, with the exception of only one station (No. 5), is very similar to that of the Schmidt balance.

Reich believes[4] that the greater amplitudes previously observed by the senior author are due to the greater interval of error of the magnetometers then used. We have some doubt that this is the correct explanation. If it is correct, the following should be expected.

Let us assume that a certain curve has been obtained at Clear Creek with the Schmidt balance. The maximum amplitude of the anomaly there is 165γ. If we should contemplate measurements with the Thomson-Thalén magnetometer we would conclude that it would not be advisable to use such an instrument, as the middle error of the Thomson-Thalén is at least plus or minus 100γ. We should expect that on account of this error the Thomson-Thalén curve could not resemble the Schmidt curve at all. The experience at Clear Creek showed, however, that the general trend of the two curves did agree fairly well, because the amplitude of the anomalies registered by the Thomson-Thalén was so much greater.

There is, however, no apparent physical reason why the Schmidt balance should register smaller amplitudes than other vertical magnetometers. This problem is well worth while investigating by systematic comparisons in the field. Our interpretations of the results obtained with Schmidt's balance will hardly be affected by these circumstances, because we base them on relative amplitudes of one profile as compared to another or one part of a profile as compared to another; that is, on the gradients of the curve. We believe that the Schmidt balance is very much better for placer work than the Thomson-Thalén magnetometer.

[4] H. Reich: Erdmagnetismus und glaciales Diluvium. Berlin, 1925.

In the former, subsequent readings check much better on one station than in the latter; it is better protected against temperature changes. The principal disadvantage of the latter is that the base position changes too much, as one goes from one station to another, on account of the very unsatisfactory fastening of the magnet rods in the balance ring.

Details of the Recent Survey

Measurements with the vertical intensity balance were made on 80 stations. On profiles II and III, the horizontal intensity (24 stations)

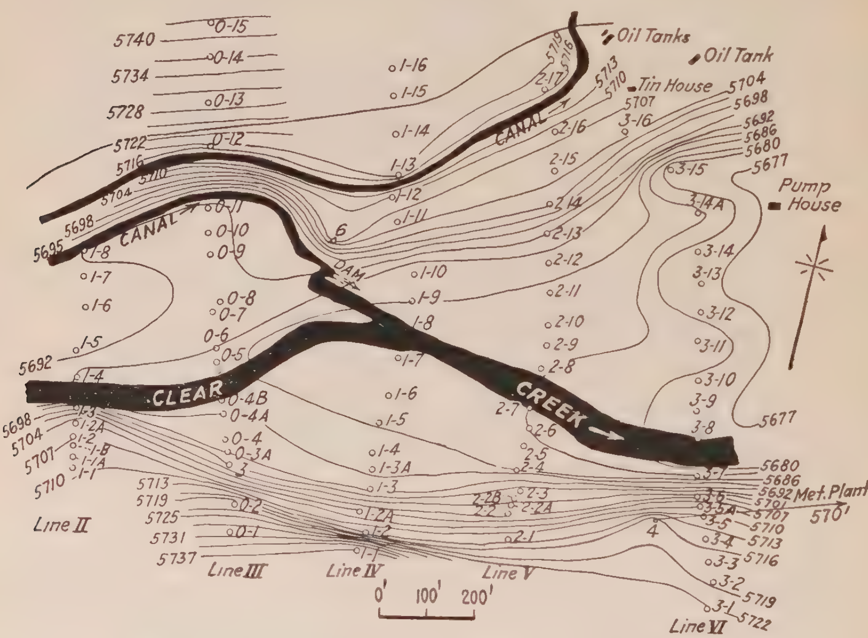

FIG. 4.—CONTOUR MAP OF AREA UNDER SURVEY. CONTOUR INTERVAL, 3 FEET.

also was observed, to prove definitely that the magnetic effects came from deposits at shallow depths. The observations and computations were made by the junior author with the Schmidt balance and by E. E. Maillot with the Thomson-Thalén instrument; the latter also did some work with the Schmidt balance. In order to correlate the magnetic effects with the topographic evidence of the glacial deposits a topographic survey was made of the area by J. L. Daly and R. J. Forsyth, who constructed the contour map shown in Fig. 4. The remainder of the charts were plotted by the senior author, who also made the geologic interpretations. In this work he was privileged in having the advice of Dr .F. M. Van Tuyl, professor of geology at the Colorado School of Mines.

The observations and computations were made as usual.[5] The base correction was determined by returning to the base twice a day. Observations were rather slow for part of the time, because the ground was frequently frozen and the instrument settled during the observations, probably due to thawing by the metal-pointed tripod legs. This interference required more observations on one point to obtain the desired accuracy. On account of the magnitude of the anomalies, only an average daily variation correction was applied. This average variation was derived from observations at the base station with both instruments throughout one day.

FIG. 5.—ISODYNAMIC MAP, SHOWING LINES OF EQUAL ANOMALY IN VERTICAL INTENSITY INTERVAL, 50γ. SCALE SAME AS IN FIG. 4.

The results of this work are represented by one map and six profiles. The isodynamic map is shown in Fig. 5 and contains contours of equal anomaly in vertical intensity of 50γ intervals. The profiles are illustrated in Fig. 4 and Figs. 6 to 11. They are drawn approximately in the direction of magnetic south to north. The section of the ground is shown below the magnetic profile and is derived from the results of the topographic survey; absolute elevations in feet are given on the left hand

[5] See C. A. Heiland: Directions for the Use of the Vertical and Horizontal Magnetometer. Askania-Werke, Houston, Tex.

margin; 20 ft. in the ordinate correspond approximately to 50 ft. on the abscissa.

A glance at the isodynamic map reveals that the steepest magnetic gradients are found in the vicinity of the steepest topographic gradients on the south rim of the Clear Creek basin. (Along the south rim, where the strongest magnetic disturbances occur, most of the gold has been taken out.) An inspection of the profiles shows also the close relation of vertical intensity and topography on the south rim.

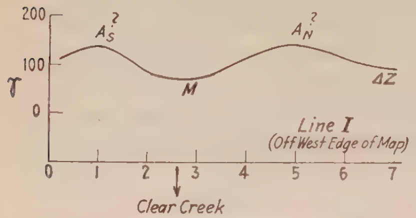

FIG. 6.—PROFILE OBTAINED AT CLEAR CREEK.

Otherwise, the magnetic curves seem to be rather irregular and do not show relation to the topography, especially not in the middle of the stream bed. However, both phenomena may be readily explained by one common cause; in fact, almost every peak in the magnetic curves may be correlated to the geologic conditions. Before we can do this, however, it

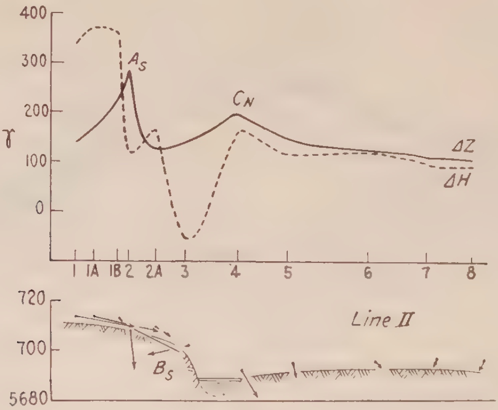

FIG. 7.—PROFILE OBTAINED AT CLEAR CREEK.

will be necessary to enter into a discussion of the geological details related to the origin of river channels and to determine their magnetic effects. For more details regarding the geologic features of placer deposits, the new textbook by Lindgren should be consulted.[6]

DISTRIBUTION OF MAGNETIC ANOMALIES

When rocks undergo the natural decay due to water, wind and ice, their constituent minerals will act in a different way. Some of them

[6] W. Lindgren: Mineral Deposits, Ed. 3. New York, 1928. McGraw-Hill Book Co.

will show very little resistance against destroying influences; thus the more resistant minerals will be set free in individual grains of different sizes and will retain their dimensions for an appreciable time. Such minerals of the latter type are quartz, gold, platinum, magnetite, ilmenite, cassiterite, garnet, etc. In the process of erosion, the minerals will be

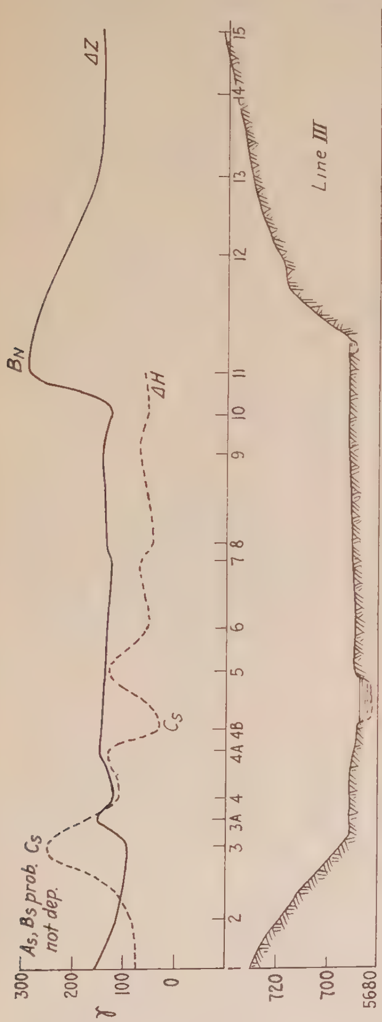

FIG. 8.—PROFILE OBTAINED AT CLEAR CREEK.

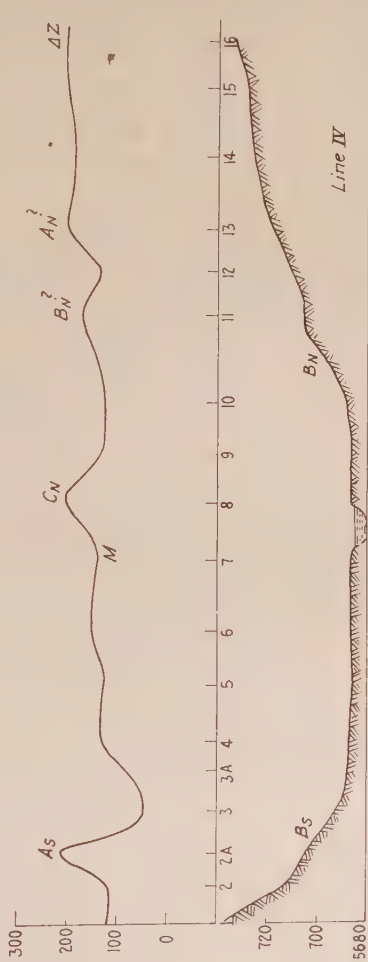

FIG. 9.—PROFILE OBTAINED AT CLEAR CREEK.

carried away by the water of creeks and rivers and will be redeposited on other locations, which are often an enormous distance away from their point of origin. During the transportation by the water a natural process of mechanical separation takes place. There is always a natural grade in a river, depending on the difference in altitude of river bed and nearest base level. The grade is steeper in higher than in lower parts of

the river. The velocity of the water varies, among other factors, as the square root of the grade. On the other hand, the transporting power of the water increases as the sixth power of the velocity and is directly proportional to the area of the particles exposed and indirectly proportional to their specific gravity. The particles lose from one-half to one-third of their weight in water; their friction on the ground decreases as the

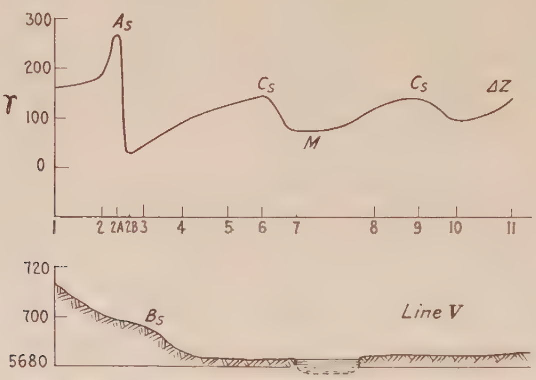

FIG. 10.—PROFILE OBTAINED AT CLEAR CREEK.

cosine of the angle of slope of the river bed. As a consequence of this, the river may drag and roll in its upper parts boulders of considerable size, while in its lower part it may carry only clayey particles in suspension.

From the transportation power of a stream, the conditions of deposition of the detritus may readily be seen. At a decrease of velocity, the

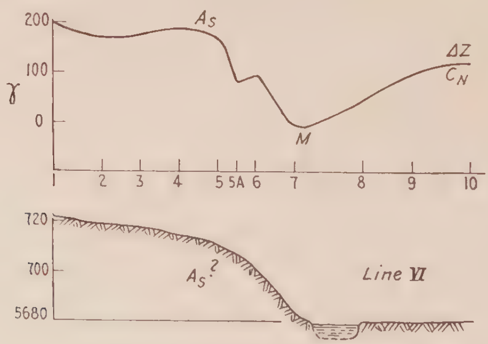

FIG. 11.—PROFILE OBTAINED AT CLEAR CREEK.

larger particles will be deposited in the upper parts of the river, the more clayey particles closer to the mouth; the colloidal substances are coagulated by the electrolytic action of the sea water. Also, at any given locality the particles with the greatest specific gravity will tend to sink to the bottom of the sediment; hence the concentrations of gold, platinum,

magnetite, ilmenite, etc., are always found in the lowest parts of the gravel bars. The weight of the heavier particles however, is not the only factor, because the velocity of the water decreases as the bottom of the channel is approached.

It is of great importance for the correct interpretation of the magnetic anomalies which are observed above river channels, and which are frequently very intricate, to consider further the geologic factors that lead to the distribution of gold and magnetite. As we have seen before,

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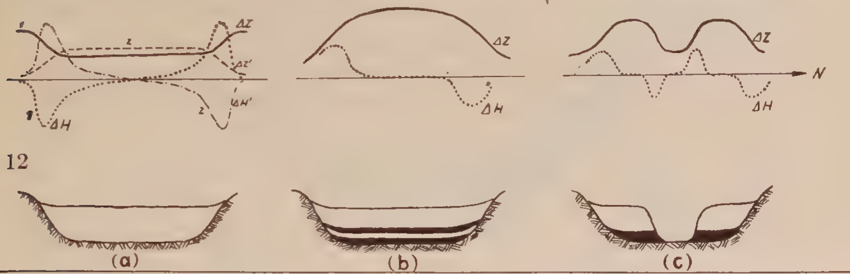

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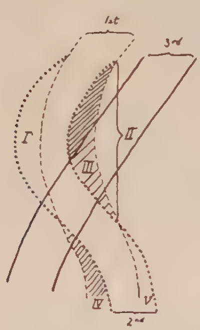

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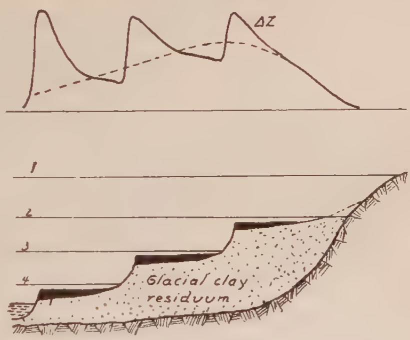

FIG. 12.—*a*. CHANNEL WITHOUT CONCENTRATIONS. 1. BEDROCK MORE MAGNETIC; 2. CHANNEL MORE MAGNETIC. *b*. CHANNEL WITH ONE OR MORE CONCENTRATIONS IN UNDISTURBED CONDITION (DEPRESSED CHANNEL). *c*. CONCENTRATIONS DIVIDED BY CHANNEL (ELEVATION OF CHANNEL OR CHANGE IN WATER SUPPLY).

FIG. 13.—HORIZONTAL INTERFERENCE OF CHANNELS.

FIG. 14.—TYPE OF PLACER WITH TERRACE CONCENTRATIONS.

the velocity of the water is the principal factor in the deposition; we shall therefore proceed to derive a few geologic and magnetic conclusions which result directly from the consideration of the behavior of the velocity in channels.

Without taking into account the effects of capillarity, because we deal with great dimensions, it follows directly from the behavior of the velocity that we must expect stronger magnetic anomalies above the narrower portion of a given channel than above the wider part. As the resistance to the flow is inversely proportional to the perimeter of the wetted surfaces, the velocity is greatest if the wetted perimeter is least. Since the velocity increases as the slope, which is greater at the upper

portion of a channel, and since the magnetite may be carried by relatively great velocities only and will be dropped in portions of the creek where this velocity is somewhat decreased, it follows that magnetic effects are most probable at the narrower portions of a channel.

It may happen that under certain conditions the magnetite occurs in very fine particles. This may be due either to the constitution of the rocks from which the mineral has been derived or to the grinding influence of the boulders in the river bed, or to both causes. In such case, no concentration will occur in the bottom parts of the river bed, but the

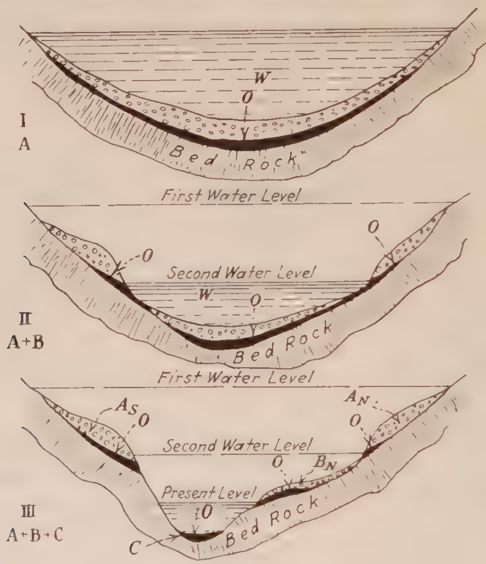

Fig. 15.—Evolution of channels at Clear Creek.

fine particles will be carried together with others of a clayey nature and will be deposited together with them, evenly distributed. If the bed rock is nonmagnetic, the magnetic conditions will be such as are demonstrated in Fig. 12a, curve 2. In the northern hemisphere and in high magnetic latitudes, where most of the placers occur (inclination 60° to 80°), the magnetic vertical intensity will be at its maximum above the channel. The horizontal intensity will be at its maximum close to the south edge and at its minimum in the vicinity of the north edge. If the thickness of the magnetic clay has been reduced later on in the center of the channel by stream erosion, the intensity will decrease toward the center as illustrated by the dotted line in Fig. 14. Such conditions have actually been observed by K. C. Laylander at Quesnel River, British Columbia.

If, however, the magnetite particles have been coarser, they have been deposited very likely in the upstream portion of the channel, so that at a lower location the gravels are completely free of magnetite.

If the bed rock is nonmagnetic, no magnetic effect will be noticed; if it is magnetic, the channel gravels will behave north-magnetic in comparison to the magnetic bed rock, as illustrated in Fig. 12*a*, curve 1. There will be a minimum in vertical intensity above the channel, a minimum in horizontal intensity above the south edge and a maximum above the north edge.

In the upper portions of the channel, the magnetite, ilmenite, etc., will be concentrated. This will occur where the velocity is least. Figuring out theoretically the change of velocity as depth in a channel, it results that the curve representing this relation is a parabola concave to the depth ordinate; the velocity is at its maximum at a certain distance below the surface, somewhat smaller at the surface and attains a minimum at the bottom. The velocity also decreases toward the sides, but the exact amount of it depends on the slope of the banks. Therefore the heaviest concentration will lie in the center of a straight channel (Fig. 12*b*). The behavior of vertical and horizontal intensity is then similar to that previously illustrated.

Deposition of Additional Concentrations

After the deposition of one concentration, there may arise other conditions whereby another concentration will be deposited on top of the first. Such factors are: (1) increase of water and detrital supply; (2) depression of the valley so that the grade is increased (this must be accompanied by an ample supply of sediments so that the degrading effect of the increase in velocity is compensated by the aggrading action of the sediment, thus in turn lowering the velocity by the increased friction still more); (3) by uplift of the base level, or other processes producing temporary base levels (obstructions, etc.). This will lead eventually to a deposition of a leaner concentration; that is, a concentrate which previously, during the deposition of the first concentration, would have been deposited at the location of a lower concentration. The magnetic conditions above two concentrations are similar to those above one concentration, the amplitudes being somewhat larger. The curves shown in Figs. 13 to 15 are to illustrate only the general appearance of the magnetic disturbances, and should not be used to derive quantitative conclusions about the effect of one type of deposit in reference to another.

Effect of Water Velocity

Geologic factors opposite to those described in the last paragraph will produce the magnetic conditions illustrated in Fig. 12*c*, which may also be derived from the behavior of the water velocity.

The channel excavated by a straight water course does not have an arbitrary shape under ideal stratigraphic conditions. Complications arise naturally in reality because there are vertical differences in the

petrologic composition, hardness, etc., of the sediments. If we take a river channel in a homogeneous medium, the shape of the channel will not be arbitrary. First, the bank angle will depend on the stability of the bank material; it increases as resistance of the bed rock. With these side angles and a given section of the water volume, the river will tend to develop a channel by which it can drain the water in a minimum of time. As noted previously, the maximum velocity, other things being equal, will be obtained in a channel that has a minimum of wetted perimeter. This means that the river will tend to produce a shape of channel in which it may have the maximum velocity or minimum wetted surface. Such a channel, naturally, has a cross-section that may be excavated with a minimum of effort. By differentiation of the hydraulic equation for the velocity in such channels, we find that for a channel of maximum velocity the hydraulic mean depth (ratio of water area divided by perimeter) must be one-half of the actual depth, or the cross-section of the channel must be a semicircle, a half-square or a half-hexagon, the latter obviously being the most frequent.

With the conditions illustrated in Fig. 12, after the water volume that deposited the first filling was decreased (for climatic reasons, for instance) the water covered a relatively large surface at shallow depth. This condition is not stable, because, as proved before, the perimeter of the wetted surface is too great. To obtain the optimum of this perimeter, the river must cut deeper and consequently decrease in width. This is what we actually observe; it is the most common cause of terracing. Another possibility is that the supply of sediments has decreased, thus increasing the velocity of the water. Both causes have played an important part during the glaciation of the areas that furnished the material for placers. An uplift of the river valley together with a decrease in water supply may also have increased the tendency of the new water course to cut deeper, the decrease of the velocity thus produced not being sufficient to convert the degrading effect of the water immediately into an aggrading effect. If the second channel scours out the first concentration to bed rock and the velocity and sedimentary material in the new channel is such as to deposit practically no concentration, we have the magnetic condition illustrated in Fig. 12*c*.

Generally, however, there are concentrations deposited in each channel, as illustrated in Fig. 15. Subsequent channels are usually narrower than the preceding ones. If the stream bed undergoes no uplift, no change in local base level or in general character of sediment supply, the deepening of the channels will work toward a decrease of grade. Here the decrease in velocities in subsequent channels will give rise to a slight lessening of the strength of concentration. The experience proves that in placers which have been formed in such a manner, the highest benches are usually the richest in gold and also most magnetic (Fig. 15, *A*).

As different channels are marked in the topography of river beds by benches, it follows that the magnetic anomalies must show a definite relation to the configuration of the benches. The question then arises: On what portion of the bench may we expect the strongest magnetic anomalies? According to the results so far known, we have to distinguish two decidedly different types.

Types of Magnetic Anomalies

One type is illustrated in Figs. 15 and 16. When the water was at the first level, the gravels A were deposited. Then the water dropped down to the second level and eroded the major part of A, reworked the concentration and deposited gravels and concentrations B. Then the level dropped again and deposited C. Fig. 16 shows that the concentration

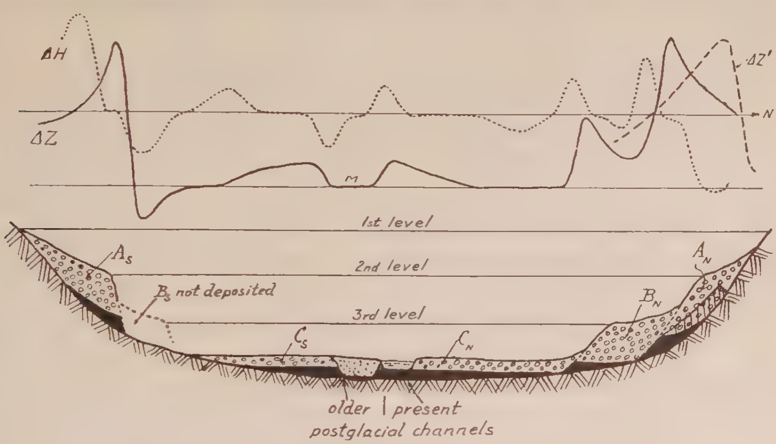

FIG. 16.—IDEALIZED GEOLOGIC AND MAGNETIC PROFILE AT CLEAR CREEK.

is strongest just where it is cut off and that this point lies approximately *below the steepest grade between A and B, off to the middle of the river.* Therefore there is a strong maximum in the vertical intensity just above this point. If the bench B has not been deposited (meander effect) so that the instrument is lower than the bulk of the concentration, a strong minimum in the vertical intensity will result. Where the youngest concentration C is interrupted by the present channel there is a drop M in the vertical intensity. This minimum must not necessarily coincide with the present location of the river. It may be due to another recent but relatively older bed while the very youngest may not have reached the concentration C_N as yet in the process of downcutting. In reality, the bed rock is also cut down somewhat below the various concentrations as shown in Fig. 15, a condition which purposely has not been considered in Fig. 16. The horizontal intensity shows "plate" effects at the edges of

the concentration C_S and C_N. As the older concentration A_S is inclined toward the north, there is a stronger maximum of H on the south edge and a smaller minimum on the north edge of this concentration. Opposite conditions in regard to horizontal intensity occur on the north rim. Should the concentration not increase so much in thickness toward the middle of the river as shown in Fig. 16, but be equivalent to a layer of uniform thickness, the maximum of the vertical intensity lies away from the edge of the bench toward the rim, as indicated by the dashed line in Fig. 16.

Another type of placer deposit gives quite different magnetic effects (see Fig. 14). The valley was filled with glacial clay when the water level was at the position 1. Then the level dropped to a position just below line 2 and excavated the first terrace without any appreciable aggregation of sediment. At intervals of greater water supply the basin was overflowed to inundate the flood plain; the highest water level is indicated in Fig. 14 by No. 2.

On the flood plain, gravels and concentrates were deposited. Close to the river bed the overflowing was more frequent, so that some sort of a natural levee was formed which also accumulated more concentrates. Therefore, on account of the uniform increase in thickness of the concentrates toward the middle of the stream, as well as on account of that natural levee (not shown in Fig. 14) a maximum in the magnetic vertical intensity was produced, which in this type of placer, therefore, is *at the outer edge of the bench.* In the same manner, the other terraces are formed and act magnetically in the same way. Sharp minima in the vertical intensity are produced if the instrument is set close to the foot of the bench, because the instrument is below the concentration. The amplitude of the anomaly seems to increase toward the center, because the older deposits have been more subjected to erosion and wear. The behavior of the horizontal intensity may also be drawn for this case.

Effect of Meandering Channel

So far we have considered only vertical alterations of an original channel that was straight. If the original channel is curved, horizontal differences in the deposition of magnetic sediments take place, which are illustrated in Fig. 13. There is some doubt whether or not straight channels are possible in nature. A small irregularity in the configuration of the bottom of the river bed, or the local presence of sediments may cause a horizontal deflection, and even the deflecting influence of the rotation of the earth may have something to do with the "meandering" of rivers.

Suppose a river has a course like that illustrated by No. 1 in Fig. 13 and is deflected into the new bed No. 2. There will be wear on the outer

edge I of the bench and deposition of sediments and concentrates at the inner bend II, and the same will occur at V and IV respectively. Later on the river may follow the course 3, whereby the portion III of the original concentration II is either completely removed or reconcentrated in the third channel. The channel of a river is never on one location, but in constant horizontal migration. If the flood plain is wide enough, the river may drain not only in one but in a multitude of channels, between which the bars are located ("braided" stream courses of Alaska, California, etc.). Still further complications arise if this system of three subsequent channel locations is submerged and later on another independent channel system is produced on top of the old ones. The magnetic disturbances then become very complicated, but it is possible in case of such superposition of concentrations to separate anomalies due to the upper channels from those caused by the lower ones by using relative depth determinations. These determinations may be made by using both horizontal and vertical intensities, or, if only the latter has been observed, by measuring the horizontal distances in the Z-curves from the maximum to the points where the intensity has dropped to one-half of the maximum amplitude. Complications arise frequently, on account of irregular concentrations in pockets in the stream beds, irregular (eddy) currents, etc.

If the channel system has been submerged and is now covered by strongly magnetic strata (basalt flows, for instance, in some Californian placers) the application of the magnetic method for their location is totally impossible, but otherwise buried placers may often be discovered successfully by the magnetometer.

There exists a very common misconception that in the formation of a meander the deposition in the inside bend takes place on account of lack of velocity and the wear on the outside because of an increase of velocity. Fifty years ago, Thomson proved by experiment and theory that the conditions are much more complicated.[7] Very recently, also, Albert Einstein discussed the mathematical phases of the formation of meanders.[8] If the water moves around in a semicircular bend, for instance, it takes, under the influence of gravity, a motion like that in a free vortex; that is, the velocity is greater on the inside bend than on the outside. As in a vortex, there will be a slope of the free surface of the water from the inner side upward on account of the centrifugal force. As the water moving along the bottom and banks is retarded on account of friction, its centrifugal force is not great enough to balance the excess of pressure exerted by the higher water column on the outside turn; therefore a

[7] J. Thomson: On the Origin of Windings of Rivers in Alluvial Plains, with Remarks on the Flow of Water Round Bends in Pipes. *Proc.* Roy. Soc. (1877) **3** 228.

[8] Reference not available at time of writing this paper.

transversal current will result, which is directed from the outside turn against the inside on the bottom of the water and up the inside bank, and which corresponds to a transversal outward movement on the surface. The outward surface current carries the excavating water against the outside bank; the bottom current moves the detritus and deposits it at the inside bank. Thus a lateral shift of the stream bed is accomplished.

Results of Present Investigation

Having explained in great detail the close relation of geology and magnetic disturbances, we may now proceed to a very brief discussion of our results, which indicate that the disturbances are caused by a type of placer of which the ideal condition is illustrated in Fig. 16.

Line I.[9]—There is a minimum M due to the present creek and two maxima which are probably due to the oldest benches. The concentrations probably lie deeper than those occurring on the next lines.

Line II.—The maximum A_S is very likely to be produced by the oldest bench A, which reaches quite a distance under the second bench B. The youngest concentration on the other side C_N produces also a maximum due to inundation of the flood plain. There is no magnetic effect produced by bench B_S; it contains probably no concentration, as it lies on the outside bend of the river. The disturbance vectors indicate clearly that the cause of the magnetic effects is very shallow.

Line III.—This also shows no effect of bench B_S, because it is located on the outside bend of the river. There has probably even not been a deposition of a concentration in A_S, although the slight rise of the vertical intensity seems to indicate that there may be a thin layer of magnetite on the bed rock below A_S and B_S. There is a rise of the vertical intensity again toward the river above the flood plain concentration C_S, which is also indicated by maxima and minima in the horizontal intensity. Very pronounced is the maximum above B_N on the other side, being off to the middle of the river, as theoretically expected.

Line IV.—This has the very pronounced maximum due to the oldest concentration A_S. Here also, B_S has no effect. We believe that the river was not following a course similar to its present one when B was deposited but that its course was approximately that indicated in the contour map by the contour line 5692 on the south rim. The behavior of the contour lines in the north rim seems to confirm this view. In addition, there is a slight maximum on the north side, probably due to B_N, and A_N also seems to produce an indication there. In the vicinity of the stream channel there is a minimum M, but off to the south, which also seems to support the idea that the stream has tended to migrate to the north.

[9] Lines I to VI are shown in Figs. 6 to 11, inclusive.

Line V.—This may be called an ideal profile. Both young concentrations C_S and C_N are well marked and separated by a minimum somewhat south of the present channel. A_S produces a strong maximum but B_S is again without effect. It must be admitted, however, that the repeated lack of effect of B_S might be explained by a type of deposition such as illustrated in Fig. 14—the deposition on flood-plain remains. If this is the case, the maxima must be above the short horizontal stretches between subsequent banks A and B. Against this view, however, may be quoted the strong effect of B_N on line III, which is off toward the middle of the stream, and also the fact that the amplitudes above the older benches A are generally much greater than those of the younger ones. With the present material, this question is difficult to decide.

Line VI.—This is difficult to interpret. The minimum M in this case is not due to recent stream-bed migration but to the fact that station No. 7 is so much lower than the preceding ones that the bulk of the concentration is probably above the instrument. The slow increase of the vertical intensity toward south may be due to a slow rise of a fairly uniform magnetite layer in that direction, which must be, according to the Z-curve, at some distance away from the surface, and which probably corresponds to bench A. The reason for the maximum C_N being so much off to the north is seen from the contour maps; there is a shallow depression on stations 8 and 9 of this line, indicating that the stream may have been a little farther north previously. In general, some caution is advised in correlating the maxima C_N and C_S too closely with the present stream bed, because it is shifting constantly.

Magnetic Effect of Glacial Sediments

A few words may now be said about the magnetic effect of glacial sediments. Placer deposits as terrestrial sediments are generally subjected to fast removal; therefore nearly all placer deposits are of Tertiary, Pleistocene or Recent age. The formation of the Clear Creek placer during the Glacial Period seems to be well established because the river gravels rest on Tertiary strata in the vicinity of Golden. There can be no doubt that these river sediments produce the magnetic effects observed. Any other interpretation of the results seems impossible, especially that of a more deep-seated cause, first on account of the very shallow depth of the disturbing layers, as determined by the character of the magnetic curves, and second because any effect that eventually could be produced by changing magnetic properties of the bed rock, etc., must produce completely different magnetic curves. The bed rock is mostly of Pennsylvanian and Cretaceous age and consists of formations which are virtually nonmagnetic.

In addition to this, the measurements made by Gibson[10] proved the magnetic effect of Californian placers, the majority of which are of Quaternary age. W. Lindgren[11] says: "The great bulk of this output (of gold) came from the Quaternary deposits, and about $300,000,000 (20 per cent.) is a conservative guess for the amount obtained from the Tertiary gravels."

The geologic conditions in the Cariboo-District, where K. C. Laylander made his measurements on Quesnel River, are described in reference to the origin of the placers by W. L. Uglow and W. A. Johnston as follows:[12]

"The gold, liberated in the zone of Tertiary weathering and concentrated in the old stream valleys, was reconcentrated . . . In Pleistocene time . . . little erosion . . . ; a part of the Tertiary gravels, therefore, was preserved. Some of the placers are postglacial in age and are the result of the concentration by streams of the gold in the gravels, which were caught up, by means of ice erosion, in the glacial drift."

According to this statement, the terrace gravels of Quesnel River are postglacial, but were derived from glacial sediments, which, therefore, must have been magnetic also. As stated before, Laylander observed actually a rise of the vertical intensity above the glacial clay which constitutes the general fill of the valley. Relative to this, Uglow and Johnston say:

"The gold content of the postglacial and glacial gravels was derived partly by stream erosion of glacial drift . . . and partly from erosion of the preglacial gravels."

At Fairbanks, Alaska, magnetic measurements are now being made successfully, the results of which are not yet published. L. M. Prindle[13] writes the following about the age of the auriferous gravels:

"Deposition most probably began in the Pleistocene or earlier and continued to the present time, the most active period probably being during the time when the glaciation of the areas about the Upper Yukon made so large an amount of silt available."

Reich[14] aims to prove that in Germany deposits of the Glacial have no magnetic effect. He criticizes measurements previously made by the senior author which gave negative magnetic anomalies above valleys

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[10] A. Gibson: *Op. cit.*

[11] W. Lindgren: The Tertiary Gravels of the Sierra Nevada of California. U. S. Geol. Survey *Prof. Paper* 73 (1911) 81.

[12] W. L. Uglow and W. A. Johnston: Origin of the Placer Gold of the Barkerville Area, Cariboo District, British Columbia, Canada. *Econ. Geol.* (1923) **18**, 541.

[13] L. M. Prindle: A Geologic Reconnaissance of the Fairbanks Quadrangle, Alaska. U. S. Geol. Survey *Bull.* 525 (1913) 67.

[14] H. Reich: *Op. cit.*

in the Tertiary surface filled with glacial material. The senior author stated only that the boundary surface of Tertiary against Glacial sediments was effective in such a manner that Glacial sediments showed a north-magnetic behavior in reference to the Tertiary strata. This is the geologic condition illustrated in our Fig. 12*a*, curve 1. These results can well prove Reich's theory that in that locality the glacial beds were without magnetic effect; Reich has only misunderstood the author's statement about the negative anomalies to mean that the glacial strata had an effect of their own, north-polar magnetization, or something like that. Later on, Reich himself found on other localities magnetic maxima above Tertiary deposits and minima above other nonmagnetic sediments (Pennsylvanian Age)—of which the principal character was quartzitic, similar to the glaciofluvial sediments above which the senior author observed his minima.

While we do not doubt that the deposits of the glacial period which have been tested by Reich in Germany did not show any magnetic effects, we note that the types of sediments investigated were only glacial till, terminal moraines and outwash plains. He has not examined the fluvioglacial sediments in the narrower sense, especially not the river deposits.

From the results of our measurements and previous magnetic observations on placers, we therefore reach the conclusion that the view can not longer be held that sediments of the glacial period are without magnetic effect, at least not for the North American Continent.

Reich says that even if glacial deposits produce magnetic effects, they would be locally very limited, such as are produced by the effect of individual boulders in the till, etc. We entertain a somewhat different view, because the anomalies surveyed at Golden cover a length of approximately 1 km., and it is to be expected that the anomalies may be followed several kilometers farther, because the valley continues for an appreciable distance. The placers in California cover areas of several square miles, and so do the placers in Alaska.

It is of great practical importance to prove that glacial deposits may have magnetic effects, because upon this fact rests the magnetic exploration of gold placer deposits.

Publication of Results Desirable

We agree with Reich that for the extended regional anomalies in North Germany a different cause must be assumed; namely, the different depth and magnetization of igneous massives. Magnetic prospecting for such formations has been applied since 1925 in this country on a large scale, primarily on suggestion of the senior author, because the buried crystalline formations show a very definite relation to the struc-

ture of oil-bearing formations. About two to three thousand stations are being made at present every day, according to a very cautious estimate. The results of this vast prospecting, unfortunately, are kept very secret by the various organizations.

Many serious mistakes made in the interpretation of the results could be avoided if more examples showing the relation of magnetic anomalies to geologic structure were published. To contribute our small share to the knowledge of the relations that exist between geology and earth magnetism has been the principal aim of this paper.

A Demonstration of the Reflection of Geologic Conditions in Observed Magnetic Intensity*

By H. R. Aldrich,† Madison, Wis.

(Boston Meeting, August, 1928)

This paper is not a treatise on the theory and practice of magnetic surveying. It presents a diagram upon which have been plotted observations taken with the simplest form of magnetic instrument, the lowly dip needle, over a large area astride the Lake Superior syncline of northern Wisconsin. The exposures of the underlying formations are not sufficiently great in number nor fortunate in distribution to have provided the fuller understanding of the geology which has been secured by the use of the magnetic observations as an auxiliary tool or accessory in the hands of the geologist.

The diagram (Fig. 2) in large part speaks for itself if studied in connection with the geological map (Fig. 1) for which it is the main control. It is presented to demonstrate the extent to which the use of such methods may aid in the solution of geological problems in a covered country. The diagram is probably unique in assembling the expression of such a wide range of formations, affected by so wide a range of secondary processes, and deformed by so many different structures in terms of magnetic variation.

Few geologists outside of the Lake Superior country have had any first-hand experience in the taking of magnetic observations and the interpretation of the results in terms of geological conditions; nor have the Lake Superior men gone with the methods far afield from the mere tracing of iron formations. In more recent years the oil companies have gone heavily into investments in magnetometers, balances, and dip needles in the quest of concealed structural anomalies which are the favorable loci of petroleum accumulation. But these geophysicists come back from the fields with their collections of observations facing the question of interpreting the geological meaning of their magnetic anomalies.

It is considered probable therefore, and certainly it is the hope of the writer, that the appended charts, or magnographs, which show how character of the country rock and its changes, structural strike and dip and their changes, depth of burial and its variation, contact metamorphic

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† Assistant State Geologist of Wisconsin.

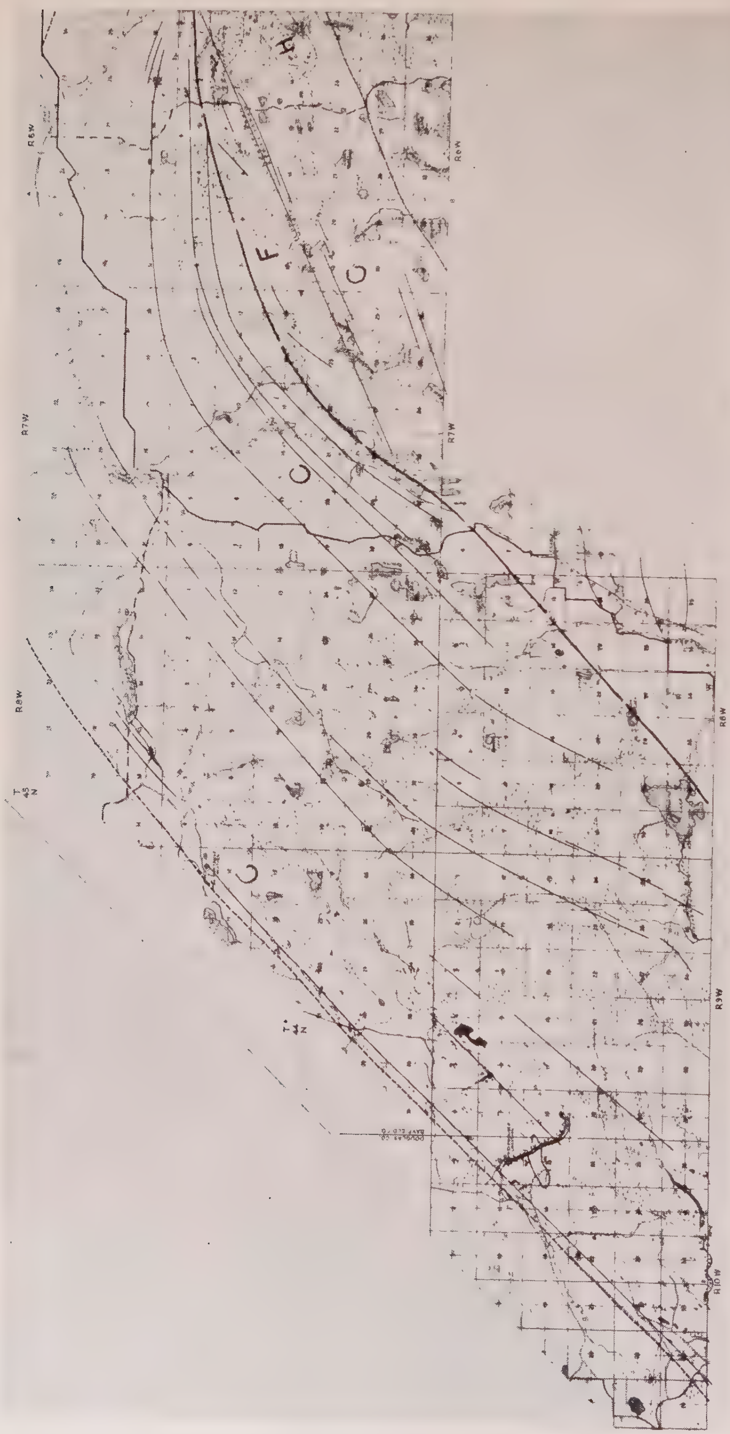

FIG. 1.—GEOLOGIC MAP OF GOGEBIC IRON RANGE AND THE KEWEENAWAN FORMATIONS IN PART OF NORTHERN WISCONSIN.

A. Iron formation.
B. Tyler slate.
C. Middle Keweenawan lavas.
D. Intrusive gabbro.
E. Felsite.
F. Conglomerates.
G. Lake Shore traps (lavas).
H. Granite intrusive.

FIG. 2.—DIAGRAM OF DIP-NEEDLE READINGS ACROSS THE GOGEBIC IRON RANGE AND THE KEWEENAWAN FORMATIONS IN PART OF NORTHERN WISCONSIN.

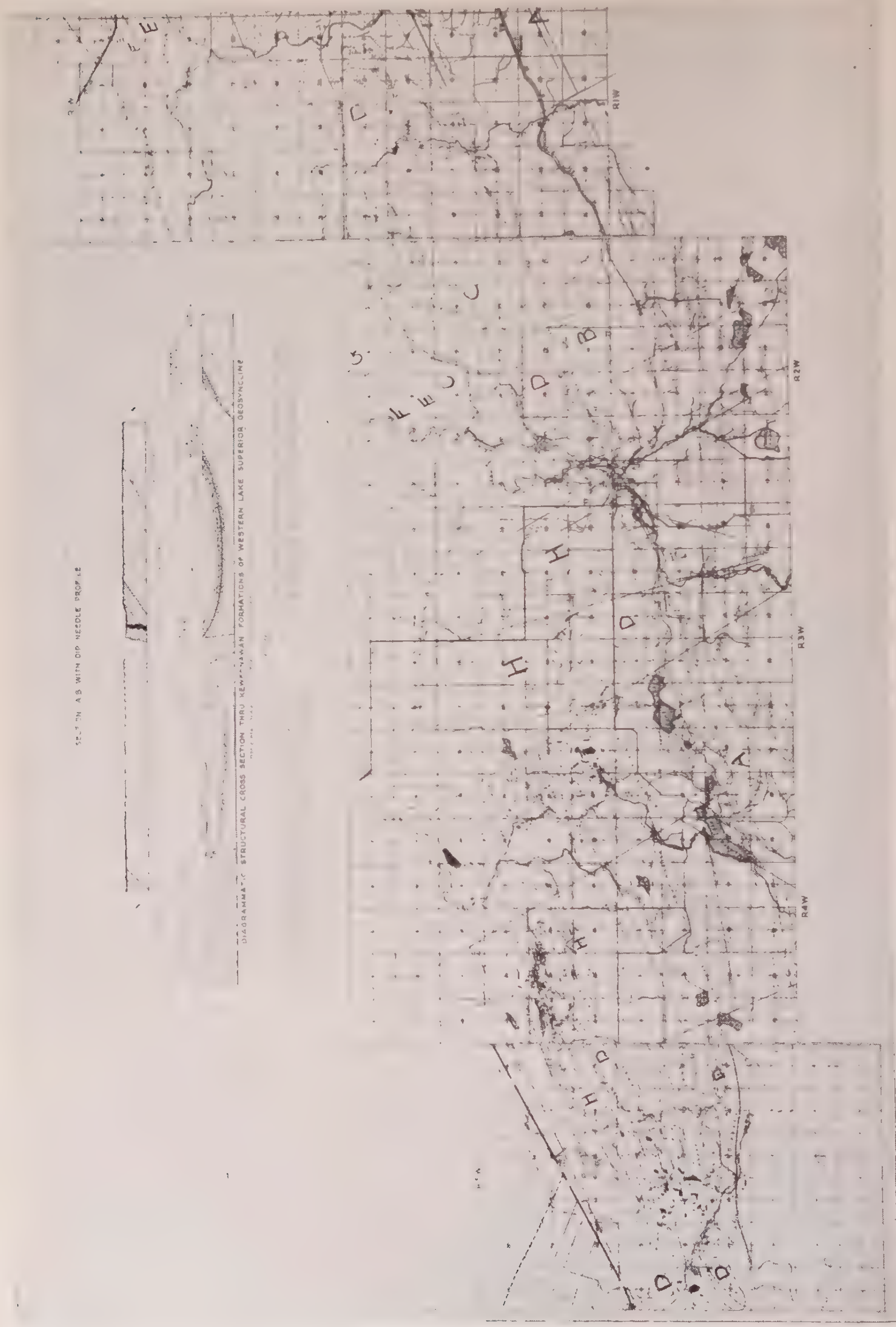

FIG. 1.—*Continued.*

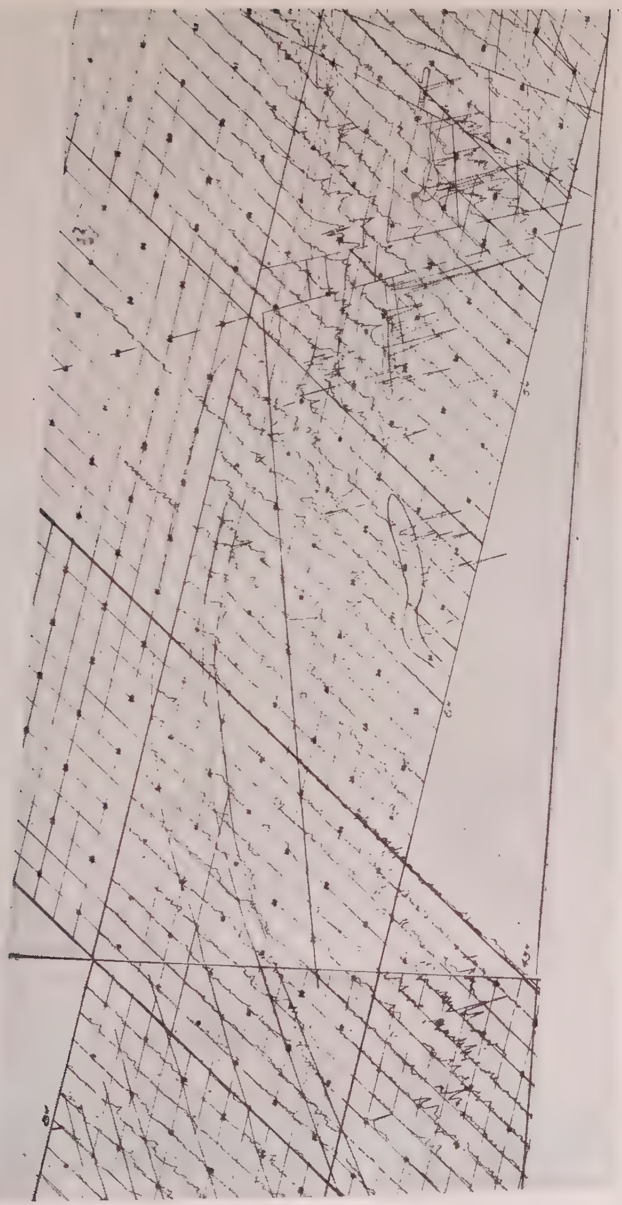

FIG. 2.—*Continued.*

effects, normal faults and thrusts, joint control of buried topography and other essential geologic features and phenomena reflect themselves in magnetic observations, will be of some service in the promotion of these methods and the means of inspiring some degree of confidence that

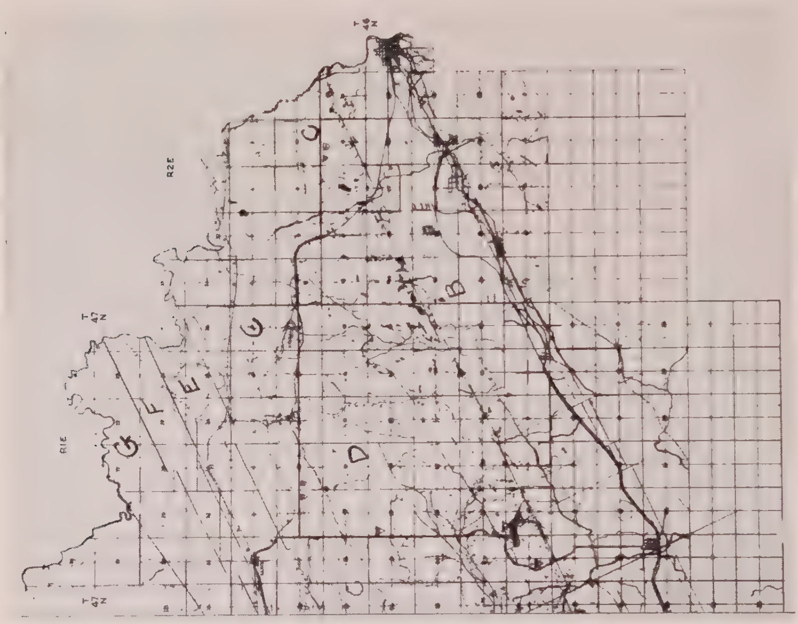

Fig. 1.—*Continued.*

there is a high probability that the results obtained with a physical instrument may be interpreted with some degree of accuracy.

The Geographic Location and Setting

The region in which these observations were taken includes northern Iron County, central Ashland and southern Bayfield counties of northern Wisconsin.

The region is glaciated and the ratio of exposure to total area surveyed is probably not greater than 1:100,000.

The youngest rock formations in the area surveyed are of Upper Keweenawan age. Next below are the Middle Keweenawan lavas, which are western extensions of the Michigan copper formations. Beneath these is the Lower Keweenawan, a mere 100-ft. collection of arkose and conglomerate. This formation is negligible in the magnetic record.

The Tyler succeeds the Lower Keweenawan in depth. It is conformable and is assignable to the Upper Huronian. It is a slate or graywacke. Conformably beneath the Tyler is the Ironwood, an iron formation, which

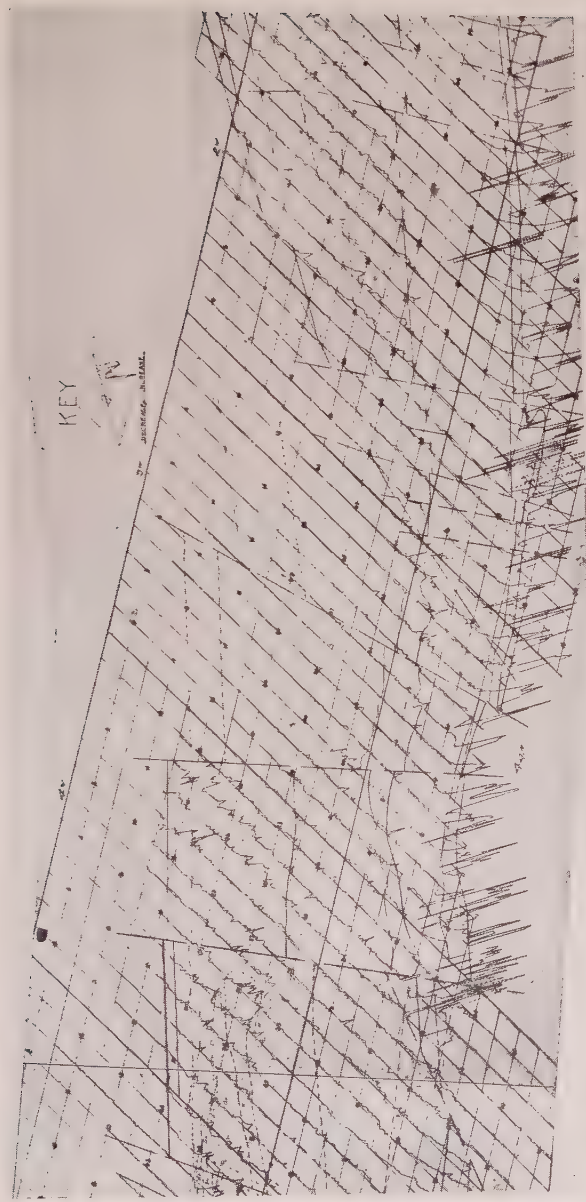

Fig. 2.—*Continued.*

is succeeded in depth by the conformable Palms, also of Upper Huronian age. Below the Palms, unconformably, and existing only in isolated patches, is the Bad River dolomite of Lower Huronian age.

The great basement of the region is largely Kewatin greenstone schist, but within this are large masses of intrusive Laurentian granite. The diagram does not include areas farther south than a mere fringe of the Archean south of the Gogebic iron range.

The column thus described holds good for the southern area although the Huronian formations are not known or reflected magnetically west of T. 43 N., R. 7 W. West of this township the Middle Keweenawan lavas probably lie in thrust relationship upon the Archean.

The glacial drift includes terminal and ground moraine with a variety of material ranging from red clay and rock flour to cobbles and boulders of granites and trap rock of Middle Keweenawan origin measuring 10 ft. or more as a maximum size. There is a great abundance of outwash, principally sands and gravels, and there is lake clay more or less varied; there are eskers, kames and boulder trains. In depth the drift ranges from nothing to more than 200 ft., particularly in the preglacial valleys and in the main synclinal trough.

The Upper Keweenawan sediments are in general well-oxidized sandstones, graywackes, shales and conglomerates. The latter are composed to a large extent of felsitic pebbles and cobbles although amygdaloid and dense trap are well represented. Toward the base these are interbedded alternately with lava flows. The entire thickness is estimated to be no less than 20,000 feet.

The Middle Keweenawan series consists dominantly of basic lava flows which vary widely in individual thickness, composition, and texture. The tops of the flows are usually amygdaloidal, and they are frequently well broken, with a fine sediment filling the interstices between the fragments. Petrographic examination shows that the tops, and more especially the bases, are richer in magnetite than the central portions. Some alkalic flows occur and these are practically devoid of magnetite. At the top of the series, and separated from the main body of the flow sequence, is a succession of basic lavas known as the Lake Shore traps. The formations accomplishing the separation of these are the Great Conglomerate and the Chippewa felsite. The entire Middle series is of the order of 25,000 ft. thick.

The Lower Keweenawan consists of arkose and conglomerate containing quartz, quartzite, slate, schist, and iron formation. It is not over 100 ft. thick and is insignificant in interpretation.

The Tyler slate is a graywacke or slate with beds of quartzite. It is of the order of 10,000 ft. thick. Toward its base it is highly ferruginous with magnetite prominently represented west of Range 1E. East of there iron carbonate is notable in quantity in the base.

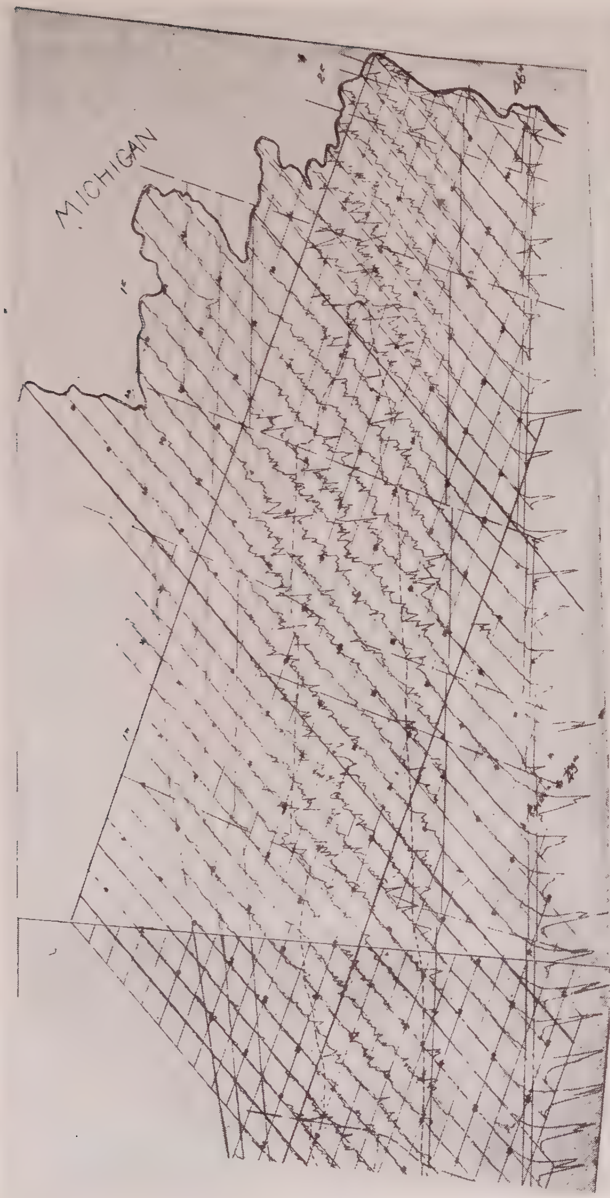

Fig. 2.—*Continued.*

The Ironwood is the iron formation of the Gogebic Range. It is composed of silica as quartz or very fine chert, depending on the degree of recrystallization, and iron minerals in the ratio of 65 per cent. silica to 35 per cent. iron. The iron minerals are siderite, hematite, and magnetite with some amphibole in the west where recrystallization has been highly important. The formation is divisible into several more or less continuous members which fall into two classifications, one a thin, evenly bedded cherty carbonate, the other a coarse, irregularly bedded ferruginous chert. It is recrystallized with increasing tensity toward the west. The entire formation is about 600 ft. thick.

The Palms is a feldspathic slate, toward the base not greatly unlike the Tyler, with an uppermost member composed of extremely pure quartzite. The thickness is of the order of 450 ft. Magnetite and other iron minerals are unimportant.

The Bad River dolomite is of variable thickness and occurs only in patches beneath the unconformity. It is generally a tremolitized dolomite with chert members. This formation is negligibly ferruginous.

The Laurentian granite is a normal granite varying in color from gray to pink. Magnetite is unimportant.

The Kewatin greenstone schists are of variable composition and structure but magnetite is neither important in quantity nor segregated into bunches, so far as can be discovered. In part at least, it appears to have been of extrusive origin.

The column is complicated by the injection of magmas of late Upper Keweenawan age. These are gabbros and "red rock" differentiates. In Range 2 W. and to the eastward red rock is negligible. The gabbro is confined to a single thick laccolite in which, however, are generously represented the earlier flows as inclusions, horses, pendants, or shreds. The laccolite is confined to the series of lavas, but it is noteworthy that its base rides gradually higher stratigraphically from west to east.

In Range 3 W., the laccolithic aspect of the intrusion is lacking. The gabbro is confined to a narrow belt in the northwest of T. 44 N., R. 3 W. A porphyritic granite has taken prominence in the Keweenawan section. It is bounded east, south, and west by fault planes.

In Range 4 W. the gabbro is again in force. In T. 44 N. it makes contact with the Ironwood near Mineral Lake. The base of the Keweenawan here is a wide anorthosite.

In Range 5 W. the intrusion is complicated belts of "red rock" and gabbro alternate. A few more or less continuous belts of the lavas are identifiable. The contact with the Huronian is igneous following a thrust plane. The Tyler is practically missing.

In T. 44 N., R. 6 W., in the northeast, near Davis Hill, the intrusions come to an end. Here gabbro makes its way into the Great Conglomerate of Upper Middle Keweenawan age.

Structural Features

The diagrams show conditions only on the south limb of the Lake Superior syncline. The strike varies from east and west to N. 45 E. Dips vary from vertical to 30° north.

During the formation of the syncline, which studies indicate to have been occasioned by the foundering of the roof of the magmatic chamber or the downwarping of the flat-lying flows and sediments into the magmatic wedge on account of the overloading, stresses were relieved to a considerable extent by the normal differential movement of upper beds over lower; but these movements passed over into numerous thrusts. At the same time, it appears that the rate of foundering was not uniform along the axis of the fold. In consequence of this, torsional strains were set up and a great number of transverse, approximately dip faults of normal type which have been found by the magnetic methods have been ascribed to this torsion. The thrusts are numerous and well established. The well-known Keweenaw fault, which is the southern boundary of the Middle Keweenawan on the Point, is one. This appears to have been the controlling factor in the injection of the great laccolite in Wisconsin. A second thrust is to be seen in the vicinity of Davis Hill in T. 44 N. 6 W. Another, the so-called bedding fault of the Ironwood in Wisconsin and Michigan, is probably of this same age. It is also deemed probable that the western extension of the Keweenaw fault bevelled across the Tyler slates and brought the Keweenawan into contact with the Ironwood from Mineral Lake in T. 44 N., R. 4 W. to Trappers Lake in T. 44 N., R. 6 W.

With this brief review of the location, geology, and structure, the diagram and the geological map should speak for themselves.

The Dip-needle Readings

The data plotted on the diagrams are dip-needle readings. The instrument responds to changes in inclination as well as to changes in intensity of the magnetic field. It is small, measuring 4 in. dia. and 1 in. thick. It weighs about 1 lb. It is held in the hand while making an observation. Observations were taken at intervals of $1/40$ mile along lines $1/2$ mile apart. They were taken during several field seasons from 1915 to 1925, and represent no less than 26 men. Therefore the personal equation is a variable and there is no influence of prejudice.

The diagram consists of an isometric projection of the ground plan. Along the north to south lines half a mile apart the readings are plotted in a way analogous to the plotting of a topographic profile. The section lines are taken as the datum line; intensities greater than normal are plotted above, intensities less than normal below, the datum line. Since these profiles are also thrown into isometric projection, the greater than normal intensities fall to the east, those less than normal to the west of the traverse lines.

Geologic Factors Reflected in the Diagram

1. *External Formation Boundaries.*—The external boundaries most clearly demonstrated are the southern boundary of the Ironwood and the southern boundary of the Middle Keweenawan series. See Ranges 2 E. to 3 W.

2. *Internal Formation Boundaries.*—The diagram shows how individual flows of the lava series may be traced where they are not complicated by intrusives. See 44 N. 6 W., 7 W., 8 W., 43 N. 8 W., 9 W., 10 W. The lines following the structure trace individual flows.

3. *Transverse Faults.*—Since the external and internal boundaries of a formation may be traced, wherever these are abruptly offset a discontinuity or dislocation is indicated. These are clearly indicated on the diagram. See in particular Ranges 1 W., 1 and 2 E. Also see Figs. 3 and 4.

4. *Longitudinal Faults.*—In the vicinity of Davis Hill, in the north of 44 N. 6 W., is seen a crescentic area of variations having a linear character trending from northeast in the west to east and west in the east. South of this is an area of erratic variation more or less elliptical in shape. South of this in turn is a belt of variation whose linear characteristics trend northeast. The area of no variation correlates with a late Middle Keweenawan conglomerate over which older Middle Keweenawan lavas have been thrust. See structural cross-section.

5. *Folding.*—The fact that internal boundaries can be traced is proof that folds can be followed along the strike. The overthrust segment referred to north of Davis Hill is an example of this.

6. *Dip.*—The dip of a formation can not be determined magnetically in terms of degrees. However, on the diagram of Douglas County,[1] which represents formations on the north limb, the most prominent variations are less than normal. The structural dip here is to the south. On the diagram of the south limb the variations are greater than normal and here the dip is to the north. In general, if structural dip is downstream on the magnetic flux, the magnetic intensity increases. The converse is also true. Change of dip can be recognized by the change in the width of the belt of variation. For example, see the change in width of the belt representing the Ironwood across Ranges 2 W. to 2 E. The dip flattens to the westward with abruptness at the cross faults. The fault block containing the Berkshire property in sections 9 and 10 of T. 44, R. 2 W. is an outstanding example of this.

Fig. 4 is an enlargement of the part of Fig. 2 that contains this fault block. The footwall and hanging boundaries are shown.

The surface width of the formation between the two cross faults is more than double that beyond the faults. There is no abrupt thickening

[1] Not presented herewith.

of the formation, hence change in dip must be held accountable. Fortunately there is a fair number of exposures. Along the footwall the

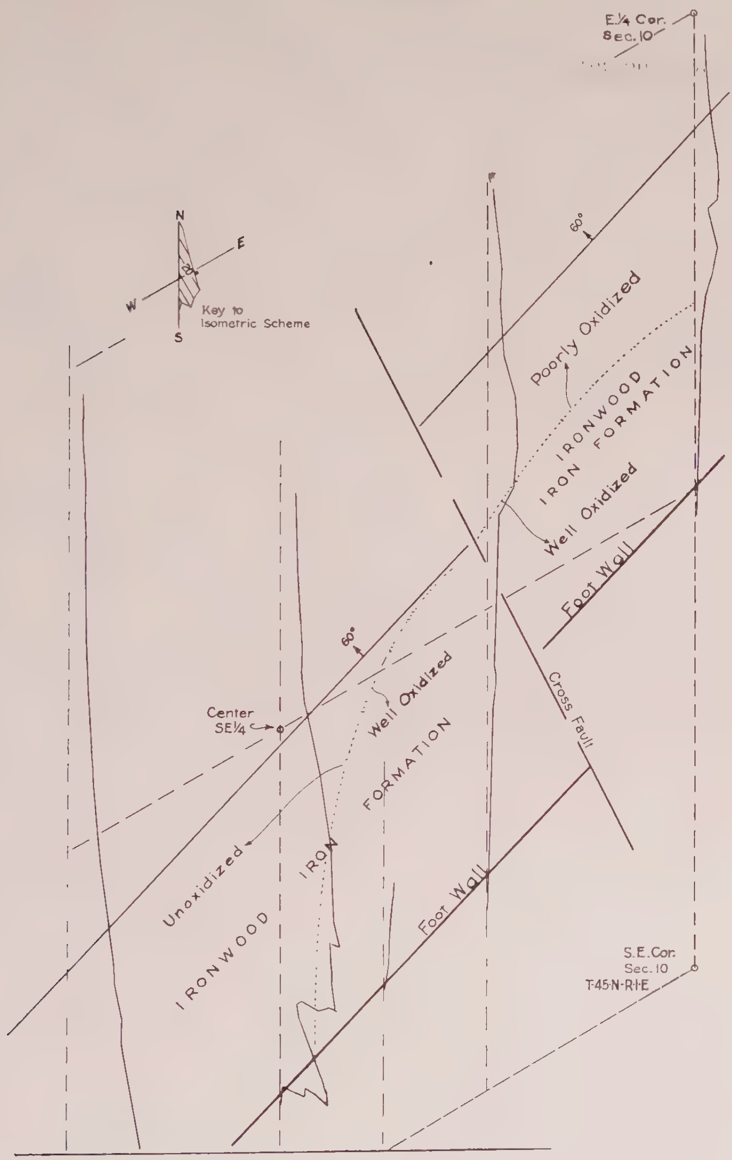

FIG. 3.—USE OF DIP-NEEDLE READINGS.
Location of footwall, detection of oxidation, determination of cross-faulting.
Scale: 40 in. = 1 mile.

structural dip is but a few degrees to the northwest. In the southeast of section 10 the exposures show that the basal numbers are drag-folded to

produce a synclinal structure which pitches westward. This accounts for much of the abnormal width. Exposures along the quarter line of section 9 show that the upper members are dipping about 60° north. The outermost member strikes through the east quarter corner of section 9. Hence the additional width of the magnetic belt remains to be accounted for.

It is noted that the four consecutive profiles from the west line of sections 9 and 16 to the quarter line of section 10 show distinctive and isolated cusps through which the north boundary is drawn. These are

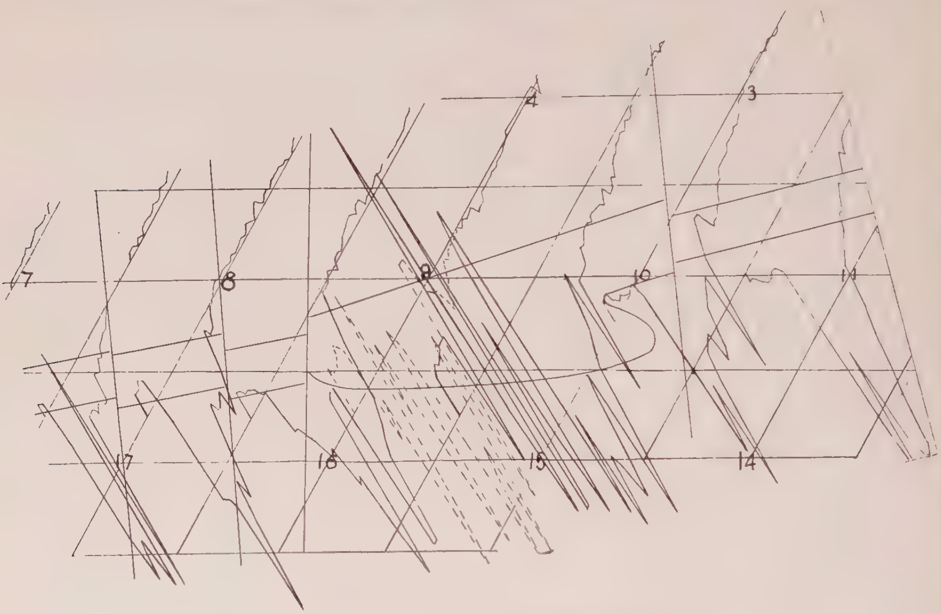

FIG. 4.—ENLARGEMENT OF A PORTION OF FIG. 2.

peculiar to these four profiles. Fortunately, there is an exposure in the northeast of the northwest of section 10 which identifies the formation responsible for these cusps. Incidentally the cusps were traced across the fault block and found to be a continuous magnetic belt. The exposures show this to be produced by a drag fold of the upper member of the Ironwood produced in the development of the main Lake Superior synclinal structure.

The diagram also shows on a smaller scale the manner in which the footwall is reflected in the magnetic observations. It shows the way in which cross faults may be detected although evidence for determining the direction is found along the footwall of the Keweenawan formations to the north (see Fig. 2).

7. *Nature of a Formation.*—In unknown country some idea of the nature of the underlying formation can be obtained from the character

of the magnetic variation. For example, if the variations range from a fairly constant minimum to a fairly constant maximum, the probability is that the country rock is a series of bedded formations, either sedimentary or extrusive igneous. If the variation is devoid of linear characteristics, but their range is rather wide, the chances are good that the underlying formation is a basic intrusive. If the variations are not great but are without linear features, the country rock is probably granitic. Examples on the diagrams are the conglomerate which has been referred to already in connection with the Davis Hill conglomerate, the gabbro of the Middle Keweenawan series in 1 E., 1, 2 W. and the porphyritic granite in T. 45 N., R. 3 W.

8. *State of Oxidation.*—In connection with the study of the Ironwood formation which contains iron ores, it was desirable to locate regions of high oxidation, for here the chances of ore are best. In detail this is shown on Fig. 3. Note that on the east line of section 10, the maximum observed dip-needle reaction was only 33°. On a traverse 750 paces west, the maximum attraction was 75°. Furthermore, the 33° reading on the east line was taken more than halfway across the formation, whereas the 75° observation fell less than 150 ft. north of the footwall. In narrowing the limits of the area of oxidation traverses were run in the following order: (1) 250 paces west of the east line; (2) 500 paces west; and (3) 375 paces west. The first shows even less attraction than the section line. The second showed high attraction, hence abundant magnetite, hence lack of oxidation. The third was not completed, since it showed at once a state of high oxidation.

It is noteworthy that, as shown on the large-scale diagram, the dip-needle variations increase from east to west. This is because the carbonate of the original formation has been recrystallized to form magnetite with increasing intensity to the westward. In the east, surface waters have oxidized the original carbonate to hematite. In this connection, note how the increase in magnetic intensity over the Ironwood corresponds to the closer approach and thickening of the intrusive gabbro in the Keweenawan to the north.

Conclusions

In general, all of the broader gaged problems of geology are amenable to the use of magnetic instruments. Some of the more important ones have been demonstrated on the magnograph. In general, also, the only way to determine whether or not the instruments will be of specific aid is to try them out and carefully study the results. The experience of the Wisconsin Geological Survey has been that in most cases the instruments have produced results which throw light upon the problems in hand, and in some instances have been the only means of gaining an understanding of the situation. To date the Survey has covered some

300 townships, in round numbers, and within these the instruments have revealed what could have been gained in no other way. The second outstanding service has been in enabling us to extend the information obtained from an exposure into the unknown covered country adjacent. In other words, the instruments not only provide an avenue by which to approach totally unknown country, but they have been of inestimable service in interpolating between known situations.

There are on the charts plottings of many thousand points. The preparation of this is a considerable undertaking. As presented herewith, Fig. 2 is but the reproduction of a work sheet upon which are carried the field results. For the inadequate addition of sufficient cross-sections to make clear the situation at all points and for the quality of the magnograph in general, the writer offers his apologies.

However, knowing that many geologists are asking what can be done with magnetic observations after they have been collected, the writer has come to feel that only by taking observations systematically over a great area in which the major features are known, and in which the minor details can be discovered with some degree of precision, can confidence be inspired in the uninitiated that the results have significant value. And since, to the writer's knowledge, there is no private interest or public organization that has started the collection of such a range of observations, or is very likely to undertake such work, he has taken what was available to date, with the hope that this demonstration may stimulate interest in the most promising and most rapid, most neglected but most interesting and simple, of the rapidly growing auxiliary tools of the geologist.

Geomagnetics is but an accessory tool in the kit-bag of the geologist. Interpretation of results requires geological background and field experience in correlating observations with topography, physiography, formations and structure. In the lack of broad training with geological sciences the magnetic methods are mere physical instruments to be used in collecting statistical data which turn out to be an end in themselves.

DISCUSSION

A. C. Lane, Cambridge, Mass.—Mr. Aldrich mentioned lines of equal magnetic force. Some weeks ago I suggested the term "isontic," which means "being equal," for such curves, but Miller, of Madison, called my attention to the fact that Galton in 1889, 30 or 40 years ago, used the term "isogram." These lines of force could be called "magnetic isograms."

Professor Stauffer has shown that what has been called Upper Keweenawan has Cambrian trilobites in it; and I have a letter from Schuchert saying that Ulrich has made up his mind that some part of what has been called the Keweenawan is Cambrian. Aldrich's paper says that there are big granites which invade the lower but not the upper part of the Keweenawan. I should not be surprised, therefore, if we had to split the Keweenawan.

A New Micromagnetometer

By Frank Rieber,* San Francisco, Calif.

(New York Meeting, February, 1928)

The discovery that strongly magnetic bodies localized near the surface of the earth could be detected by the distortion which they produced in the resultant magnetic field marked the beginning of magnetic surveying methods.

From an original ability to detect bodies of magnetic iron ore and the like when located very close to the surface, these methods have been extended until today they promise to give much information with regard to the structure of such slightly magnetic material as sedimentary rock containing only a very small proportion of iron.[1]

The recent extensions have been made possible through the development of more delicate instruments, and of methods for their use. From the original compass and dip needle, the design of apparatus has progressed until equipment is typified by magnetic balances capable of measuring the components of the earth's field to an extremely high degree of accuracy.

It seems unlikely that the limit of such instrumental development has been reached. Just as the change from compass and dip needle has enabled the location of relatively minor changes in magnetic materials, so additional developments in the direction of increased sensitivity, as well as in greater dependability and constancy of operation, should reveal positively many slight changes which either escape notice in the present surveys or are so small that they cannot with certainty be separated from uncontrollable errors due to inconstant operation of the equipment.

The ability to read smaller and smaller variations does not mean, of course, that deductions as to the causes of these variations will be made as accurately and definitely as is possible with the more pronounced anomalies.

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* Frank Rieber, Inc., consulting engineers.

[1] For example, certain strata in the more recent fresh-water deposits of the Tulare Series, in the San Joaquin Valley, California, which contain material only slightly more magnetic than the adjacent strata, show sufficient magnetic anomalies in the vicinity of known faults and folds to give promise of extensive usefulness of magnetometer surveys in the exploration for oil. Information of this nature will continue to prove of increasing value in many other fields as well, especially if we can hope to work successfully with smaller and smaller effects.

For example, a body of concentrated iron ore near the surface will cause a most pronounced distortion in the magnetic field in the vicinity. Finding such an anomaly in the measurements, the presence of the ore-body can be deduced with almost absolute certainty. On the other hand, a minor anomaly readily readable with a sensitive instrument might in some cases be due to an anticline containing a slightly magnetic stratum, or a syncline partly filled with magnetic material, or simply a very slight concentration of magnetic material due to some tendency towards classification during deposition. Increased sensitivity, therefore, may prove of less advantage than would be expected, due to the vastly greater difficulties in interpreting the more complex picture which the more detailed readings would present.[2]

Concerning the desirability of greater dependability and constancy of operation, however, there can be little question. All work with magnetic instruments is bound to be simplified if it is definitely known that the readings which are being recorded are due to changes within the earth, rather than to changes within the instrument.

This paper describes a new device, which even in its present experimental form gives promise of extending to a considerable degree the accuracy and dependability of magnetic instruments. The paper does not take up in detail the possible methods of interpreting pictures which may be obtained from magnetic surveys, but deals with the instruments with which the readings are taken.

Before describing the operating principles and details of construction of this new instrument, however, it may prove of advantage to review briefly the relation of some of the simpler structures to the magnetic anomalies which they may cause, and likewise to form an approximate estimate of the consistent accuracy with which one may hope to measure such anomalies with the type of instrument now in common use.

Underlying Principles

The distorted magnetic field in the vicinity of magnetic bodies near the surface of the earth may be considered as being in reality a combination of two separate fields. The first, which may be termed the primary field, is due to the magnetic properties of the earth as a whole and may be considered as being uniform both in direction and in strength. The field due to the local magnetic body may be termed the secondary field. It is the latter which, superimposed on the uniform primary field, gives the distorted result.

[2] If the sensitivity can be improved, without any essential sacrifice in dependability, there is justification in placing some importance on it, since it is highly possible that other types of geophysical investigation may be developed which will supplement a highly detailed magnetic survey in such a manner as to render even the smallest readings of great value.

Several diagrams of such fields are given here (Figs. 1–4). Instead of drawing these diagrams empirically, the author has used the old familiar device of making the fields draw their own pictures. A large solenoid with a relatively uniform internal field was used to represent the primary magnetic effect of the earth. Small steel bars, introduced into this solenoid in various positions, represented local bodies with magnetic properties.

Pictures of the combined fields from both the bars and the solenoid, and of the fields due to the bars alone, were made by covering the bars with a piece of bromide paper, dusted over with fine iron filings. A diagram of the assumed structure was then sketched in over the photograph of the fields involved.

This experimental method is so rough as to have practically no value in the study of details of magnetic anomalies, where the variations are so slight that no distortion could conceivably be shown in an actual field diagram. The illustrations are intended rather to illustrate the possibility of inferring the existence of simple structural changes, if pictures of the secondary fields associated with them are obtainable, and if the primary field of the earth can be removed in some manner.

These figures will serve to illustrate some of the possibilities and some of the obvious limitations of magnetic surveying. They also show plainly that the final magnetic picture, from which the anomaly is to be deduced, should show the secondary field only. The primary field is so strong that in most cases it would obscure or mask the resultant field.[3]

Of course the earth's field cannot actually be removed while the field of the anomaly is being investigated. However, it can be neutralized in the instrument which then becomes responsive only to the anomalous field. Or the total strength of the resultant field can be measured and a calculated portion of this can be subtracted as being the earth's field. The remaining field will then be due to the anomaly.[3]

Practically, the method of neutralizing the effect of the earth's magnetic field on the instrument is the most widely used. The magnetic balances now in general use will serve as typical examples. Such balances are constructed to measure the component strength of a field in some definite directions with respect to the instrument. Obviously, if both vertical and horizontal components of the field of the anomaly can be measured and the vertical and horizontal components of the earth's field can be neutralized in the measurements, an accurate picture of the direction and strength of the secondary field at the place of the instrument can be made.

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[3] No reference has yet been made to the variations in the earth's field from time to time. Such variations probably apply equally well over large areas, and hence can be compensated for by recording at a base station on a continuous record the variations from time to time, to be applied as corrections to field readings taken at known times.

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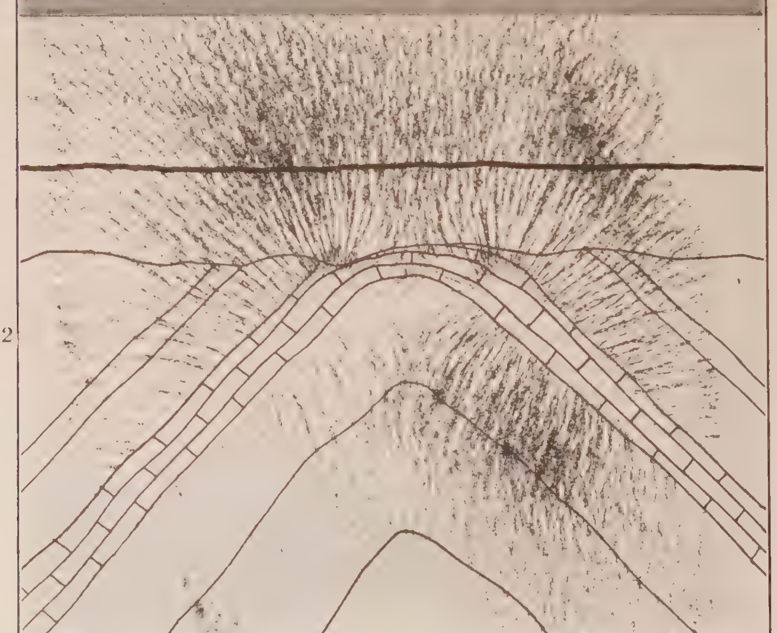
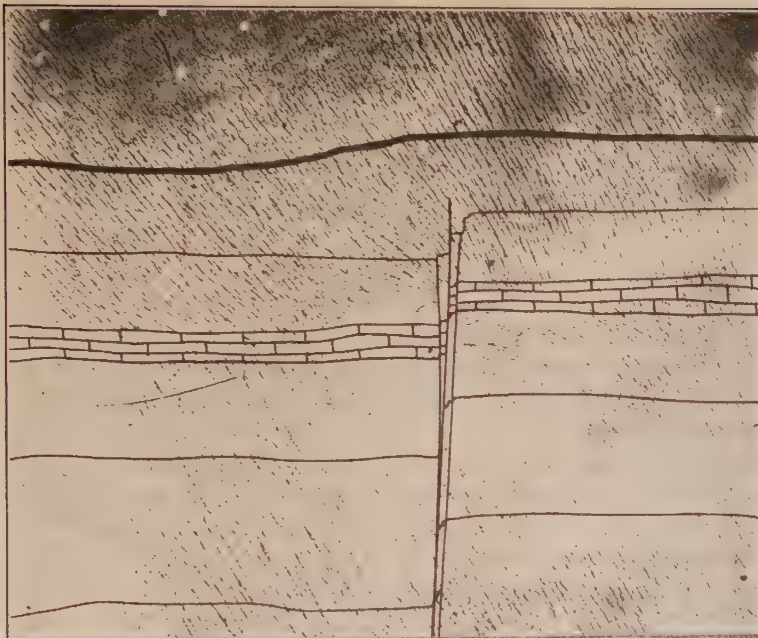


FIG. 1.—PRIMARY AND SECONDARY FIELDS OVER A FOLD CONTAINING A PERMEABLE STRATUM.

FIG. 2.—SAME AS FIG. 1, BUT SHOWS SECONDARY FIELD ONLY.



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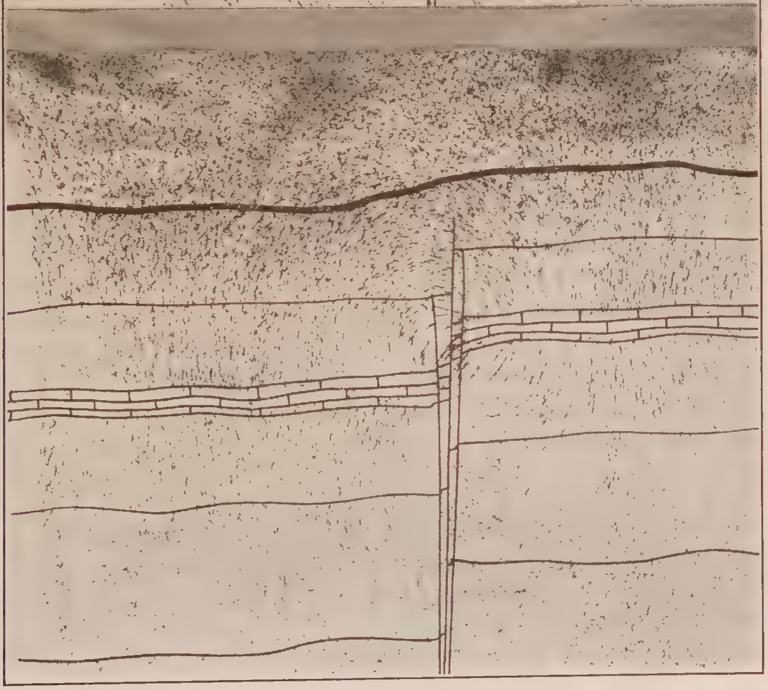

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FIG. 3.—PRIMARY AND SECONDARY FIELDS OVER A FAULTED STRUCTURE CONTAINING A PERMEABLE STRATUM.

FIG. 4.—SAME AS FIG. 3, BUT SHOWS SECONDARY FIELD ONLY.

A Typical Instrument

A typical magnetic balance for vertical component use consists of a light portable tripod, on which is mounted a double-walled housing, interlined with cork. A permanently magnetic balance beam, of about the size and shape of the beam in the usual analytical balance, is supported by quartz knife-edges resting on quartz cylinders, giving a very small area of contact.

The motion of the beam is observed by the reflection of a small engraved scale in a mirror attached to the beam. This image is viewed through a microscope, enabling extremely small beam deflections to be utilized.

Levels are provided for accurately setting the instrument, the direction of the knife-edges being preferably magnetic north and south, as determined by an auxiliary compass; also, a thermometer is mounted in the instrument housing.

Mounted in a brass tube which hangs below the instrument, between the legs of the tripod, is a small permanent magnet, which can be adjusted vertically.

In use, the instrument is set up at a base station, where the assumed value of the anomalous field may be taken as zero. The vertical component of the earth's field at this point tends to tip the balance beam—a tendency which is neutralized as nearly as possible by adjusting a small screw on the beam which provides a reverse moment due to gravity. When this adjustment is as close as is practical, the balance is brought to an exact zero scale reading by adjusting the compensating magnet below the tripod.

This position of the magnet is left constant throughout the ensuing survey. If at any station a deflection is noted in the balance beam, it is assumed that this deflection is due solely to the field of some anomaly.

By making such measurements for both vertical and horizontal fields, the apparent direction and strength of the anomalous field at the place of observations may be obtained by combining the measurements. By making observations at a number of points, a picture may be drawn showing the characteristics of the field of the anomaly as this field emerges from the earth.

Sources of Error

In order to estimate the possible value of a new instrument, in the matters of accuracy and consistency, it is necessary to form some general estimate of the degree of precision which can be maintained with a typical balance such as has just been described.

Unfortunately, one can not simply take the minimum scale reading possible with such an instrument as a measure of this accuracy. Such readings often are not justified by the construction employed, and are occasionally beyond the scope of some of the basic principles of the

instrument, so that no degree of care in construction would suffice to permit consistent repeated readings at the minimum scale value.

A magnetic balance is essentially an instrument for weighing (by comparison with a gravity component) the reaction of the earth's field on a permanent magnet. Its total accuracy thus depends on the combined accuracy of the weighing apparatus, the permanence of the magnet, the elimination of stray forces, etc., and the leveling and stability of the instrument. These are best considered separately.

Weighing

Since the damping provided to stabilize a magnetic balance precludes the mean-swing method of reading,[4] and since the balancing weight must be set once and for all at the base station, and cannot be removed for null check[5] to accompany each reading, one can not possibly hope to maintain the same degree of accuracy that could be expected from a fine analytical balance. Also, rougher field handling requires more rugged and less sensitive construction than that used in laboratory instruments.

An arbitrary estimate of the possible accuracy for such a field instrument might place its sensitivity, for weighing, at about 1 part in 50,000.

A vertical component instrument weighs a total magnetic moment of about 40,000 gammas at San Francisco. The above accuracy in the weighing operations, therefore, might be sufficient to justify a minimum scale reading of 1 gamma.

Magnets

The term "permanent magnet" is somewhat of a misnomer. No magnet, even of the best steel available, can be considered as at all permanent, within the limits of error and under the conditions of use required for a magnetic balance. Repeated temperature changes, and the continual jarring due to transportation, cause a gradual loss in strength of the magnets in balance beams.[6] While such a gradual loss may be compensated for by daily or periodic standardization of instruments, a more serious effect has been noted. This takes the form of a loss during use and a partial recovery during rest periods.[7]

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[4] Analytical balances of very fine construction are dependable, under full load, to about the sixth place. To maintain this accuracy, such a balance must be read while swinging slightly, and the mean positions between extreme swings taken as null point.

[5] The null point under no load should be checked each time the beam is set upon the knife-edges, for full accuracy.

[6] Out of a number of instruments, in continual daily use in investigating possible oil structure, where rough handling and fluctuating temperatures were the rule, the largest loss over one year's time was 1 per cent.

[7] A loss of 50 gammas, or 50 parts in 40,000, would not be unusual for one day's rough use. About 90 per cent. of such a large loss would be restored during an overnight rest of the instrument.

Such fluctuations can not, unfortunately, be distributed evenly as corrections to the day's readings. It is highly possible that a single abrupt jar would cause at least half of the total change observed, leaving a day's readings worthless.

Treatment with liquid air,[8] long recognized as adding to the permanent qualities of magnets, is said to minimize these fluctuations, but any instrument employing a permanent magnet without some new type of precaution would seem to be basically untrustworthy beyond an accuracy of 1 part in 5000. In terms of vertical field measurement at San Francisco, this would mean a minimum dependable scale reading of about 8 gammas.

Stray Forces

Circulating currents of air within the instrument case will exert some effects on the balance. These are in part provided against by the construction, where a cork-interlined case prevents rapid transfer of extremes of temperature (such as the shadow of the observer interrupting strong hot sunlight) and a copper inner case distributes more or less evenly the changes which do pass through the cork. However, an extremely small force will deflect such a balance beam slightly from its true position of rest, and an arbitrary estimate of possible error might fairly assign 1 gamma inaccuracy in the vertical component measurement at San Francisco, or about 1 part in 40,000, to this cause.

Leveling

The instruments in common use are mounted on collapsible-leg tripods, similar to those used for well constructed camera supports. In discussing the errors due to improper leveling, therefore, the accuracy of the level bubble tube itself and the ability of the tripod to maintain this accuracy must both be taken into account.[9]

In use, the instrument for the vertical component measurement is placed with its knife-edges in the magnetic north-south direction. A slight error in this direction will have a negligible effect. The instrument is balanced and read and then rotated through exactly 180° and read again, the mean of these two readings being taken as the true vertical component value. As a matter of fact, this mean average is really the component along the axis about which the instrument has been rotated. If this axis is not actually vertical, because of inaccurate leveling, a change in the axial component will take place.

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[8] J. Dewar and J. A. Fleming: Changes Produced in Magnetised Iron and Steels by Cooling to the Temperature of Liquid Air. *Proc.* Roy. Soc. (1896–97) **60,** 57.

[9] Naturally, no exact estimate can be made of the error due to any one type of tripod construction. Makers of good instruments, however, discourage the use of extension-leg tripods, even of extremely heavy construction, for mounting precision instruments for triangulation and leveling to 20 seconds.

At San Francisco, an error of about 15 gammas will be caused for each 0.01° inclination of the instrument axis from the vertical, as measured in the magnetic north-south plane. It is extremely difficult to imagine a sustained accuracy of leveling within 0.01° with the bubbles and tripods in common use on these instruments. Therefore a possible error of 15 gammas may be assigned to this cause.

Total Error

Summing up these rather arbitrary estimates, it is hard to place the maximum possible error for such an instrument as has been described, under average conditions of use, at less than 24 gammas, or a possible average working error of 12 gammas for use at San Francisco. This error, in terms of the total field is ± 0.03 per cent.

Further, inherent limitations of some of the basic principles of such instruments—notably, the variability of the magnetic properties of the beam—would seem to render further refinements of design of little avail in improving the sensitivity and consistency.

The New Instrument

In an attempt to overcome the inherent limitations of the type of instruments now in use, a new instrument has been designed, operating on a different principle. It was realized that this instrument must first of all maintain the vertical and horizontal component measurements in an absolutely vertical and horizontal direction. The levels and tripod mountings and other parts of present magnetometers being thought inadequate for this purpose, a heavier form of tripod has been adopted, together with the level bubble tubes of the type employed by the Coast and Geodetic Survey. These levels, which are the most accurate available, can be read to 2 sec. of arc,[10] or approximately 0.0005°.

Principle of the String Galvanometer

The principle adopted for measuring the magnetic field is that of the vibrating-string galvanometer, in common use for recording electrical vibrations, notably in the electrocardiograph for recording human heart beats. In this instrument, as commonly used, a very fine wire is held under tension and a beam of light is concentrated upon it. By means of a projecting lens a magnified image of the shadow of the wire is projected on a moving photographic film.

A microscope may be focused on the wire and observed directly, a certain amount of illumination being supplied to render the wire visible.

[10] In the earth's field at San Francisco, an error in leveling of 0.0005° would mean the introduction into the vertical field measurements of a component of approximately ½ gamma.

A strong magnetic field is imposed on the wire, with its direction along the axis of the observing microscope. If we now pass even a very slight alternating current through the wire, it will move transversely in the magnetic field, the motion being easily visible in the microscope.

This device becomes a very sensitive means of detecting small alternating currents, when the wire is subjected to such tension that its own vibratory period coincides with that of the alternating current which is being measured, a very slight trace of alternating current being sufficient to cause the wire to vibrate appreciably.

Adaptation of Vibrating Wire to Magnetometer

In adapting this principle to the new magnetometer element, the operation has been reversed. Instead of subjecting the wire to a strong magnetic field and then using it to detect weak alternating currents, a strong alternating current is passed through the wire and is used to detect the presence of weak fields. For example, if such a wire is tuned

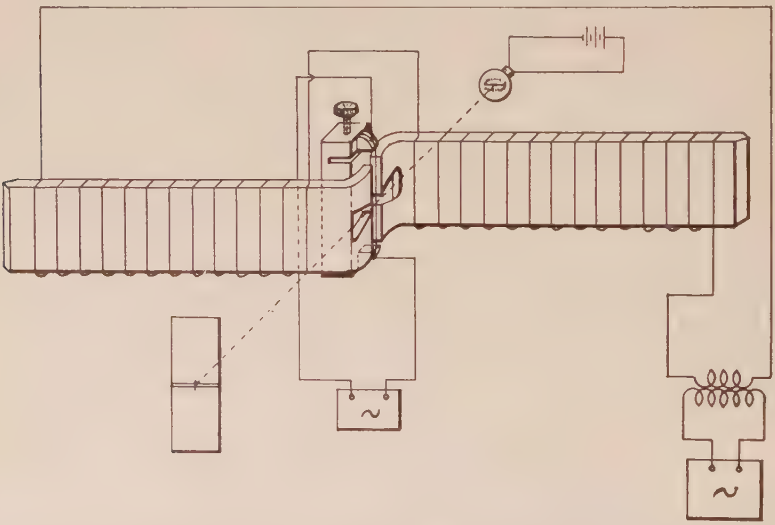

FIG. 5.—CONSTRUCTION OF THE FIRST MODEL OF THE MICROMAGNETOMETER. Two bars of permeable alloy are bent near opposing ends, and a fine wire is stretched in the gap between the bars. Notches in the pole pieces permit light from a small lamp to cast a shadow of this wire on a microscope objective (not shown) which projects a magnified shadow image of the wire on a strip of film.

accurately to alternating current (say of 100 cycles) which can be obtained from a small vacuum-tube oscillator, and if a microscope is then mounted so that transverse vibrations of the wire can be noted, it will be found that on turning the assembled parts so that the axis of the microscope is in the direction of the earth's field, the wire will vibrate to a rather large amplitude.

The sensitivity obtainable with this arrangement alone, however, is not nearly sufficient to serve as a means for measuring the earth's field. In the new instrument the following expedient for intensifying the effect has been adopted.

Two bars, of a magnetic alloy that is extremely permeable at low field intensities, are arranged end to end with a very small air gap between the opposed ends. If the vibrating wire is stretched across this air gap, and means for viewing it along the axis of the bars are provided, it will be found that this arrangement is extremely sensitive to the slightest variation in the earth's field (Figs. 5 and 6).

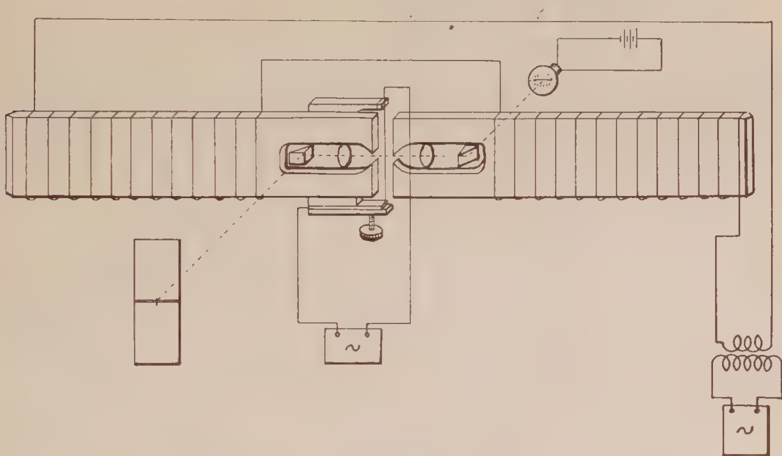

FIG. 6.—MODIFIED FORM OF THE INSTRUMENT SHOWN IN FIG. 5, NOW UNDER CONSTRUCTION.

The alloy bars are straight, and small prisms deflect the light through lenses and project the wire image either upon a film or into an eyepiece. A small oscillator supplies current which vibrates the wire, while a second small oscillator of higher frequency passes current through coils surrounding the bars, to overcome hysteresis.

If the instrument is placed with the axis of the bars accurately transverse to the direction of the earth's field, the magnetism in the bars will fall to zero value, and the wire will cease to vibrate. In this manner the device may be made to serve as an accurate indicator of the direction of the earth's field.

Preliminary Tests

To test the accuracy with which this adjustment of direction could be made, an experimental model of this device was mounted on a rigid tripod, as illustrated, on which was likewise mounted a telescope provided with cross hairs (Fig. 7). By focusing this telescope on a properly divided scale at a considerable distance, it was possible to determine to a very high degree of accuracy the angle through which the instrument was turned.

The instrument was set repeatedly to the apparent null position where the fine wire ceased to vibrate when viewed in the microscope, and the angular setting was then noted by viewing the distant scale through the telescope. Repeated trials of this procedure placed the accuracy with which the original experimental instrument could be employed as well within 0.01°.

Current to vibrate this wire was provided from a small vacuum-tube oscillator which could be constructed in a highly portable form for field use. The exact strength of the current is immaterial as long as it is sufficient to provide the required degree of accuracy in obtaining the null. Current strength does not enter quantitatively into the readings.

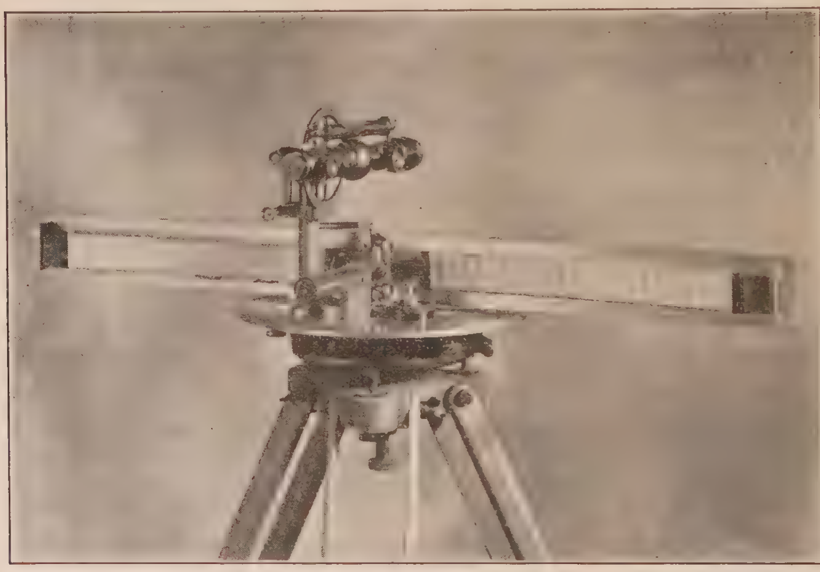

FIG. 7.—THE FIRST EXPERIMENTAL INSTRUMENT, SET UP FOR VISUAL TESTS. A transit telescope, mounted on the instrument, measures small rotations in the horizontal plane, by readings taken on a graduated scale at 100 ft. distance.

In this instrument, additional alternating current of higher frequency was passed through coils of wire wound about the magnetic bars, in order to overcome the slight magnetic lag, which would otherwise give a different null value as the instrument approached the null from one side or from the other.

Photographic Recording

Modifications of the original construction immediately suggested by this experiment were made on the initial machine, which was then set up for a photographic test of its possible accuracy (Fig. 8).

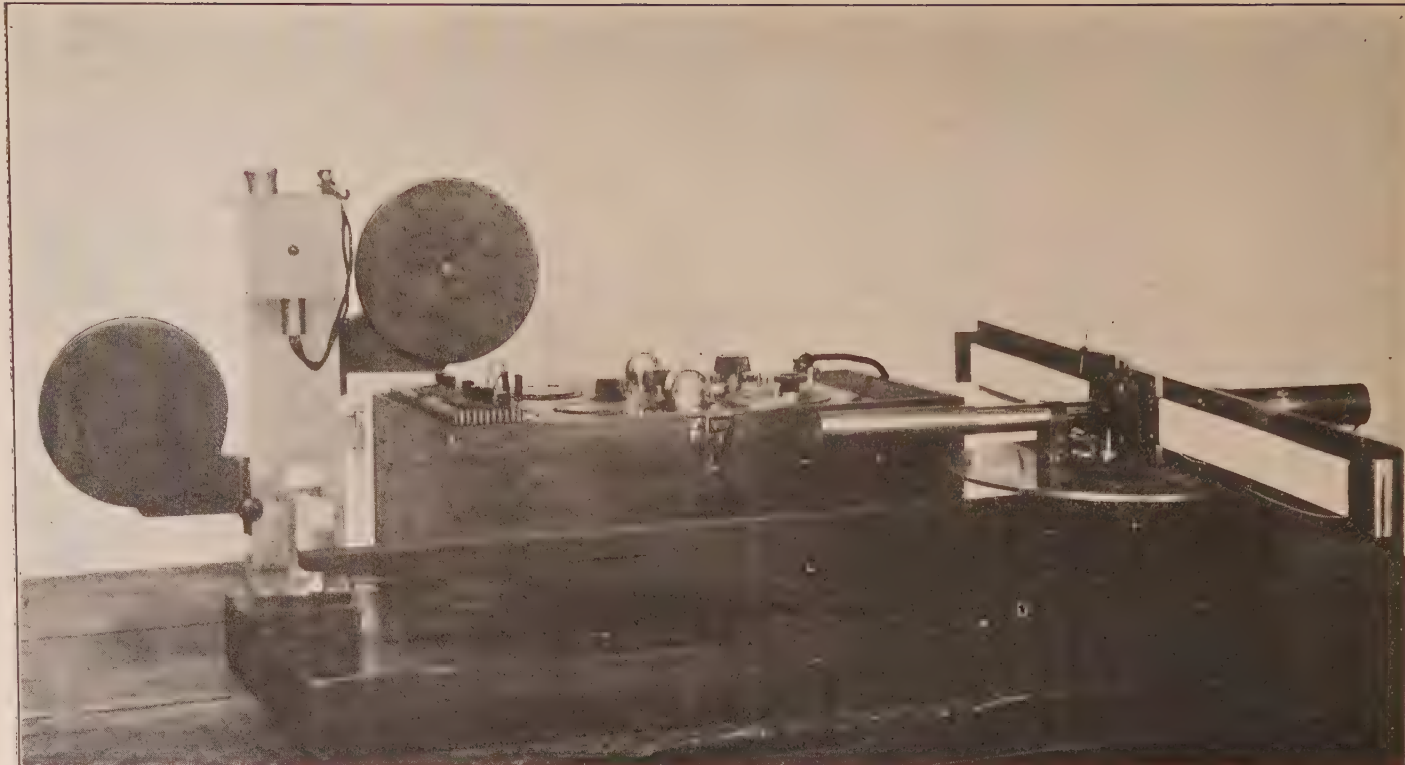

Fig. 8.—Instrument set up for photographic test.

The magnetometer is shown at the right, mounted for small rotations in the horizontal plane. A magazine camera at the left records the vibrations of the magnetometer wire, as shown in Fig. 9. The portable oscillator at the rear supplies current to vibrate the magnetometer wire.

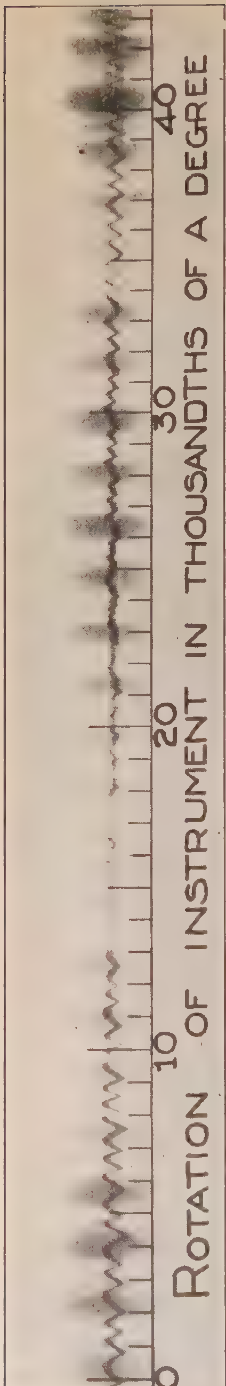

FIG. 9.—THIS PHOTOGRAPHIC RECORD OF THE VIBRATIONS OF THE MAGNETOMETER WIRE WAS MADE WITH THE APPARATUS SHOWN IN FIG. 8.

Instead of viewing the vibrating wire through a microscope, in this test, the image of this wire was projected on to a moving film, of which the rate of motion with respect to the angular change in position of the magnetometer was known (Fig. 9).

It is to be noticed that the degree of accuracy with which the null can be estimated in the plate obtained in this way is somewhat interfered with by vibrations of the crude photographic set-up employed, which does not allow the image of the wire to come precisely to rest at the null position. An accuracy of about 0.001° is easily possible on this rough test, although the photographic record gives a rather poor idea of the extreme precision with which settings may be duplicated.

It is interesting to note that, with the field components at San Francisco, this means that an estimation of magnetic intensity to within 0.4 gamma is possible with the first experimental model of the instrument.

The effects of the length and dimensions of the vibrating wire and the arrangement of the magnetic bars were next given some attention, with the result that a few small changes were made and a second model is now under construction.

The chief differences in the second model are that the magnetic bars are arranged in a straight line instead of being bent to permit the passage of the line of sight and that a longer vibrating wire of large cross-section and a closer spacing of the pole pieces are adopted.

A dependable accuracy of 0.0005° is hoped for with this device, when used as a null instrument.

The two bars, the vibrating wire, and the microscope may be considered as a unit and may be employed in a number of various types of instrument construction. For example, they may be mounted to swing about a vertical or horizontal axis and thus be used to determine the direction of the field in the vertical and horizontal planes to a high degree of accuracy.

Direct current of a known value may be passed through the coils surrounding the bars when these

are aligned in the true direction of the field, until the effect of the field on the bars is precisely neutralized. The value of this direct current will then be proportional to the intensity of the field at this point. Such currents may be read to a high degree of accuracy with the aid of a potentiometer.[11]

The bars may be mounted as a unit in a manner similar to that used with the balance, in order to measure either the vertical or horizontal components of the earth's field, in which case a permanent magnet may be rigidly mounted with respect to the bars and adjusted to neutralize the earth's field, leaving the measurement of the unbalanced component to be carried out by the magnetometer.

This estimation of the value of the unbalanced component may be made by passing a very small known current through the coil surrounding the bars, or by adjusting a second control magnet, which has been properly calibrated and placed at a distance from the bar.

The instrument itself should be almost entirely free from such troubles as are experienced with pivoted or suspended instruments, since the ratio of the mass of the wire to its strength is so favorable that no possible jar in service could break or damage the wire, which constitutes the only movable part of the device.

To take full advantage of the possible precision of this instrument, a portable potentiometer, accurate to 1 part in 100,000, may be carried as part of the field apparatus. In practice, such equipment might prove too cumbersome and delicate. A permanent magnet, used as a standard of comparison, has many attractive advantages. Such magnets, used as balance beams, can not be protected from strong demagnetizing effects when jarred. However, a neutralizing magnet for the new apparatus could be designed so that a magnetic shield surrounding it during transportation would practically prevent the demagnetizing effect of jars.

Conclusions

The development of magnetic surveying would seem to have reached an instrumental limit, beyond which little progress may be expected unless some new principle of instrument construction can be introduced. The instrument described here is believed to embody such a principle. The experimental model first constructed shows an apparent accuracy of over 30 times that of typical field balances.

Due to the lightness and the strength of the wire which constitutes the one moving part, and to other features of construction, greater dependability as well as greater accuracy may be hoped for.

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[11] Laboratory measurements with precision potentiometers are accurate to about 1 part in 1,000,000.

The Eötvös Torsion Balance Method of Mapping Geologic Structure

By Donald C. Barton,* Houston, Texas

(New York Meeting, February, 1928)

The theory of gravitation is based on Newton's law that any two bodies exert a mutual attraction which is proportional to the product of their masses and inversely proportional to the square of the distance between them; *i. e.*,

$$A = K\frac{Mm}{R^2} \tag{1}$$

where K is a constant, M and m are the respective masses of the two bodies, and R the distance between them.

In the earth's gravitational system, the earth is one of the bodies; the other may be any body within or without the earth. In theoretical discussions and calculations in regard to gravity, that other body usually is taken as a unit of mass: in the C. G. S. system, a body of one gram in mass.

Gravity and Level Surfaces

Gravity is defined in geophysics as the force of attraction exerted by the earth, towards itself, on a body of unit mass. The intensity of gravity is usually expressed in dynes of force or in centimeters of acceleration. In geophysical work in America, most commonly it is expressed in dynes (per gram) with the "per gram" understood and not directly expressed. In geophysical work in Europe, most commonly it is expressed in centimeters per second per second (cm./sec.^{2}) and less commonly merely as centimeter-gram-second units (c. g. s.). At sea level, the intensity of gravity amounts very nearly to 980 dynes.

The vertical is the direction along which the attraction of gravity is exerted at any point. In most cases, it is very nearly, but not quite, perpendicular to the earth's surface. The path that would be taken by a body falling freely through space toward the earth is vertical at each point and may be spoken of also as "the vertical," or as the line of the vertical, or as a line of force of gravity. At the earth's surface it commonly is very nearly, but not quite, a straight line.

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* Consulting Geologist and Geophysicist.

A level surface[1] is a one that is everywhere perpendicular to the vertical. There are an infinite number of level surfaces, but for purposes of brevity in discussion and clarity and ease in graphic representation, an arbitrary set of level surfaces at some arbitrary interval often is used as though only these surfaces existed. The surface of a body of water at rest and acted on only by gravity is a level surface. A level surface is not a plane and it is not a surface of equal gravity.[2]

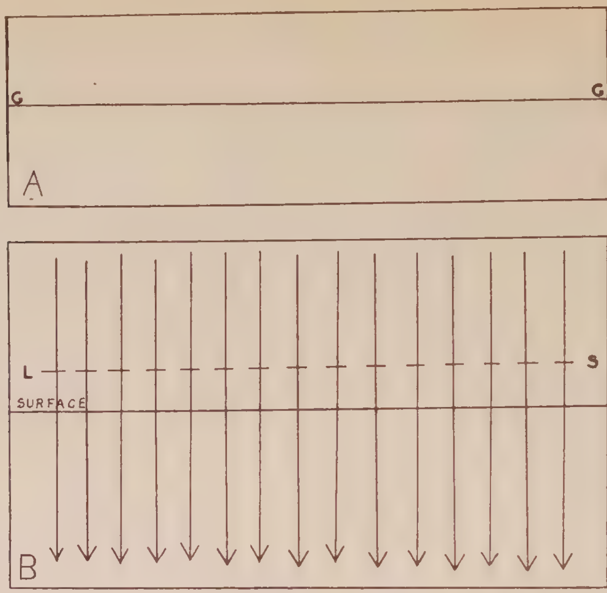
A horizontal surface, in contrast to a level surface, is a plane surface tangent to the level surface and perpendicular to the vertical at some observation point or some other point which for the moment is being used as the center of reference.

If the earth were a homogeneous sphere at rest, gravity would be the same everywhere over the surface of the earth and would vary only radially; the lines of force of gravity would be radial, and the level surfaces would be spherical and concentric with the earth's surface. But the earth is not at rest; it is not a sphere; and its outer crust is in no way homogeneous. On account of the rotation of the earth, centrifugal force tends slightly to counteract the earth's attraction. The effect is at a maximum at the equator and decreases to zero at the north and the south poles. As the earth is a spheroid flattened at the poles, and therefore the radial distance of the surface at the poles from the center of the earth is less than at the equator, the force of attraction of the earth is at a maximum at the poles and at a minimum at the equator. As the result of those two factors, the value of gravity at sea level is at a maximum at the poles and a minimum at the equator. The increase of the value of gravity northward in the northern hemisphere is known in torsion balance work as the "Normal Northward Gradient" and amounts to 7 Eötvös units (or 7×10^{-9} dynes) at latitude 30. It is a distinctly appreciable quantity and has to be allowed for in the calculations. The level surfaces are warped concomitantly and the amount of the warping is a function of latitude. It is of a very appreciable magnitude and the "normal differential curvature," as it is called in Eötvös torsion balance work, has to be allowed for in connection with the "differential curvature or R" values, to be described later.

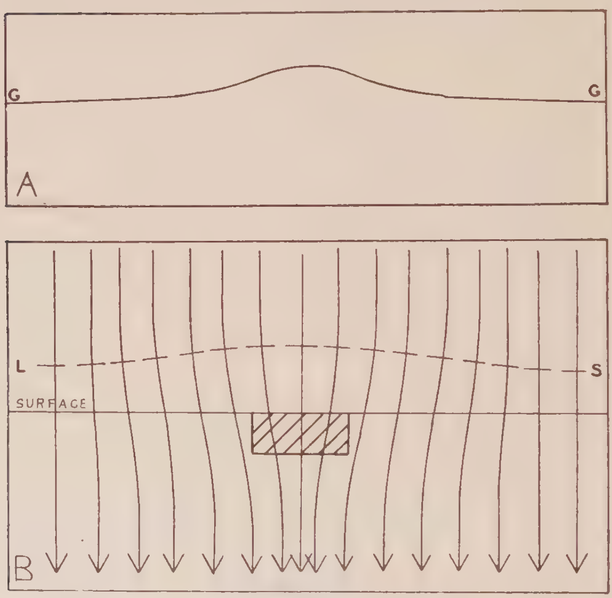
The earth's crust is distinctly inhomogeneous. The continents are supposed to be composed of lighter rocks than are the ocean basins. Plutonic igneous complexes and metamorphic complexes are heavier than most masses of sedimentary rocks. Limestone and anhydrite are heavier than other sediments, and the older sands and shale tend to be

[1] "Niveau surface" in some of the torsion balance literature; "equipotential surface" of geophysics and of the potential theory.

[2] The level surface being an equipotential surface, it is a function of $\frac{1}{R}$, and a surface of equal gravity is a function of $\frac{1}{R^2}$, where R is the distance to the attracting body.



a. With a homogeneous distribution of mass.



b. With a body denser than the surrounding country rock.

FIG. 1.—DIAGRAMMATIC SKETCHES SHOWING A SMALL SECTION OF THE EARTH'S CRUST AND GRAVITY RELATIONS.

heavier than the younger. Horizontal uniformity of distribution of sediments of uniform density is destroyed by diastrophism. The horizontal distribution of mass in the earth's crust within a few miles of the surface, therefore, is very irregular; bodies of rock of high specific gravity rise into or beside bodies of rock of lesser density and the magnitude of those bodies ranges from that of a mountain system down to that of a glacial boulder. Topographic relief also causes a horizontal opposition of rock of relatively high specific gravity against air of relatively no specific gravity. This irregularity of the horizontal distribution of mass causes a deformation in the gravitational system. A body of density

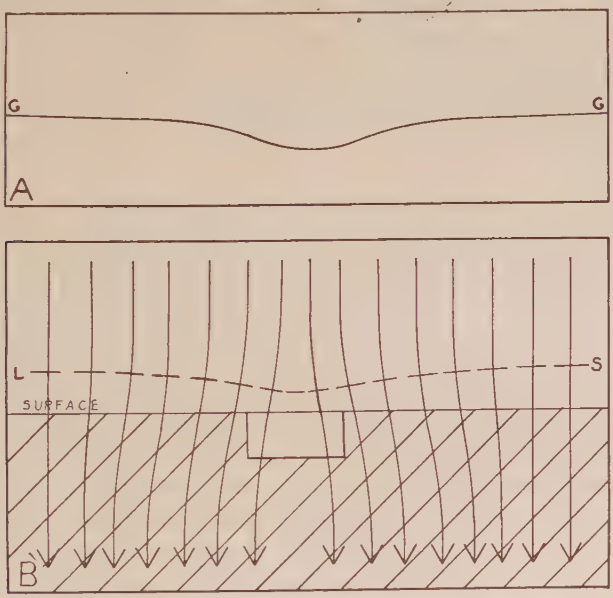


FIG. 1.—c. With a body lighter than the surrounding country rock.
gg = profile of gravity.
LS = level surface.
Arrows = lines of the vertical

greater than the average will exert a definite gravitational attraction approximately toward its center of gravity. The intensity of the attraction will be proportional to the excess of mass. There is no repulsion possible in the gravitative system, but the practical effect of a body less dense than the average is the same as if the body exerted a force of repulsion proportional to its deficiency of mass. Gravity at any point is the three dimensional vectorial sum of the normal gravity and of the respective positive and "negative" forces of attraction of the various bodies with excess or deficiency of mass.

As the result of the irregular distribution of mass in the earth's crust, the intensity of gravity locally is at a maximum over the areas of excess

of mass and at a minimum over the areas of deficiency of mass; the lines of the vertical tend to crowd from the bodies deficient in mass into the bodies with excess of mass—that is, the vertical is deflected toward the excess of mass and away from the deficiency of mass; the level surfaces are warped up, convexly, over the areas of excess of mass and are warped downward, concavely, over the areas of deficiency of mass. (Fig. 1.)

Elements Measured by the Torsion Balance

The torsion balance measures directly two elements of that deformation in the gravitational system: the rate of the horizontal variation of the intensity of gravity, and a function of the curvature of the level surface. Indirectly from the rate of variation of gravity, it is possible to calculate the total variation of gravity, if a sufficiently close net of stations has been occupied.

The rate of horizontal variation of gravity is known in torsion balance work as the "gravity gradient" and is defined as the difference in the intensity of gravity per horizontal centimeter. Unless definite specification is made that a component of the maximum gradient is meant, the term "the gradient" refers to the maximum or "total" gradient. The convention in regard to the direction of the gradient is that the gradient is positive in the direction of the increasing intensity of gravity; that is, it is toward the relatively heavier mass and away from the relatively lighter mass. The symbol *Gr g* was used by Eötvös to designate the gradient; on maps, the gradient is represented by an arrow flying in the direction of the increase of gravity and proportional in length to the magnitude of the gradient. The measurement of the gradient by the torsion balance is due to the fact that the warping of the vertical, or what is the same thing, the divergence of the level surfaces, produces at each weight of the balance a small horizontal component of gravity, which is equal to the horizontal gradient of gravity.

The differential curvature that is measured by the Eötvös torsion balance is not the actual curvature of the level surfaces but is a function of the difference in the curvature of the level surface in the directions of greatest and least curvature. Any level surface taken over a considerable area is a very irregularly warped surface and can not be represented by a simple mathematical surface, but within the small area of the torsion balance, it can be represented by either an ellipsoidal or a saddle-shaped surface of very large radius of curvature. Within the area of the torsion balance, the trace of that surface in vertical planes approximates a small portion of a very large circle. The differential curvature is the difference between the reciprocals of the radii of curvature of that surface in the vertical planes in the directions of the greatest and the least curvature.

The curvature value, or the R value of the literature which was used by Eötvös and is used in most work with the torsion balance, is gravity times that difference between the reciprocals of the radii of greatest and least curvature (minimum and maximum radii of curvature); that is:

$$R = g\left(\frac{1}{r_{\min}} - \frac{1}{r_{\max}}\right). \tag{2}$$

The measurement of R by the torsion balance is due to the fact that the warping of the level surfaces produces a small component of gravity at each weight of the balance. The resulting moment of torsion is a function of R and the azimuth of the balance beam. The orientation of R by convention is taken as that of the axis of algebraically minimum curvature (maximum radius of curvature) and the azimuth of that axis is designated by the symbol λ. On maps, R is represented graphically by a line oriented with an aximuth of λ and with a length proportional to the magnitude of R.

Torsion Balance Units of Measurement

The unit of measurement for the gradient in work with the Eötvös torsion balance is 1×10^{-9} dynes (per gram) per horizontal centimeter. That unit is coming to be called an "Eötvös" or an "Eötvös Unit." A gradient of 1 E means an increase in the intensity of gravity per horizontal centimeter of about one thousand-billionth (1×10^{-12}th) the value of gravity. The values of the gradient actually measured in the field range most commonly from 5 to 30 E. The mean maximum value for the gradient observed in work with which the writer was connected is about 150 E.

The unit of measurement for the differential curvature, strictly speaking, should be in units of 1×10^{-12} radians per centimeter, as the significance of the magnitude lies in its indication of the difference of the warping of the level surface in different directions. The R that is actually used in practice, however, is gravity times that difference, and is measured in terms of 1×10^{-9} dynes per gram per centimeter (Eötvös units). But as the reciprocal of the value of gravity in most places is within a few per cent. of 1×10^{-3} and as the field measurements of R are not accurate within a few per cent., the numerical value of R in terms of 1×10^{-9} dynes per gram per centimeter (Eötvös units) is the same for most practical purposes as the numerical value of the differential curvature in terms of 1×10^{-12} radians per centimeter. The values of the differential curvature actually observed in the field commonly range from 5 to 50 Eötvös units. The mean maximum value for the largest observed values of R is about the same numerically as that for the gradient: 150 Eötvös units. Large values for R are very much commoner than large values for the gradient.

Working System of Eötvös Torsion Balance

The working system of the Eötvös torsion balance consists essentially of a torsion wire suspended at its upper end and carrying at its lower end a freely swinging horizontal aluminum bar. A gold or platinum weight is fastened at one end of the bar and an equal weight is suspended by a fine wire from the other end of the bar. If a horizontal moment of rotation acts on the weights, the bar tends to rotate and to twist the torsion wire; the latter tends to resist the torsion; and the balance bar comes to rest when the resistance of the wire to torsion is equal to the torque.

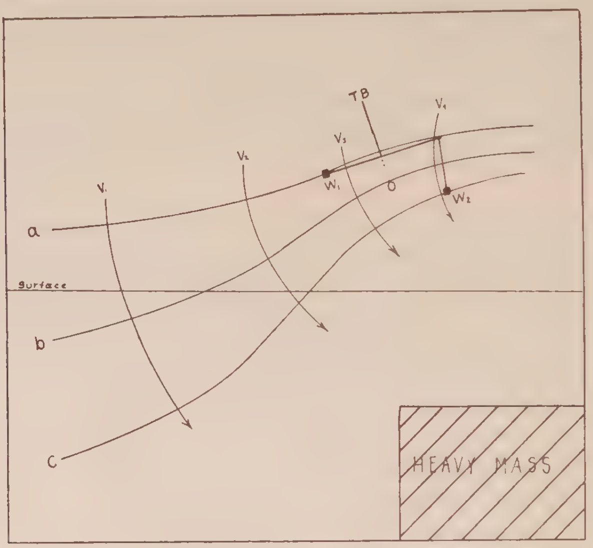


Fig. 2.—Eötvös torsion balance in a vertical section through a gravitational field.

The phase of the curvature of the level surfaces is represented more in detail by Fig. 3 and the phase of the curvature of the vertical is represented by Fig. 5.

V_1 to V_4 = lines of the vertical; a, b, c = level surfaces; TBW_1W_2 = Eötvös torsion balance: W_1, upper weight, W_2, lower weight.

If the rotation of the wire is small, the resistance of the wire is directly proportional to the angular amount of the torsion. The torque necessary to twist the torsion wire through any given angle can be determined in the laboratory and a coefficient of torsion can be calculated for the wire. If an unknown torque causes a rotation of the bar and if the torsion coefficient of the torsion wire is known, the magnitude of the torque can be determined by measuring the angle through which the balance bar has turned. The only practical improvement that has been made in Baron Eötvös' original design of the essential working system of the torsion balance is the Z-shaped balance bar introduced by Schweydar of Berlin. The balance bar has roughly the shape of a "Z" on its side in a vertical plane; and the weights are fastened rigidly to the ends of the upper and

lower arms of the "Z;" the torsion wire is thereby lowered without lowering the center of gravity of the instrument. Various other modifications of the balance have been designed and are being worked on, but no other modification of the essential working system of the balance has been perfected.

Measurement of the Differential Curvature

In its measurement of the differential curvature, the Eötvös torsion balance virtually is a special form of a much older type, the Coulomb

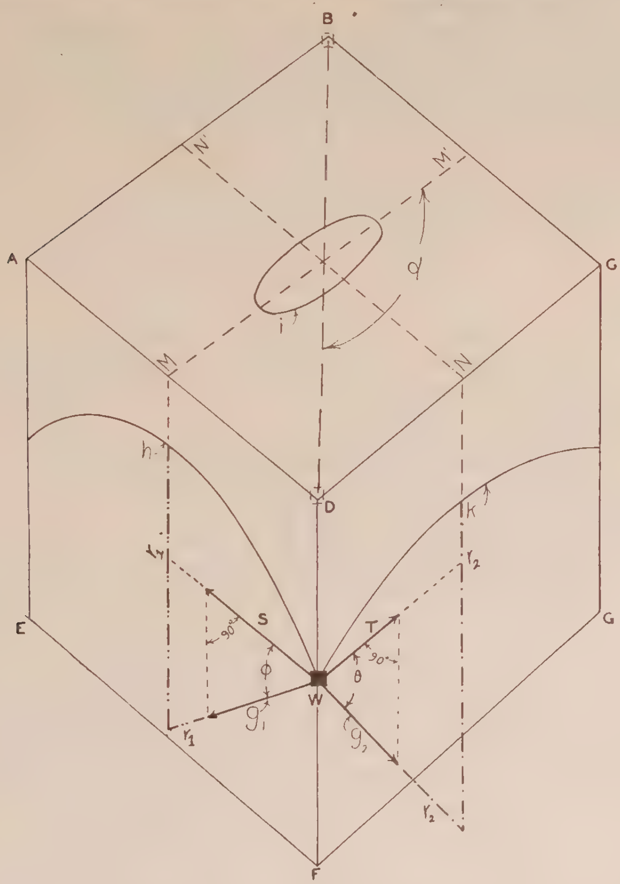


Fig. 3.—Diagrammatic block diagram of the space system around an Eötvös (or Coulomb) torsion balance, representing the curvature of the level surfaces.

balance; in which both weights are at the ends of a straight bar and therefore in the same horizontal plane. The theory of the measurement of the differential curvature is as follows:

In Fig. 3, let i be the trace of a level surface in the horizontal plane $ABCD$; let $DCGF$ be the perpendicular plane parallel to the major axes, MM', of that level surface; and $ADFE$ be the corresponding plane parallel to the minor axes NN'. Let k and h be the respective traces of that level surface in $DCGF$ and $ADFE$, BD be the projection on $ABCD$ of the balance beam, l be the half-length of the balance beam, W be one of the weights of the balance and be in the level surface, hik, m be the mass of the weight, and α be the angle of azimuth of the balance beam. Let r_1 and r_2 be the radii of h and k respectively, and g_1 and g_2 be the projections respectively on $ADFE$ and $DCGF$ of an arrow representing the value of gravity at the position of W.

As the angle between g_1 or g_2 and the vertical is extremely small, and as g_1 (or g_2) $= g$ times the cosine of that extremely small angle, g_1 and g_2 may be used as equal to g. From the relation of the vertical to any

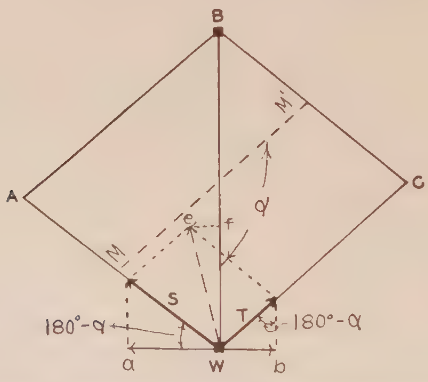


FIG. 4.—REPRESENTATION OF THE SITUATION IN THE HORIZONTAL PLANE THROUGH THE WEIGHT W OF FIG. 3.

level surface, g_1 and g_2 are perpendicular to h and k respectively and therefore are no longer perpendicular to the horizontal plane, and each has a horizontal component, S and T respectively.

Remembering the assumption that the profile of the level surface is the small portion of a very large circle,

$$S = g \cos \phi = g \cdot \frac{MD}{r_1} = g \cdot \frac{1}{r_1} l \sin \alpha \tag{3}$$

$$T = g \cos \theta = g \cdot \frac{ND}{r_2} = g \cdot \frac{1}{r_2} l \cos \alpha \tag{4}$$

Let Fig. 4 represent the horizontal plane through the position of W; aW and bW will then be the components of S and T respectively, acting

perpendicularly to the balance bar, and their difference times (ml) will be the torque F tending to rotate the balance bar. Then,

$$aW = S\cos(180 - \alpha) = g\frac{l}{r_1}\sin\alpha\cos\alpha \tag{5}$$

$$bW = T\sin(180 - \alpha) = g\frac{l}{r_2}\cos\alpha\sin\alpha$$

$$F = (ml)(bW - aW) = (ml)lg\left(\frac{1}{r_2} - \frac{1}{r_1}\right)\frac{1}{2}\sin 2\alpha \tag{6}$$

The magnitude $\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$ is a constant for any given level surface and the magnitude $g\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$ is the R value or "horizontal directive force" (Richtkraft) of Eötvös. Geometrically F is represented by ef. The line eW is the horizontal projection of g at W, and ef is the component of eW perpendicular to the balance bar.

If the axes of reference do not coincide with the major and minor axes of curvature, and if λ is the angle between the two sets of axes and if β is the aximuth of the balance bar in reference to the axes of reference,

$$\alpha = \lambda - \beta \tag{7}$$

$$F = m\frac{l^2}{2}R\sin 2\alpha = m\frac{l^2}{2}R\sin 2(\lambda - \beta) \tag{8a}$$

$$= m\frac{l^2}{2}R(\sin 2\lambda\cos 2\beta - \cos 2\lambda\sin 2\beta) \tag{8b}$$

$$= ml^2\left[\left(\frac{R}{2}\sin 2\lambda\right)\cos 2\beta - \frac{1}{2}(R\cos 2\lambda)\sin 2\beta\right] \tag{8c}$$

$\left(\frac{1}{2}R\sin 2\lambda\right)$ and $(R\cos 2\lambda)$ are constants for any given level surface and for any given set of axes. $(R\cos 2\lambda)$ can be shown geometrically to represent $\left(\frac{1}{r_x} - \frac{1}{r_y}\right)$ where r_x and r_y are the radii of curvature of the trace of the level surface in the vertical planes of the Y and the X axes respectively. There is no simple graphical representation of $\left(\frac{1}{2}R\sin 2\lambda\right)$. By differential geometry it is possible to show[3] that:

$$\frac{1}{2}R\sin 2\lambda = \frac{d^2U}{dxdy} \quad \text{and} \quad -R\cos 2\lambda = \left(\frac{d^2U}{dy^2} - \frac{d^2U}{dx^2}\right)$$

[3] The terms $\frac{d^2U}{dxdz}$, $\frac{d^2U}{dydz}$, $\frac{d^2U}{dxdy}$, and $\left(\frac{d^2U}{dy^2} - \frac{d^2U}{dx^2}\right)$ often shortened in torsion balance work to U_{xz}, U_{yz}, U_{xxy} $(U_{yy} - U_{xx})$ or U_Δ), are the shorthand symbols of calculus for those and two other quantities to be discussed shortly where U is the Newtonian gravitational potential. Any one approaching the theory from the geometrical side and not understanding the calculus of the potential function has to accept them merely as arbitrary symbols.

From those relations:

$$R^2 \sin^2 2\lambda + R^2 \cos^2 2\lambda = 4U_{xy}^2 + U_\Delta^2 \text{ and } R = \sqrt{4U_{xy}^2 + U_\Delta^2} \quad \text{(9a)}$$

$$+\frac{R \sin 2\lambda}{R \cos 2\lambda} = \frac{2\,U_{xy}}{-U_\Delta} \text{ and } \tan 2\lambda = -\frac{2\,U_{xy}}{U_\Delta} \quad \text{(9b)}$$

As β is known, U_{xy} and U_Δ can be calculated from equation (8) if at least two independent values of F have been obtained by observation. The values of R and λ can then be calculated by the formulas of equation (9).

The Eötvös torsion balance becomes a special type of the Coulomb balance if the assumption is made that within the very small area of the working system of the Eötvös torsion balance the curvature of all level surfaces is identical. The upper level surfaces in general will tend to have slightly flatter curvature than the lower level surfaces, but the difference between the curvature of the upper level surfaces and the lower level surfaces is very small compared with the difference between the curvature in the plane of the major axis and in the plane of the minor axis except where the level surfaces approach a plane or spherical form. The vertical difference in the curvature, therefore, can be neglected. If the curvature is the same at all levels of a system, the horizontal directive force is the same at similar points at all levels, and the force acting on the weights of a Coulomb balance is the same within the accuracy of the instruments whether the weights are at the ends of the beam, as in the original Coulomb balance, or one of the weights is suspended perpendicularly below the end of the beam, as in the Eötvös torsion balance.

Measurement of the Gradient

The great contribution made by Baron Eötvös was in showing that if the weights of the Coulomb balance are suspended in two level surfaces, one lower than the other, the balance measures the horizontal gradient of gravity as well as the differential curvature. The use of the Eötvös torsion balance to measure that gradient depends on the two theorems: (1) That if there is a horizontal gradient of gravity, the line of the vertical is curved; and (2) That if the line of the vertical is curved, there is at each weight of the balance a small horizontal component to gravity that is equal to the horizontal gravity gradient; also, as with the measurement of the differential curvature, on the assumption that as the area of the torsion balance system is very small compared to the distance to the attracting system, the variation of gravity is uniform within the torsion balance system.[4]

[4] For experimental proof of the justification of this assumption, see, Karl Mader: Zur Verwendung der Drehwage von Eötvös bei nahen grossen Massen. *Sitzungsb.* d. Akad. d. Wissenschaft in Wien, Math.-Naturw. Klasse Abt. IIa (1924) **133**, Pts. 3 and 4.

The proof of the first theorem is as follows: In Fig. 5, let AA' and BB' be any two level surfaces; PQ and $P'Q'$ be any two lines of the vertical, PP' being their intersections with AA' and QQ' their intersections with BB'; dd' the distances respectively between PQ and $P'Q'$; g be the value of gravity along PQ, and $(g + \Delta g)$ the value of gravity along $P'Q'$. By the nature of the relation of the vertical to all level surfaces, PQ and $P'Q'$ must be perpendicular to AA' and BB'. Then:

1. If PQ and $P'Q'$ are straight lines, AA' must be parallel to BB' and d must equal d'; or
2. If PQ and $P'Q'$ are curved, AA' and BB' are not parallel and d is greater or less than d'.

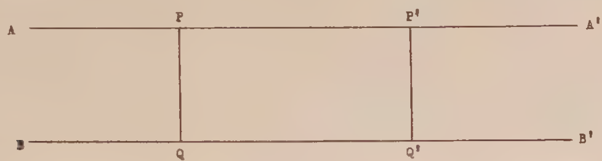


FIG. 5.—LEVEL SURFACES FOR PROOF OF FIRST THEOREM FOR USE OF EÖTVÖS BALANCE IN MEASURING GRADIENT.

The work done, if a unit of mass is moved from Q to P', will be the same whether the mass is moved along $QP - PP'$ or $QQ' - Q'P'$ (no work is done moving a mass at right angles to the vertical) and will be,

From Q to P,	dg	From Q' to P'	$d'(g + \Delta g)$
From P to P'	0	From Q to Q'	0

$$dg = d'(g + \Delta g)$$

If PQ and $P'Q'$ are straight lines, $d = d'$ and $\Delta g = 0$, the value of gravity is the same along PQ and $P'Q'$, and there is no gradient between P and P' and between Q and Q'.

If PQ and $P'Q'$ are curved, $d \gtrless d'$ and then $(g + \Delta g) \lesseqgtr g$, Δg has a definite value, and there is a gradient from P' and Q' to P and Q.

The proof of the second theorem is as follows: In Fig. 6 let

aa' be the level surface through the center of gravity of an Eötvös torsion balance system.
$abca'$ and $aeda'$ be two vertical unit squares (1 cm. on a side),
V be the line of the vertical,
g the value of gravity at a,
$gr(g)$ the gradient from a to a'; then the $(g + gr\ g)$ will be the value of gravity at a'

The vertical will be perpendicular to aa' but on account of its curvature will make an angle, α, with bc and ed.

The work done, if a unit of mass is moved from d to a, is the same whether the path of the movement is via $da'a$ or dea and will be:

From d to a', $g + gr(g)$	From e to a, g
From a' to a, 0	From d to e, $g \cos \alpha$

$$\text{(adding)} \quad g + gr(g) = g + g \cos \alpha$$
$$\text{and} \quad gr(g) = g \cos \alpha$$

($g \cos \alpha$) is the small horizontal force, which is set up because, above or below the level of the center of gravity, the vertical is not perpendicular to the horizontal and gravity therefore has the small horizontal component ($g \cos \alpha$). This small horizontal force acting on one of the weights

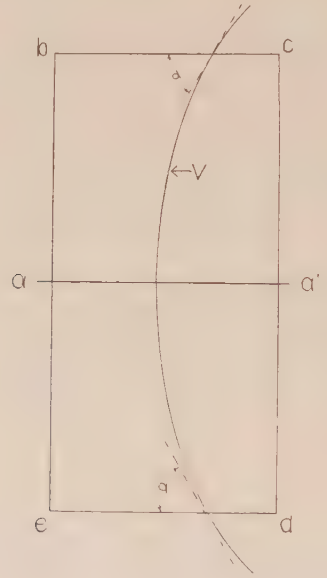


FIG. 6.—VERTICAL SECTION THROUGH AN EÖTVÖS TORSION BALANCE SYSTEM REPRESENTING THE CURVATURE OF THE VERTICAL.

of a torsion balance will tend to rotate the balance. As the situation in $abca'$ is symmetrically reversed from that in $aeda'$, the force acting on the upper weight of an Eötvös torsion balance is equal to that acting on the lower weight, and the same in rotational effect, but opposite in absolute direction. As α is very close to 90°, $\cos h{\cdot}\alpha = h \cos \alpha$ and as h is the vertical distance between the weights, that small horizontal force acting on each weight $= \frac{h}{2} g \cos \alpha$. As $g \cos \alpha = gr(g)$, the torsion balance is able to measure the gravity gradient.

The actual magnitude of the force F acting on the weights of the balance in any particular position is proportional to the maximum gradient ($Gr\, g$) and to the sine of the angular difference between the azimuths

of the balance bar and of the gradient. If the azimuth of the balance bar referred to the direction of the gradient as the axis of reference is $+\beta$, then

$$F = -mlh(Gr\ g) \sin \beta. \tag{10}$$

If the axis of reference is transferred to north and if, in the derivation of the transformation equations, the gradient is considered to be in the northeast quadrant, and if α = the new azimuth of the balance and if γ = the azimuth of the gradient:

$$\beta = \alpha - \gamma \text{ and } F = -mlh\,[(Gr\ g)\ (\sin \alpha \cos \gamma - \cos \alpha \sin \gamma)] \tag{11}$$

The quantities $(Gr\ g) \cos \gamma$ and $(Gr\ g) \sin \gamma$ are constants for any particular gradient and represent the north-south and the east-west components respectively of the gradient. In the notation of the calculus of the theory of gravitation

$(Gr\ g) \cos \gamma = \dfrac{\partial U}{\partial x \partial z}$ and $(Gr\ g) \sin \gamma = \dfrac{\partial U}{\partial y \partial z}$ or in the abbreviated notation in use in the torsion balance work, U_{xz} and U_{yz} respectively. Equation (11) then becomes:

$$F = -mlh\ U_{xz} \sin \alpha + mlh\ U_{yz} \cos \alpha \tag{12}$$

CALCULATION OF THE DIFFERENTIAL CURVATURE AND THE GRADIENT

The balance beam of the torsion balance in any particular position of the balance is affected by a torque which is the vectorial sum of two torques, the one the function of R, and the other the function of the horizontal component of gravity that is equal to the gradient. The value of the total torque, F, acting on the balance system is given by the following equation:

$$N = \frac{K}{2} U_{\Delta} \sin 2\alpha + KU_{xy} \cos 2\alpha - MU_{xz} \sin \alpha + MU_{yz} \cos \alpha \tag{13}$$

Where: K is an instrumental constant depending on the mass of the weights, the square of the half length of the balance bar, and on the reciprocal of the torsion constant of the torsion wire; and M is an instrumental constant depending on the mass of the weights, the half length of the balance bar, the vertical distance between the weights, and the reciprocal of the torsion constant of the torsion wire; and τ is the torsion constant of the torsion wire in scale divisions of the instrument; and

N = angular rotation of the balance in scale divisions; and $F = N\tau$

The torque F causes an angular rotation of the balance bar until counterbalanced by the resistance of the torsion wire. The angle through which the beam turns is determined by reading the deflection in a mirror on the stem of the balance of a fixed point without the rotating system.

If four values for N are determined by observation, the equations can then be solved for the values of the four unknowns, U_{xz}, U_{yz}, U_{xy}, and

U_Δ. From the nature of the torsion balance, however, the zero point of the instrument enters the equation as a fifth unknown in the case of the obsolete, single-balance type of instrument and then at least five determinations of N are necessary. As the double-balance modern type of instrument consists of two complete independent balance systems mounted closely side by side but with a difference of orientation of 180°, the zero point of each system enters the equation as an unknown; and therefore with the modern instrument at least six independent determinations of N are necessary. As two determinations of N, one by each system in the instrument, are made in each position, the observation of the modern instrument in three positions is sufficient mathematically for the determination of all six variables, but as a matter fact, at least four and usually five positions are observed, the last two a repetition of the first two for the purpose of a check on and an increase in the accuracy of the observations.

The solution of those equations for the gradient, the curvature, and the zero points of the two systems of the instrument, in practice, is replaced by the use of simple formulas. These very commonly are used in the form of a table, the fundamental form of which is somewhat as follows:

$n_0' = \frac{1}{3}(n_1' + n_2' + n_3') =$				$n_0'' = \frac{1}{3}(n_1'' + n_2'' + n_3'') =$		
I	II	III	IV	$R_1' = (n_1' - n_0') =$	$R_2' = (n_2' - n_0') =$	$R_3' = (n_3' - n_0') =$
$(R_2' - R_3')$	$R_2' + R_3'$	$R_2'' - R_3''$	$R_2'' + R_3''$	$R_1'' = (n_1'' - n_0'') =$	$R_2'' = (n_2'' - n_0'') =$	$R_3'' = (n_3'' - n_0'') =$
				I − III =		II − IV =
$V = 2R_3' + R_1' =$		$VI = 2R_3'' + R_1'') =$		VI − V =		$R_1 - R_1'' =$
$VII = 2R_2' + R_1' =$		$VIII = 2R_2'' + R_1'' =$		VIII − VII =		
$U_{xz} = B \cdot IX =$		$U_{yz} = A \cdot X =$		IX = Mean =		X = Mean
$Gr(g) = + \sqrt{U_{xz}^2 + U_{yz}^2} =$				Azimuth $Gr(g) = \arctan \frac{U_{yz}}{U_{xz}}$		

where n_0' and n_0'' are the zero points respectively of the two systems of the instrument; n_1', n_2', n_3', and n_1'', n_2'', n_3'' are the respective scale reading of the two systems read in the 0°, 120°, 240° azimuths measured clockwise from north; and A and B are instrumental constants. The Roman numerals indicate the sequence of operations.

The curvature values are calculated by similar extension of this table.

This and the similar tables lead through a simple, indicated routine to the north-south and east-west component of the gradient, and to U_Δ and U_{xy}. The calculations involved are the simplest operations of addition, subtraction and multiplication. The last step of obtaining the total gradient and its azimuth, or R and its azimuth, is then performed easily by entering simple tables or graphs respectively with U_{xz} and U_{yz} or U_Δ and U_{xy}.

USE OF THE EÖTVÖS BALANCE IN GEOLOGY

The values recorded by the instrument respectively for the gradient and the differential curvature at any station are the vectorial sums of the effects of all the irregularities of distribution of mass around the instrument that are of sufficient size and proximity to the instrument sensibly to affect it. Those effects, genetically, are of three types: the topographic, the planetary ,and the geologic.

Topographic Anomalies

The topographic anomalies are caused by the irregularities in the distribution of mass due to the hills and valleys, mounds and depressions

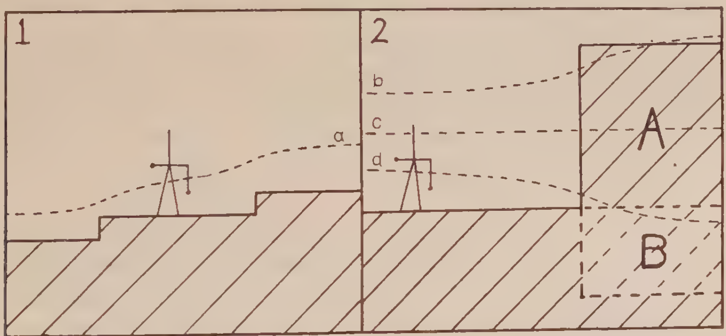


FIG. 7.—DIAGRAMMATIC REPRESENTATION OF THE VARIATION OF GRAVITY IN THE VICINITY OF AN EÖTVÖS TORSION BALANCE.

1. Due to a depression of the surface on the left and a slight rise of the surface on the right of the balance.
2. Due to a hill or cliff, *A*, rising above the level of the instrument and a depression, *B*, which would produce identically the same gradient at the instrument as *A*.

a, b, c, d are profiles of the variation of the value of gravity.

surrounding the instrument, rising above or sinking below the horizontal plane through the foot of the instrument. If the terrane in which the instrument is set up is a flat horizontal plain, there is no gradient or curvature effect; but if there is a depression below that plain, gravity is less than normal above the depression, a gradient is set up away from the depression and the level surfaces are warped down, concavely, above the depression (Fig. 7–1). If the surface rises above the plain, gravity is greater than normal above the elevation, a gradient is set up toward the elevation, and the level surfaces are warped up, convexly, above it. But if a cliff or hill rises more than a meter above the center of gravity of the instrument, the mass of the hill or cliff above the level of the center of gravity of the instrument exerts an attraction upward and therefore tends slightly to counteract gravity, to set up a gradient away from the cliff or hill, and to warp the level surfaces down into the hill or cliff (Fig. 7–2).

Corrections for the respective topographic anomalies of the gradient and differential curvature are applied to the values observed at each station. The level of the ground around the instrument is determined with an alidade or transit at a definite net of points, most commonly at distances of 1.5, 3, 5, 10, 20, 50, 100, and very rarely 200, 500, and 1000 m. from the instrument on azimuths of 0°, 45°, 90°, 135°, 180°, 225°, 270°, 315°. The density of the soil is measured at two or three points near the instrument in the more accurate work, but for most purposes sufficient accuracy is obtained by using a mean density for the area, and furthermore, in many places, indeterminable irregularities in the zone of weathering make unnecessary great refinement in the determination of the soil density. The calculation of the respective topographic effects of gradient and differential curvature is made in practice by one of several simple rule of thumb formulas which have been derived and cast in simple tabular form. The elevations obtained in the levelling are entered in the proper spaces in the table; and a few operations of simple addition and subtraction and of multiplication by coefficients give the total anomaly due to the topography. That effect is then subtracted algebraically from the respective observed value.

The effect of a small mass on the gradient observed varies both with the horizontal and vertical position of the mass in reference to the instrument. There is no effect on the gradient, if a small mass is on a level with or perpendicularly below the instrument; the effect is at a maximum when the mass is 40° to 60° above or below the horizontal plane through the center of gravity of the instrument. The effect of irregularities of mass within 2° of that horizontal plane are negligible except in very refined work and within ±5° in most work. The effect also varies inversely as the cube of the distance of the mass from the instrument. In regions of low to moderate relief, in most torsion balance work it is possible to choose station sites so that all large irregularities of topography are within ±5° of the horizontal plane through the center of gravity of the instrument, are moderately distant from the instrument, and therefore can be neglected. If the instrument is set up on an extensive hill sloping uniformly 1° the effect of the slope on the gradient amounts to 14 Eötvös units, and for slightly steeper slopes increases directly as the number of degrees of slope. Unless an impracticable number of drill holes are sunk for samples for the determination of the density of the soil and the subsoil, the value used may be in error to the amount of 10 per cent. If a surficial zone of weathering is present, the surface of the unweathered rock acts like another topographic surface, but as it conforms only very roughly to the surface of the ground, its effect on the gradient and the differential curvature is only very roughly proportional to the effect of the actual surface of the ground, and in practice can be determined only by an impracticable number of core holes through the zone of weathering. As

the gradients due to geologic structure commonly are of the order of 7 to 25 E, it is advisable to choose station sites where the inclination of the ground is less than 1° and preferably less than 0.5°. In regions of low or moderate relief, station sites usually can be chosen to have slopes of less than 1°, except where stations have to be close together. In regions of rugged or mountainous relief, usually it is difficult to pick station sites where the slope of the ground is less than 3° and where hills do not arise more than 5° above or valleys extend more than 5° below the horizontal plane of the center of gravity of the instrument and where those hills and valleys are not relatively near the instrument. If a reasonably accurate topographic map is available, a value for the effect of the topography beyond 1000 m. can be obtained by rather laborious and tedious calculation, but it and the value for the effects within the 1000-m. zone of leveling may easily be very much in error. In rugged country, therefore, it is not practicable to use the torsion balance for surveys of gravity gradients of structure unless the gradients caused by such structures are very large. The terrane correction formulas now in common use for the zone within 100 m. are not valid for elevations of more than 0.5 m. above the level of the base of the instrument.

The effect of a small mass on the differential curvature value observed varies with the horizontal and vertical position of the mass in reference to the instrument. The effect is proportional to the effect of the attraction of the mass in deflecting the vertical, and is at a maximum when the small mass is in the horizontal plane through the center of gravity of the instrument and at a minimum when the small mass is vertically above or below the center of gravity of the instrument. The effect varies inversely as the cube of the distance of the mass from the instrument. As the effect is very nearly at its maximum value within 10° of the horizontal plane of the center of gravity of the instrument, it is difficult even in regions of moderate relief to choose stations so that the effect of the topography will not be serious. With increasing ruggedness of topography, the difficulty of eliminating the effects of the topography increases very much more rapidly with the differential curvature than with the gradient, and the differential curvature usually is considerably more erratic than the gradient.

Planetary Effects on Gradient and Differential Curvature

The planetary effects are due to the fact that the earth is a rotating spheroid flattened along the polar axes, instead of a perfect sphere at rest. On account of the flattening and the rotation of the earth, the value of gravity at sea level increases from the equator to the poles and therefore there is a northward gradient in the northern hemisphere and a southward gradient in the southern hemisphere. As at any latitude, there is

no variation of gravity eastward or westward, there is no east or west component comparable to that northward and southward gradient. The level surfaces likewise are distorted by the flattening and rotation of the earth. If the earth were a homogeneous sphere at rest, all level surfaces would be spheres and the radius of curvature of the level surfaces would be the same in all directions, but on account of the flattening and rotation of the earth there is a change of the radius of curvature in meridional planes, and U_{Δ} therefore comes to have a value; but as our axes of reference, N-S and E-W are parallel to the major and minor axes of the level surfaces, $U_{xy} = 0$. The values of the "normal" northward gradient and the "normal" differential curvature, as the planetary effects are called in torsion balance work, vary with latitude and can be calculated mathematically when appropriate assumptions are made for the form and rotation of the earth. A table of these values usually is furnished to each observer and he has only to enter his table with the approximate latitude of his station in order to get values respectively for the normal northward gradient and the normal curvature. Those values then are subtracted from the respective observed values.

The normal effects and the terrane effects normally are subtracted in the calculation of the observed values at each station and in most torsion balance work, the term "observed gradient (or differential curvature)" is used to refer to the gradient (or differential curvature) produced by the geologic anomalies.

Geologic Anomalies

The geologic anomalies are those produced by the irregular distribution of mass in the upper few miles of the earth's crust and range from effects produced by irregularities of mass of subcontinental size to those produced by small boulders close to the instrument. These anomalies may be divided somewhat arbitrarily, on the basis of the size of the geologic structure producing them, into five orders of magnitude: first order, those due to masses of the size of large mountains or large geosynclines, such as the West Texas Permian basin; second order, those due to masses of the size of small mountain ranges such as the Amarillo buried granite ridges; third order, those due to masses of the size of the average anticline; fourth order, those due to minor geologic structure; fifth order, those due to small surficial irregularities of mass, such as glacial boulders buried in the subsoil rather close to the instrument.

The surficial irregularities of mass are due to a wide variety of geologic causes. In glaciated regions, a buried glacial boulder, the rapid variation possible in fluvioglacial deposits, the contrast between till and fluvioglacial material, or the irregularities in the surface of the bed rock buried under a thin mantle of glacial deposits, may cause abrupt changes in

specific gravity within a short distance of the instrument and thereby cause very abnormal gradient and differential curvature values to be registered by the torsion balance. Where a river valley has been filled with alluvium, a very large gradient away from the side of the valley, and very abnormal curvature values, may be set up over a steeply inclined contact of alluvium and bed rock at the edge of the valley. If the alluvium and bed rock have sufficiently contrasting specific gravities, the whole bed-rock contour of the valley floor will sensibly affect the gradient and curvature values at the surface and in such cases the torsion balance can be used to map the position and conformation of a buried valley. In some formations, irregularities of cementation, heterogeneity in the character of the formation, intercalated beds of limestone or dense sandstone, may cause a sharp change in density close to the instrument and give rise to abnormal gradient and curvature values. In the area of the Wilcox formation of Texas for example, both the gradient and curvature not uncommonly are distinctly erratic. In the Gulf Coast, a sandy pimple mound rising through a clayey soil may cause a sufficiently abrupt change in density to give abnormal gradient and curvature values. A filled-in cellar hole or other artificial excavation or small faults extending to the surface may produce sufficiently large surficial irregularities of mass to produce similar effects.

The abnormal gradient and differential curvature values caused by the surficial irregularities of mass are limited to a very small area and in most cases will not be registered if the station site is moved a few hundred feet. An abnormal gradient or differential curvature value very often will be suspected by an experienced interpreter of torsion balance results. As the surficial irregularities of mass are hidden beneath the surface and could be detected and mapped only by an impracticable number of core holes, it is impossible to calculate their effects as the effects of irregularities of topography can be calculated. The effects of the buried bed-rock-alluvium contact at the edge of a valley and the outcropping beds of limestone, hard sandstone or other dense rocks can be avoided in many places if the observer watches the geology and chooses the station site with care. The technique of minimizing those effects, where they can not be avoided, varies with the situation. On reconnaissance or on profile surveys, two, three, or in some cases four or five stations may be occupied a few hundred feet from and around a common point and the mean or median value used as the value of the gradient (or differential curvature) at that point. Where a considerable net of stations would be occupied in any case, it may be preferable to double the number of stations and then make a least square adjustment of the values.

The geologic anomalies of the second, and third and fourth order, give the geologist clues to geologic structure. But the gradients and differential curvature observed at any point will be the sum of the

sensible effects of all those orders. An anomaly is the same order of magnitude as the structure producing it. If a torsion balance survey is extensive enough and there is a sufficient net of stations, a careful analysis of the results may allow a separation of the anomalies of different orders (see discussion of the Fox oil field, page 456).

Geologic Interpretation of Torsion Balance Results

The use of torsion balance results as a guide to geologic structure depends on two assumptions:

1. That the form, depth, and relative density of a mass causing a gradient and curvature anomaly can be determined from the distribution of the gradient and curvature values in the anomaly observed at the surface.
2. That there is a direct connection between geologic structure and the anomalous distribution of mass.

The accuracy of these assumptions varies with the situation and ranges from a high order to a low order. The accuracy of the interpretation of geologic structure from torsion balance results depends on the extent to which those assumptions hold true—and on the skill of the interpreter.

Any particular body has a characteristic gradient and curvature pattern which depends on the form and size of the body and its depth below the surface. The amplitude of the gradient and the differential curvature values depends on the difference between the respective densities of the body and the surrounding medium. The gradient and curvature profiles for four common simple types of geologic oil structure are shown in Fig. 8. The structures (with the exception of that in Fig. 8e) are supposed to be infinitely long with the respective transverse cross-sections shown in the figures. The lower portion of each figure gives the structural cross-section drawn with vertical and horizontal scales equal. The upper profile gives the curve of the variation of the value of R, the differential curvature, above the structure; and the lower profile gives the variation of the gradient above the structure. On the differential curvature profile, each point on the curve above the zero line would be represented on a map by an R line of equal magnitude parallel to the line of the section, and each point on the curve below the zero line would be represented by an R line at right angles to the line of the section. On the gradient profile, each point on the curve above the zero line would be represented on a map by a gradient arrow flying to the right and each point on the curve below the zero line would be represented by an arrow flying to the left.

A vertical fault cutting a shale section of 3000 to 3500 ft. thick, overlying a very thick massive limestone section, is represented in Fig. 8a.

The fault has a throw of 300 ft. at the surface and at 1000 ft.; a throw of 400 ft. at 2000 ft., and a throw of 500 ft. at 3000 ft. The shale section is supposed to be split into three formations with specific gravities, increasing downward, of 2.15, 2.20, and 2.30 respectively. The specific gravity of the limestone was assumed to be 2.70. Both the gradient and differential curvature are symmetrically disposed in reference to the fault plane: the differential curvature is zero and the gradient is at a maximum over the fault; on the left of the fault plane the R lines are perpendicular to the fault line, and on the right are parallel to it; the gradient arrows all fly toward the right but increase to a maximum over the fault and then decrease symmetrically.

A fault with a dip of 30° cutting the same stratigraphic section as in the preceding case is represented by Fig. 8b. The gradient and differential curvature profiles are very similar to those of the preceding case but are not symmetrical, the gradient maximum and the numerical maxima of the curvature are slightly less than the corresponding maxima of the preceding case, and the zero of the differential curvature and the maximum of the gradient do not lie above the surface trace of the fault but over the upper part of the fault in the limestone basement, where the throw and the density difference are greatest. If the specific gravity had a smooth curve of increase downward, especially in the upper shale formation, both the gradient and differential curvature profiles would be very considerably modified in regard to the position of the zero points and maxima and the magnitude of the maxima.

An anticlinal or "buried" ridge of limestone overlaid by shale and with a relief of 375 ft. is represented by Fig. 8c. The top of the ridge is supposed to be at a depth of 1500 ft. and the normal level of the top of the limestone is supposed to be at 1875 ft. The ridge is supposed to be symmetrical about its crest line. The gradient and differential curvature profiles are symmetrically disposed about the vertical axial plane of the ridge; the gradient every where is toward the crest of the ridge—that is, the gradient arrows to the left of the crest would fly to the right, and on the right of the crest, they would fly to the left; the gradient is zero above the crest of the ridge; and the maxima are over the flanks. The R lines off the structure and over the foot of its flanks are toward the structure and reach a maximum over the lower part of each flank of the structure; over most of the structure the R lines are parallel to the crest of the structure and are at a maximum over the crest of the ridge. The relative magnitude and the absolute magnitude of the numerical value of the maxima of the curvature and the form of the central maximum vary very considerably with variation in the cross-section of the ridge.

A ridge similar to the preceding but with the right flank much steeper is represented by Fig. 8d. The gradient and differential curvature profiles are very similar to the corresponding curves of the preceding case

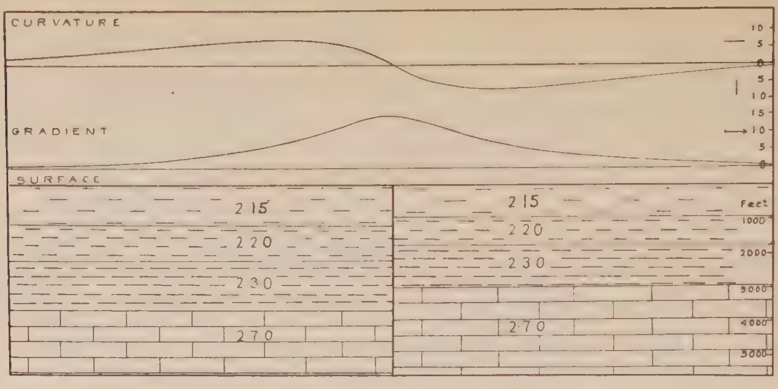


FIG. 8.—COMMON SIMPLE TYPES OF GEOLOGIC STRUCTURE AND THEIR GRADIENT AND DIFFERENTIAL CURVATURE PROFILES.
a. Vertical fault.

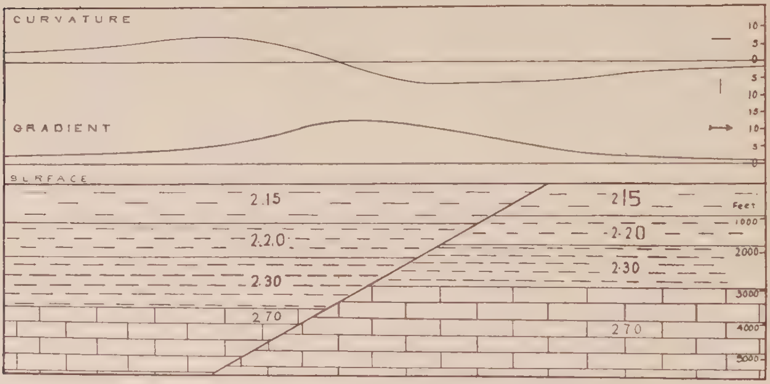


FIG. 8.—b. 30° fault.

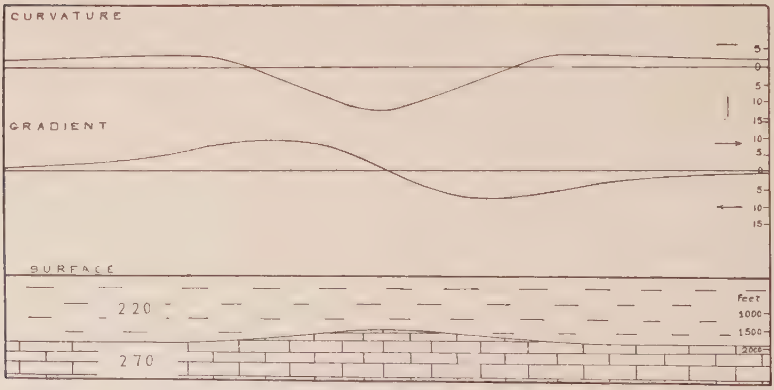


FIG. 8.—c. Symmetrical infinite (anticlinal) ridge.

but are asymmetrically warped by the asymmetry of the ridge. It should be noted that the zero point of the gradient and the maximum value of the differential curvature do not lie immediately above the point of the ridge that is structurally highest but are shifted slightly to the left down the gentler slope of the ridge. The zero point where the gradient changes direction marks the point of maximum gravity and it should be

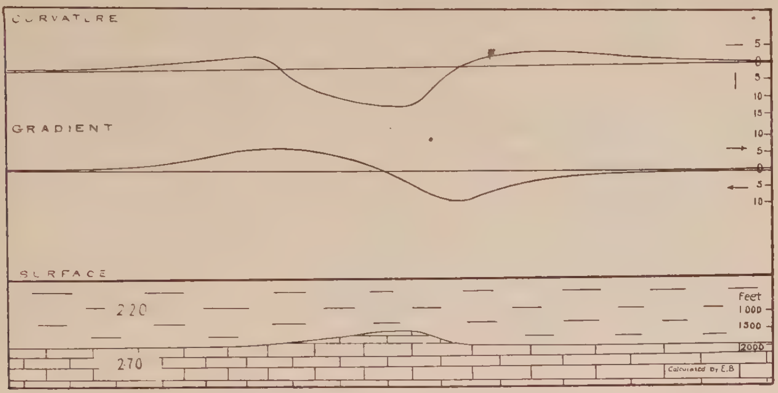


FIG. 8.—d. Asymmetrical infinite (anticlinal) ridge.

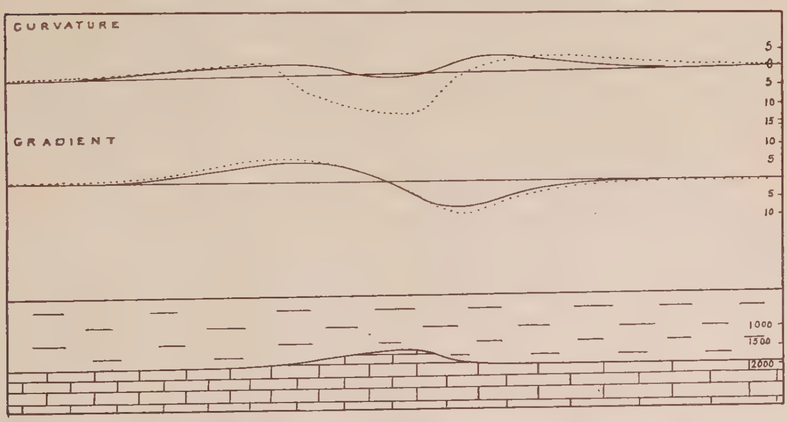


FIG. 8.—e. Ridge of the same cross-section but with a length 2 times the depth at each level. The dotted gradient and curvature profiles represent the profiles for the infinite case.

noticed that in asymmetric structures the center of the anomalous gravity high due to the structure is not exactly over the highest point of the structure; the amount of the shift of the gravity high down the gentler slope of the structure is proportional to the difference in the slope of the two flanks. In unknown structures, the shift of the gravity high due to the asymmetry of the structure can be recognized if a moderately detailed transverse profile has been run but its magnitude can not be determined

unless quantitative calculations are made. If only a scattered net of stations is available, the presence of the asymmetry may not be recognized easily, and a location made from the torsion balance data may be off just enough to miss the crest. As the gradient and differential curvature profiles of Fig. 8c show, the gradient profile is slightly modified and the differential curvature profile very considerably modified if the ridge is finite rather than infinite in length.

The dimensions of the structure sections of Fig. 8 may be taken as anything without affecting the gradient and curvature profiles. In order that the reader may visualize the dimensions readily, the unit of spatial measurement has been labelled "feet." The unit of spatial measurement does not enter the formulas for the gradient and the differential curvature, and the "feet" of the structural sections could be replaced by inches, centimeters, kilometers, or miles without affecting the gradient and curvature profiles. If the profile of gravity has been plotted, it would be affected by the change of unit of spatial measurement, as the difference of gravity between two points is the product of the gradient and the distance, and the unit of spatial measurement is involved in the distance.

Relation of Subsurface Bodies to Gradient and Differential Curvature

Any given simple body at a given depth below the surface produces a unique[5] distribution of gradient and differential curvature values at the surface; and, within the limits of practical but not mathematical accuracy any given distribution of gradients and curvature values can be produced by only one body definite in form, relative density[6] and position. If the form, relative density, and depth of a body below the surface are known, the gradients and curvature at all points can be obtained by calculations which rapidly increase in tediousness with the deviation of the form of the body from simple geometric forms, but by splitting the body up into a series of rectangular plates, the gradients and curvature values can be calculated for the most irregularly shaped body. The converse holds true only to a very limited extent. If the distribution of gradients and curvature values caused by a body are completely known, it is practically impossible accurately to determine the shape, position, and density of the body. If the gradient profile produced by an infinite rectangular block is known fairly completely, it is possible to calculate the depth to the top and the bottom of the block, the position and the relative density of the block with a good degree of approximation. For simple tabular bodies with cross-sections similar to those of Fig. 8, fair approximations can be obtained for the shape, position and relative density of the body

[5] See author's discussion on page 479.

[6] The relative density of a mass = the density of the mass minus the density of the surrounding medium.

by only moderately tedious calculations based in part on methods of the trial and error, and with only slightly greater tediousness for a block of finite length compared to that for a similar block of infinite length.

If a body producing a given distribution of gradients and differential curvature values is not homogeneous with a simple geometrical form, is extremely irregular in form, or is a composite of several simple homogeneous bodies of differing densities, or if it has considerable variation of density, it is much more tedious and in some cases impossible in practice to calculate back and obtain the data of the body.

If a geologic body is homogeneous in density, if it has a relatively simple geometrical form, if the surrounding country rock is homogeneous in density and if a sufficiently close net of stations has been occupied, it is possible to calculate back from the observed distribution of gradient and curvature values and obtain approximately the form, position, dimensions and density of the body. If a geologic body is known to be composed of two rather simple homogeneous bodies, the form, position, dimensions, and densities of the two bodies in some cases can be determined from the calculations.

If a geologic body is irregular in form and density, it is impossible to calculate its form, position, dimensions and density, but it is possible to calculate an imaginary simple homogeneous body that most nearly approximates the given irregular body. The significance of the imaginary body depends on the closeness of its approximation to the geologic body; and that is difficult of evaluation unless the geologic probabilities of the situation are known and are very limited.

Accuracy of Quantitative Calculations

All the results of such quantitative calculations, even of the most certain, are subject to varying degrees of uncertainty. Bodies differing considerably in form, position, and density may give respective distributions of gradient and curvature values that differ only by a very few Eötvös units. Except where most painstaking precautions are taken in exceptionally favorable areas, the observed gradient and curvature values may be in error by one, two, three, or even more Eötvös units on account of surficial irregularities of density and of inexact correction for the effect of topography. Furthermore, it is usually impracticable to put stations close enough together to get an exact picture of the distribution of gradient and differential curvature values. Although the general plan of the distribution of gradient and differential curvature values will be known, the detailed plan will be sufficiently indefinite so that a number of detailed profiles of equal probability may be chosen. Corresponding to each profile there will be a definite body, which will differ slightly in form, shape, dimensions, position and density from the bodies corresponding to the other of those plans. Any one of these bodies will have as great a

probability, mathematically, as the others, but some of them may not be geologically possible or as probable as the others.

A common result of such calculations is to obtain a suite of bodies that correspond to a suite of density differences, which have a distinct family resemblance to each other but differ essentially only in depth and vertical relief. The suite of bodies and corresponding suite of densities in most places have finite limits and although the calculations do not lead to a definite determination of the structure, they may define it within certain limits. In the calculation of the form of the Ordovician-metamorphic basement along a profile that had two highs, it was impossible to make any reasonable assumptions that would allow the depth to the top of the Ordovician at the crest of the first high to be greater than 1550 ft. below the surface or on the second high to be closer than 2300 ft. to the surface. A test subsequently drilled below the crest of the first high encountered the Ordovician at a depth of 1453 ft. below the surface; and a test previously drilled below the crest of the second high had found the top of the Ordovician at a depth of 2665 ft. below the surface. Calculations of that type in general should give the position of the crest of the structure with much more definiteness than the other data regarding it. If the calculations are based on a survey in which the stations were not spaced sufficiently close together completely to define the respective gradient and curvature profiles, additional uncertainty is added to the results of the calculations to approximately the same extent as the assumptions have to be made in regard to the variation of the gradient and the differential curvature between the stations at which they have been observed.

Determining Geologic Structure with Torsion Balance

As the torsion balance measures only the effect of irregular distribution of mass, the use of torsion balance results to determine geologic structure, of necessity, is dependent on the assumption of a concordance between the geologic structure and the distribution of mass. The degree of the concordance depends on the type of the geologic structure and the particular situation. Concordance relatively complete for practical purposes holds directly in many cases of salt domes, volcanic plugs, laccoliths, dikes, batholiths, intruded into homogeneous country rock, and in the case of a few types of ore deposits in homogeneous country rock. The concordance becomes increasingly incomplete as the density of the country rock becomes heterogeneous.

Indirect partial concordance prevails in most of the anticlinal, domed, and fault structures in which the oil geologist is interested. The parallelism between the structure and the distribution of mass is due to the fact that normally there is little variation in the same bed, although there may be vertical variation between beds. Deformation of the beds by

folding or faulting therefore involves concomitant deformation of the distribution of density. There is a general tendency for increasing consolidation of sediments with increasing geologic age—except in the case of limestones, anhydrite, and salt—and therefore in general for increasing density with increasing depth. The igneous and metamorphic rocks, and the early Paleozoic sediments forming the buried mountain ranges of the Mid-Continent area in general have a higher density than the overlying sediments. A general tendency therefore prevails for the association of gravity highs with structural highs; but the reverse association is possible, and will be given by a massive salt series lying not too deep below the surface in otherwise homogeneous sediments or by a massive limestone at the surface underlaid by a massive sand-shale section.

The intensity of the folding in oil structures and therefore of the deformation of the uniform distribution of density tends to increase downward. In the case of the buried-ridge type of structure, there is a tendency for the influence of the basement complex to be predominant in the gradient and differential curvature values, yet in all cases the deformation of the uniform distribution of density tends to parallel the structural deformation, and the intensity of the first tends to be of the same relative magnitude as the second.

Types of Geologic Interpretation of Torsion Balance Results

The geologic interpretation of torsion balance results may be of two types: quantitative and qualitative. If the geologic possibilities and probabilities are known well enough to show that a body possesses a high degree of direct concordance between itself and the distribution of density, and has a rather simple geometrical form it is possible to calculate backward from the gradient and curvature values and determine the form, dimensions, and density of the body, and thereby make a "quantitative" or approximately quantitative interpretation of the torsion balance results. A geophysical geologist who is familiar with the mathematical theory of interpretation, who has had considerable practicable experience and who has a fair knowledge of the geology of the area, by studying the distribution of the gradients and curvature values, the maximum and minimum values, the position of the reversals and changes in direction of the gradients and curvature values, and the rate and character of the variation of the gradient and curvature values and directions, will find in most (but not all) torsion balance surveys a fair general picture of the structural conditions of the area surveyed. He will see indications of faulting or folding; of the position of the major faults and folds; of the steepness of the dip of a fault and of the approximate size of a fold; of whether the flanks are steep or gentle; of the presence of any abnormally massive bed or formation of abnormally high or low density, and of its approximate depth.

This determination of the general and approximate form, size, position and density of the anomalous mass is what the writer calls "qualitative interpretation." In simple structural situations, a competent interpreter can make a good qualitative interpretation of the situation by simple inspection of the results; but where the structural situation is complicated or the gradients and curvature values small, refined analysis may be necessary. A map showing the contours of equal gravity (isogams) which will resemble a structure contour map, and which may be used in many respects as a structure contour map, can be constructed if the net of stations covers the area sufficiently (Figs. 12 and 13). There is, however, no constant factor between the isogams and the structure contours, even in a single structure, and the point of maximum intensity of the anomalous gravity does not lie necessarily above the crest of the geologic structure. If the gradient and differential curvature produced by larger scale structure obscure those of smaller structure in which the geologist is more particularly interested, in many cases the gradient and differential curvature anomalies of the latter can be separated out by the proper analysis (Figs. 13a, b, c).

Although satisfactory in a very great many cases, the "primary net" method of least-square adjustment of the gravity difference between stations used by Eötvös is not quite accurate enough in the more refined analysis and it is necessary to make a simultaneous adjustment of all stations. As such a simultaneous adjustment involves the solution of as many linear equations as there are stations, the calculations become extremely tedious, unless approximate formulas are available to the calculator. Anomalies obtained by such refined analyses are somewhat treacherous and should be used only by an experienced interpreter who is aware of the dangers in their use.

Results of Torsion Balance Work in the United States

Three definitely, and possibly five, salt domes have been discovered by the torsion balance in the Gulf Coast salt dome district of southeast Texas and southwest Louisiana within the three years 1924–26 in contrast to the discovery of only five new domes by geology in the preceding fifteen years.

Discovery of the Nash Salt Dome

The first of the salt domes—the first oil structure—discovered in this country by geophysical instruments was the Nash salt dome, discovered in the early Spring of 1924 by the Rycade Oil Corpn. This had been considered a faintly suspicious prospect; it was rated by the writer as having about five chances out of one hundred of being a salt dome, but apparently rated not so favorably by the geological departments of some other companies. It was blocked by the Rycade Oil Corpn. and the

reconnaissance torsion balance survey, of which the results are shown in Fig. 9, was made; this indicated the presence of a salt dome with a probability of better than ninety-nine chances out of one hundred. The dashed lines in the figure show the limits of the dome as they were interpreted by the writer from the torsion balance results entirely in advance of drilling;

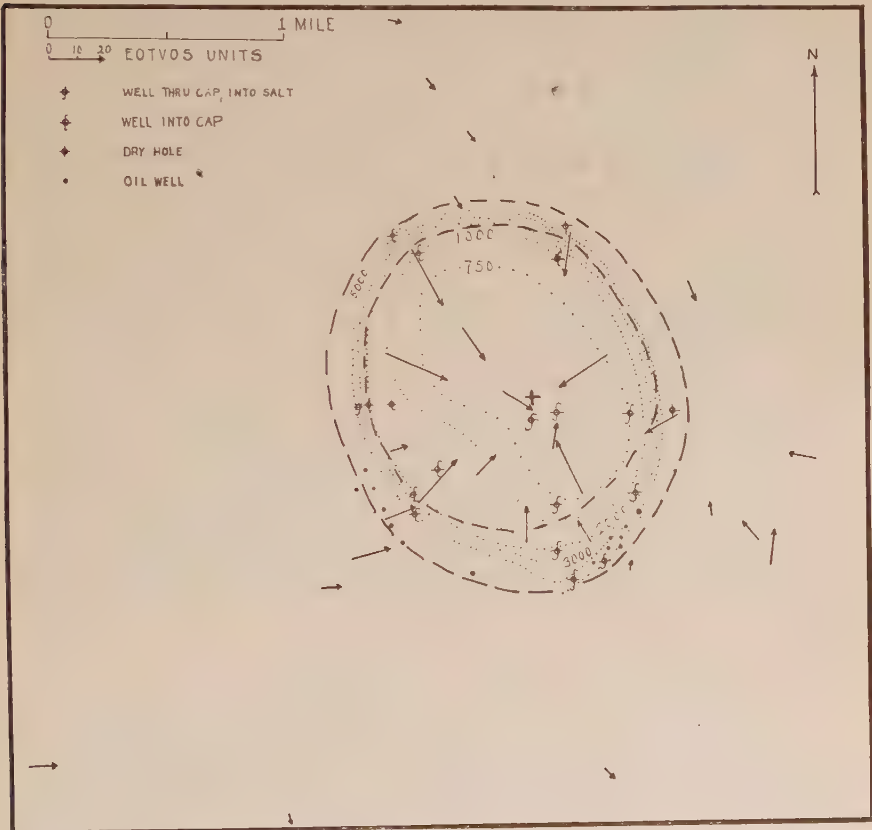


FIG. 9.—ORIGINAL RECONNAISSANCE TORSION BALANCE MAP OF THE NASH PROSPECT AND THE STRUCTURE CONTOUR MAP AS SUBSEQUENTLY DETERMINED BY DRILLING.

The structure contours are on the top of the cap or, in the absence of the cap, on the top of the salt. (Survey by the Rycade Oil Corpn. under the direction of the writer. Published by permission of E. L. DeGolyer, President, Rycade Oil Corpn.)

the inner dashed line represented the edge of the salt table at a depth estimated to be somewhere between 500 and 900 ft. and the outer dashed line represented the estimated position of the salt at 4000 to 5000 ft. The prediction of the position of the center of the dome is shown by the heavy cross in the figure. The first well, drilled just south of the interpreted center of the dome, demonstrated the presence of the dome. The second well, located on the center of the east edge of the dome, showed that the edge of the dome had been placed slightly too far out.

Our interpretation at that time was based on an empirical study of many of the known salt domes, and we had not yet made a study of the mathematical phases of interpretation and of the types of gradient profiles produced by masses of different shapes at varying depths. From our empirical studies we recognized that on any one dome the position of the maximum gradients and the rapid decrease of the gradients outward from the dome to zero seemed to be in a tolerably constant relation to the edge of the dome but that from dome to dome there was considerable variation in the relation; and in outlining the edge of the dome from the torsion balance results, we expected that the first well drilled on the edge of the dome would cause a revision of our interpretation of the position of the edge, a revision that should bring the outlined position of the edge of the dome into fair agreement with the actual for the whole of the rest of the dome. A radial contraction of 500 to 600 ft. in the position of the edge of the dome, as outlined in Fig. 9, brings the predicted position of the edge into very fair agreement with its actual position, except on the south. If consideration is taken of the facts that the Gulf Coast salt domes range in diameter size from less than a quarter of a mile to over three miles, and that they may be circular to slightly elliptical in plan, the delimitation of the edge of the dome with an accuracy of better than 1000 ft. was a very great advance on all previous methods of delimiting a dome not indicated by a well defined mound. As the result of later experience. we would not attempt now to make an accurate interpretation of the position of the edge of the dome from so sketchy a survey as that original reconnaissance survey at Nash.

Other Domes Indicated by Torsion Balance

The other four salt domes discovered by the torsion balance are the Clemens, Allen, Long Point, and Fannett domes. The Clemens dome was discovered at a locality where the presence of a dome was not suspected. The Allen dome, like the Nash dome, was at a locality where there were some very faint indications of the possible presence of a salt dome. Both domes were discovered by the Roxana Petroleum Corpn. The first indication of the Long Point salt dome was picked up by a German torsion balance crew working for the Gulf Production Co. but the seismograph more commonly gets the credit for the discovery of the dome, because it was used actually to establish the presence of the dome definitely. There had been no suspicion of the presence of a dome at that place, although it is on the edge of a large block of leases around some surficial sulfur water seeps of a type most commonly not considered as an indication of a salt dome. The Fannett salt dome was discovered by a Roxana Petroleum Corpn. torsion balance crew and a German seismograph crew working for the Gulf Production Co., almost at the same time. Which had

actual priority in the discovery, the writer does not know, but as the Fannett salt dome happened to lie wholly within the block of Gulf Production Co. leases and wholly off the block of the Roxana Petroleum Corpn. immediately to the south, the credit for the discovery of the dome usually is given to the former company and to the seismograph. The presence of either dome was not suspected until indication of the presence of a dome was given by the geophysical instruments. According to reports, the validity of which the writer does not know, indications of the presence of the Moss Bluff salt dome are now known to have been given by torsion balance surveys made by two different companies in areas closely adjacent to the dome, although they were not recognized in the interpretation of the surveys.

The presence of a deep salt dome has been indicated by the torsion balance near Dewalt, southwest of the Blue Ridge salt dome, in Fort Bend Co., Texas. The presence of the dome is reported to have been checked by the seismic method, but the prospect has not been drilled as yet.[7]

A small oil field has been developed at the Nash salt dome and a sulfur deposit has been found that is slightly too deep to be mined profitably at present. The Long Point salt dome has given evidence of being a first class sulfur dome and a half interest in the sulfur rights has been sold by the Gulf Production Co. at a price (it is reported) that would pay several times over the cost of all the torsion balance work that has been done in the Gulf Coast. Two oil wells have been completed at the Allen dome and a small amount of oil has been found on the Fannett dome.

Some eight or ten torsion balance prospects in the Gulf Coast have been drilled and condemned. In no one of the cases known to the writer did the torsion balance indicate the presence of a dome with a probability of better than twenty chances out of one hundred. The Rycade Oil Corpn. drilled three such prospects on the basis of torsion balance work done under the direction of the writer. At each of the prospects, there was some indication that in itself almost warranted drilling—three shallow water wells with authentic shows of oil at one prospect, a sulfur-water well similar to those at Nash at another, and a water well with a water of abnormal composition for surficial waters at the other—and at each prospect the torsion balance mapped a gravity high that was fainter than and not just like the gravity highs associated with the known salt domes. It was hoped that the gravity high mapped was the obscure trace of a deeply buried dome, but each of the prospects was recognized as being on the border line of favorability and unfavorability. A very distinct gravity minimum east of Welsh, La., mapped both by the Roxana Petroleum Corpn. and by the Rycade Oil

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[7] See author's discussion on page 479.

Corpn. was considered by both companies as a prospect on the border line between favorability and unfavorability. It was drilled without finding any indication of anything abnormal, but the test may not have been well located. The out and out failures of the torsion balance in the Gulf Coast area have been due, in the first place, to inexperience in interpretation; in the second place, to over-optimism in the use of indistinct indications; and in the third place, to the necessity of testing prospects of a lower order of favorability, in the absence of prospects of a high order of favorability.

Some fairly exact quantitative and semi-quantitative determinations have been made of the position and steepness of the flanks of the salt core and the cap and of the thickness and relative distribution of the cap. On account of its superior speed and its equal certainty of detection of a salt dome above 2500 ft., and its superior probability of detecting deeper salt domes, and its ability to work marsh, swamp, and shallow lake areas with considerable speed, the seismic method is the best for use in reconnaissance for new salt domes; but after a relatively new salt dome has been discovered, the torsion balance has been giving superior results in detailing the dome. From surveys very carefully made with the purpose in view, it is possible to contour the top, flanks, and thickness of the cap rock of some domes with a high degree of accuracy and of other domes with a fair degree of accuracy.

Delimiting a Salt Dome from Torsion Balance Results

The predictions and the verification of such a quantitative survey of the Hoskins Mound salt dome are illustrated by Fig. 10. The upper two panels of the figure give the gradient profile and the gradient arrows of one of the several diametral sections that were run across the dome. The calculations were tied to the depth of the top of the cap and of the top of the salt in well A and to the top of the cap in several wells in the left half of the dome off the line of the section. The lower panel gives a vertical cross-section of the dome according to the calculations from the torsion balance observations. The wells B, C, D, and E were drilled after the calculations were made, and are the only wells in the right third of the dome; the point *TC* marks the top of the cap as it was actually found in each well. The actual against the predicted depths of the top of the cap for those wells are: Well B, actual 840 ft. (256.2 m.), predicted 900 (274.5 m).; Well C, actual 947 ft. (288.8 m.), predicted 1000 (304.8 m.); Well D, actual 1222 ft. (372.7 m.), predicted 1250 (381.3 m.); Well E, actual 1565 ft. (477.3 m.), predicted 1575 (480.4 m.).

The success and the degree of error in the delimitation of a dome entirely unknown except from the torsion balance survey have been illustrated by the discussions of the Nash dome and by Fig. 9. One difficulty about the exact checking of the torsion balance predictions is that

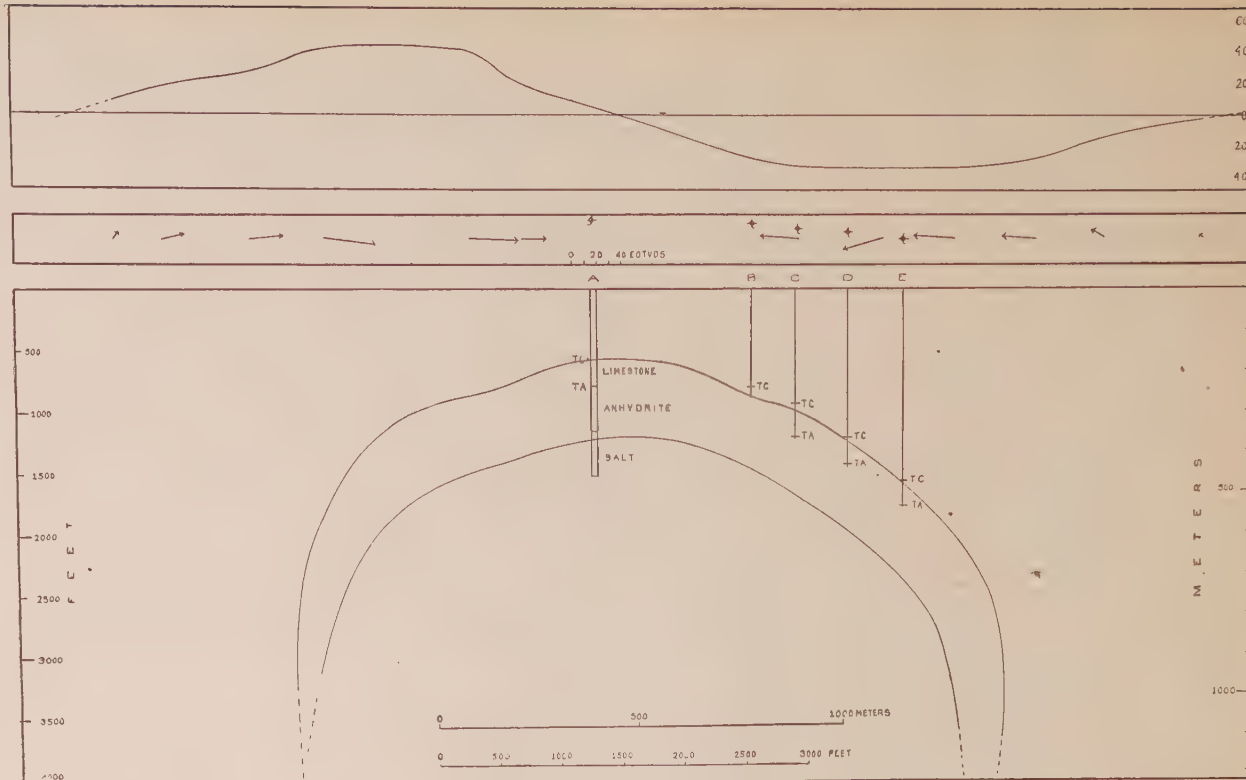

FIG. 10.—GRADIENT PROFILE, GRADIENT ARROWS AND CALCULATED STRUCTURAL SECTION FOR DIAMETRAL LINE ACROSS THE HOSKINS MOUND SALT DOME, BRAZORIA COUNTY, TEXAS, SHOWING THE CHECK OF THE PREDICTIONS BY SUBSEQUENT DRILLING.

(Survey by the Geophysical Research Corpn., Andrew Gilmour, observer and calculator under the direction of the writer. Published by the permission of P. George Maerky, Freeport Sulphur Co.)

they do not show minor irregularities and that actually the edge of the salt and cap and the top of the cap are very considerably irregular. Isolated wells, therefore, may not give a true picture of the conformation of the dome.

Mapping Faults with the Torsion Balance

The faults of the Somerset-Luling-Mexia-Powell-Sulphur River zone of faulting can be mapped with the torsion balance. Torsion balance profiles run under the writer's direction across the Sulphur River-Campbell fault, the Quinlan fault, the Powell, Richland, Currie, Wortham, Mexia, Luling fault-line oil fields, and faults southwest of Seguin in Guadalupe County and in southern Bexar County, indicated the presence and approximate position of the fault.

The indication by the torsion balance of the Luling fault is shown by Fig. 11. A single line of stations was run across the main part of the Caldwell County half of the field; Fig. 11a shows a map of that part of the field with the gradient arrows and the R lines for those stations; the upper panel of Fig. 11b gives the gradient profile for those stations and the lower panel gives the structure section along the same line (generalized after Brucks). This section shows somewhat close similarity to that of the ideal fault of Fig. 11b. Below a depth of about 2400 ft., limestone and schists are faulted against limestone and schist, except for the rather narrow band of the Trinity sand, and therefore present only a very slight density difference on the two sides of the fault. Between depths of 1600 and 2400 ft., the Edwards limestone is faulted up against the apparently lighter Eagleford, Del Rio and Austin and against the thin Buda of about the same density as itself; the Eagleford, Buda, Del Rio, and Austin are faulted up against the considerably lighter Taylor Navarro shales; and between 400 and 800 ft., the Navarro shales are faulted up against the slightly lighter Midway shales. The zone of the maximum gradient, therefore, lies over the zone of the trace of the fault in the Austin chalk, Del Rio shale, Buda limestone, Eagleford, Georgetown-Edwards limestone of the upthrow side rather than over the trace of the fault in the basement of the upthrow side, and on account of the dip of the fault to the left and the opposition of the heavier Navarro against the lighter Midway, the gradient profile is distinctly asymmetric and is fairly similar to the ideal profile of Fig. 8b. As the maximum near the right edge of the profile is based on the record of a single station, great dependence can not be placed upon it; it may signify a small fault or a concretion near the instrument, or some other surficial irregularity.

The faults of those zones have been mapped with the torsion balance by several other companies with varying degrees of success; whether the two or three failures have been failures of the method or of inexperienced interpreters, the writer is not sure. The faults in some places do not show

FIG. 11a.

a. Gradient arrow, "R" line, and structure map of a portion of the Luling (fault line) oil field.

— — — — (at the left) indicates trace of fault on Edwards limestone.

- - - - - - - (on the right) indicates trace of fault on surface.

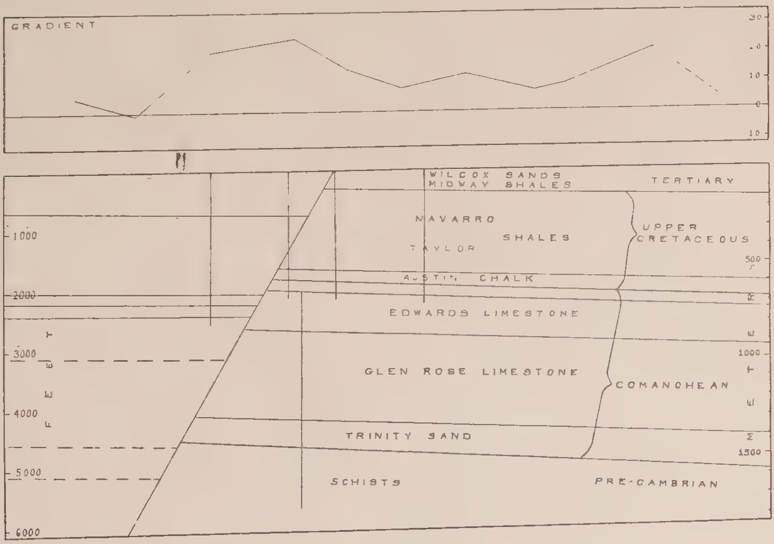

FIG. 11b.

FIG. 11.—EFFECT OF THE LULING FAULT ON THE GRAVITY GRADIENT.

b. Structure section and gradient profile.

(Survey by the Rycade Oil Corpn. under the direction of the writer. Published by the permission of E. L. DeGolyer, President, Rycade Oil Corpn.)

up in the torsion balance results as clearly as in others, and many situations would be rather difficult for an inexperienced and poorly trained man to interpret correctly.

In the Panuco district of Mexico, the torsion balance is reported to have done some rather brilliant work in making well locations along faults.

The torsion balance does much more brilliant work in detailing a fault than on reconnaissance for faults. As a single fault represents in most places a rather simple mathematical situation, rather brilliant results may be obtained in quantitative calculations of the fault, but as the gravitational effects of faults commonly are limited to a very narrow zone, stations must be placed very close together to map the fault. In reconnaissance, it is very easy to jump a fault, if any attempt is made to make speed in covering ground; and yet, taking stations every 500 ft. along a traverse across the supposed fault zone or into unknown territory is a tedious, time-consuming, and expensive reconnaissance. Furthermore, often faults are compound rather than clean-cut single breaks, and may bend or be offset or cut by cross-faults. All such irregularities can be worked and determined by taking a sufficient number of stations, but they greatly hinder an attempt to get a hasty idea of the situation by reconnaissance.

Work of Torsion Balance in Mid-Continent Field

The work of the torsion balance in the Mid-Continent district proper has been mostly reconnaissance to investigate the possibilities of the instrument and as yet very little drilling has been done on torsion balance structures. Although the torsion balance as yet has no Mid-Continent oil field to its credit as an outright discovery, the work with the torsion balance in that area shows that it has very great possibilities. Buried granite, gneiss, and Cambro-Ordovician ridges such as those in the Panhandle, as the Hambro-Nocona-Bulcher-Muenster ridge, Healdton, the Criner Hills, and some of the Kansas granite ridges, show up brilliantly in torsion balance surveys. In southern Oklahoma, a study of three long reconnaissance profiles showed: (1) that the torsion balance results indicate seven first-class prospects and eleven second-class prospects; (2) that if two wildcats were drilled on each structure on locations made wholly from interpretation of the torsion balance results, two first-class oil fields, one second-class oil field, and one or two fourth-class oil fields would be discovered, and the results on two of the prospects would not be known; the efficiency per well for the discovery of oil fields would be 30 per cent., with 30 per cent. of the wells not heard from; (3) that if two wells were drilled similarly on each of the second-class prospects, the results would be the discovery of three third-class oil fields, one small gas field, and one barren structure; one structure would prove to be

false, and ten wells would be in areas where there has been no drilling; the efficiency per well for the discovery of oil fields would be 13 per cent. with the result of 45 per cent. of the wells not known.

Although in the Seminole district, the Seminole City is not very clearly indicated in a reconnaissance survey of the area, the Bowlegs field and the eastward-northward extension of the Earlsboro Wilcox pool were predicted from the torsion balance results, but in Sec. 2, T. 7 S., R. 5 E., an A. P. C. well that is located on one of the better looking gravity highs of the area is apparently distinctly low. However, in the Seminole district, low wells are found rather anomalously in the middle of the structural highs and a single low well can not definitely condemn a surface or a gravity high. The Garber structure is faintly indicated by the torsion balance but on reconnaissance such a structure as Garber probably would not be detected unless the distance between the stations were 1500 ft. or less.

Survey of Buried Granite Ridge in Texas

The results of a reconnaissance torsion balance survey of the buried "granite" ridge of northwestern Cooke County, Texas, are shown in Fig. 12. The survey was of a sketchy reconnaissance type, comparable by analogy to a hasty Brunton-speedometer geologic reconnaissance for structure. The accuracy expected of it should be no greater than would be expected of such a hasty Brunton-speedometer reconnaissance; it should depict the high lights and major features of the structure but should not be expected to indicate accurately the details of the structure. The isogams shown in Fig. 12 are based on an approximately simultaneous least-square adjustment of the gravity differences between all the stations. A primary net of lines was laid out somewhat as in the primary net-method of Eötvös, but about one-third of the stations were used as key stations instead of merely six or seven as in the Eötvös method. This survey was tied into the Fox survey by three lines of traverse and it was possible, therefore, to give an absolute value to the isogams, which in this survey as in the Fox survey are referred to the spherical level surface through the Busby station (U. S. C. and G. S. No. 305).

The indication of the structure is as accurate as could be expected from so sketchy a reconnaissance survey. Two wells have gone into schists within the area of the map, one a mile north of Muenster at a depth of 1570 ft. below sea level, and the other about six miles west of Bulcher, at a depth of 2013 ft. below sea level. Only two other wells within the area have gone to comparable depths: one was about five miles west of the second schist well and went to a depth of 2204 ft. below sea level without encountering Pre-Pennsylvanian rocks, the other was about seven miles northeast of Muenster and encountered the Ellenburger limestone

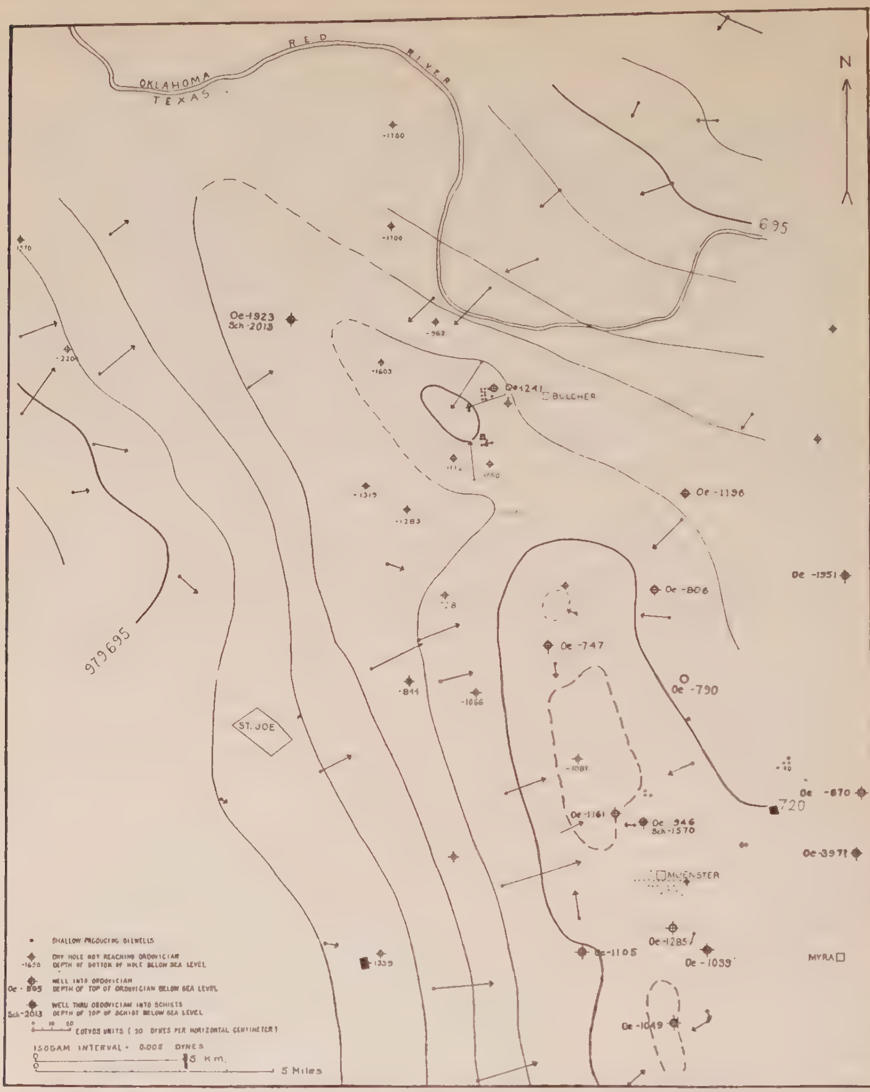

FIG. 12.—GRADIENT ARROW AND ISOGAM MAP OF THE MUENSTER-BULCHER "BURIED" ORDOVICIAN-METAMORPHIC RIDGE IN NORTHWESTERN COOKE COUNTY, TEXAS.

Isogams by the writer aided by I. R. and E. B. in a semi-simultaneous adjustment of the gravity differences between all stations. The isogams are tied into the U. S. Coast and Geodetic pendulum station No. 305, Busby, just south of the Fox oil field, Oklahoma, with, however, the elimination of the normal northward variation of gravity.

(Survey by the Geophysical Research Corpn. under the direction of the writer. Published by permission of E. L. DeGolyer, President, Amerada Petroleum Corpn.)

of the Ordovician at a depth of 1951 ft. below sea level. Both of the wells into the schists are on the torsion balance axis and both of the other two wells are some five miles off it. Thirteen wells within the area of the map have gone into the Ellenburger limestone of the Ordovician at less than 1250 ft. below sea level, and ten wells have gone to a depth of 1250 ft. without encountering the Ellenburger. All of the former lie within three miles of the crest of the gravity high; eight of them lie within one mile of that crest, and two others lie on the edge of a table of high gravity. Of the latter, two wells, the deeper of the two wells into the schist and a well halfway between it and the Bulcher high, lie on the axis of the gravity high; and the other eight all lie more than two miles off that axis.

The gravity high has a relief of some 0.038 dynes within the area of this map and as a matter of fact has a relief of about 0.052 dynes above the axis of the gravity low to the north. With the exception of the well that encountered the Ellenburger at −1951 ft. and the well immediately to the northwest of it, all of the wells that have gone into the Ellenburger are within 0.005 dynes of the axis of the ridge of gravity high. Wells have gone into the Ellenburger and one well into the granite in the southeastward prolongation of this ridge and wells have gone into granite and schist at Nocona on the northwestward prolongation of the ridge. North east at the ridge, no Ordovician or older rocks have been encountered until the Criner Hills axis of uplift has been reached. Southwest of the ridge, no Ordovician has been encountered for a very considerable distance. The highest points indicated by the torsion balance survey of Fig. 12 do not coincide exactly with the highest points of the structure as it is known at the present moment, but in spite of the very considerable numbers of wells that have reached the Ordovician, the configuration of its surface really is known only very sketchily and further drilling may very considerably alter the picture of the position of the crests of the structure, and it is quite certain that a more detailed torsion balance survey might cause considerable minor shift of the position of the highest points of the gravity high. The preceding interpretation of position of the structural crest as indicated by the torsion balance results has been based on the assumption that the points of highest gravity and highest structure coincide. That is not necessarily true but in the area under discussion is approximately true. A more detailed analysis of the torsion balance results than the writer has had time to make might show that the structural crest as indicated by the gravity relations should be shifted a mile more or less to the northeastward of the crest of the gravity high.

The survey of this area was made at a time when little was known about the Ordovician of the area. The well into the Ordovician just west of Bulcher and the two wells into the Ordovician at −1196 and −1951 ft. due southeast of Bulcher had just been drilled; the depth of the top of the Ordovician in the well immediately north of Muenster was rather

generally carried as about −1500 instead of −946 ft. and the report of the schist in the well was not generally believed. The two wells immediately north of Myra were well known but the data in regard to the top of the Ordovician in the more southerly of the two was (and is) uncertain; the axis of the structural ridge was rather generally drawn through the Bulcher oil field and the second Ordovician well north of Myra, where the Ellenburger is at −670 ft. The torsion balance interpretation of the structure, therefore, was at considerable variance with the subsurface geologists' interpretation. The result of later drilling has been in the direction of a revision of the subsurface maps toward greater conformity with the torsion balance interpretation of the structure. If no drilling had taken place in this area and if nothing were known about the geology, acreage taken on the basis of this sketchy reconnaissance survey would be well placed, and would cover the Muenster oil fields and the oil fields near Bulcher; and the Bulcher high would have been indicated as one of the most favorable points to drill first, with the discovery of the oil there as the probable consequence.

Survey of Fox Oil Fields

The results of a torsion balance survey of the Fox oil fields of Oklahoma and the results of a very detailed study and analysis of the data of that survey are represented by Figs. 13a, b, c. The gradient values of the survey are given by the gradient arrows of Fig. 13. The study of the data consisted of the calculation of the difference in the value of gravity between each pair of adjacent stations, the least-square adjustment of these values except for some of the outlying stations, the study and attempted elimination of the regional gradient due to the buried southwestern slope of the Arbuckle Mountains to the east and northeast, with the consequent obtainment of anomalous isogams which should represent the more local structure. The primary net method of adjustment of the gravity differences suggested by Eötvös was tried out but did not give quite as good results as the simultaneous adjustment of all the stations. The isogams of Fig. 13a are based on the values obtained by the simultaneous least-square adjustment of the 68 stations lying south and west of Fox, and by the linear adjustment of closed traverses to the north, east and southeast of Fox. The survey was tied into the Busby pendulum station of the U. S. Coast and Geodetic Survey, which made it possible to give absolute values to the isogams; in this case their value represents the value of gravity in the level surface through the Busby station. By using a correction based on the difference in elevation between any point within the area of the survey and the Busby station, the approximate actual value of the gravity at that point can be determined from the isogams.

Indication of the Fox oil-field structures is only indifferently visible in the gradient arrows and isogams of Fig. 13a. The general north-northeasterly trend to the gradient and the general increase north-northeastward of the value of gravity are the effects of the Arbuckle Mountains lying not far to the east and northeast and of the buried southwestward slope of the Arbuckle mass. A slight indication of underlying of structure of the Fox oil field can be seen in the small size of the gradient arrows

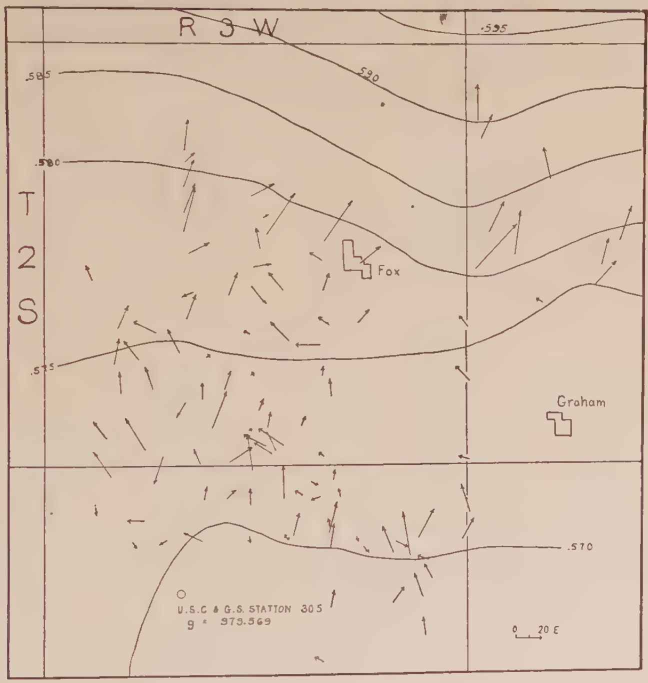

FIG. 13.—ANALYSIS OF A TORSION BALANCE SURVEY OF THE FOX OIL FIELDS, OKLAHOMA. (Survey by the Amerada Petroleum Corpn., simultaneous least-square adjustment of the gravity differences between all stations by I. Roman, analysis by the writer. Published by the permission of E. L. DeGolyer, President, Amerada Petroleum Corpn.)

a. Gradient arrow and isogam map. The isogams are tied to the Busby Station, U. S. Coast and Geodetic Survey pendulum station No. 305, and are referred to the level surface through that station.

and a tendency to reversal or rotation of their orientation about 1.5 mile southwest of Fox. As the effect of the Fox structure seemed to be so obscured by the effects of the Arbuckle mass, a further study and analysis of the situation was made with the purpose of eliminating the effect of the Arbuckle mass. The anomalous isogams of Figs. 13b and 13c represent the variation of gravity, which apparently is due to more local structural anomalies of density than the Arbuckle mass. The isogams of the two figures are the same but in Fig. 13b they are superimposed on the Bureau of Mines structure contour map of the Fox oil fields and in

Fig. 13c they are superimposed on the structure contours of a generalized A. P. C. structure contour map of the area surrounding the oil fields. The anomalous isogams give a rough picture of the structure; the agreement of the anomalous isogam map to each of the two structure maps is about of the same order as the agreement between the two structure contour maps.

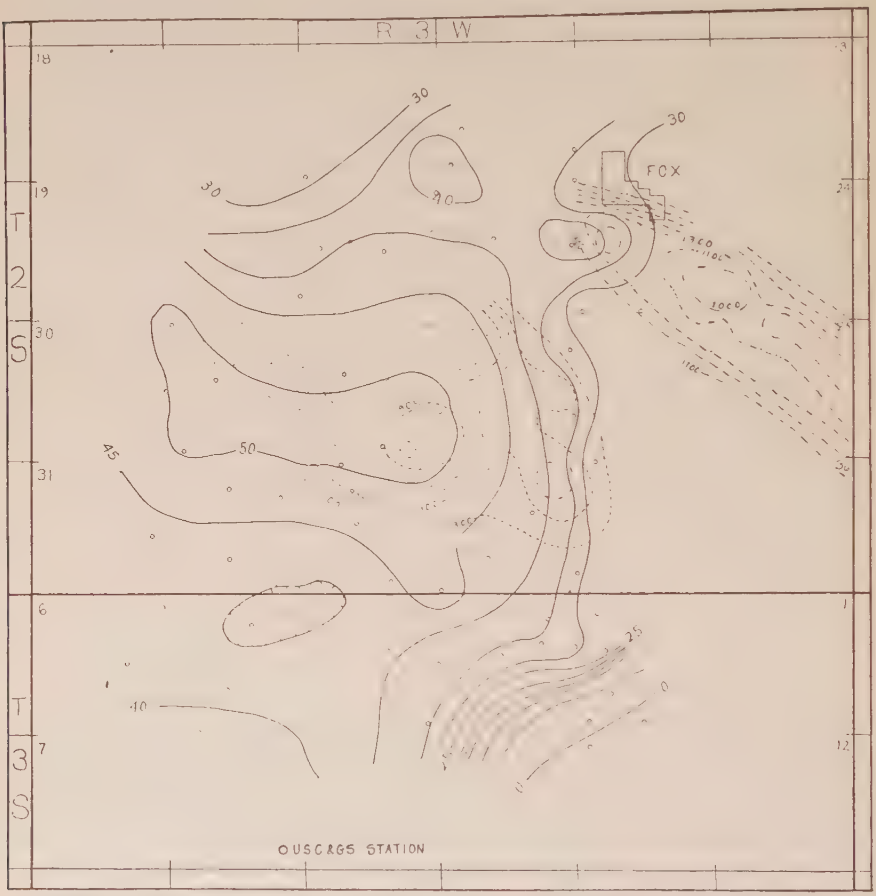

FIG. 13.—b. Map of the local anomalous isogams superimposed on a U. S. Bureau of Mines structure contour map of the Fox and Graham oil fields. The datum horizon is different in each of the three oil fields.

If this had been a wildcat prospect and acreage had been taken on the basis of the torsion balance results, a considerable part of the acreage would have been within the area that has proved to be productive. The slight shift of the anomalous isogams southwest from the apparently corresponding structure is probably due to a slight error in the estimation of the magnitude of the regional gradient due to the Arbuckle mass.

At the time that the study was made, the Bureau of Mines map was the only one available to the writer and the indication by the torsion balance of a ridge running off to the northwest was thought to be a "bust," it was not until considerably later that the A. P. C. map was received, showing that there was geological indication of a structural ridge extending off in that direction.

Although the study of the gravity relations at Fox oil field show the ability of the method to map structures having rather faint gravity

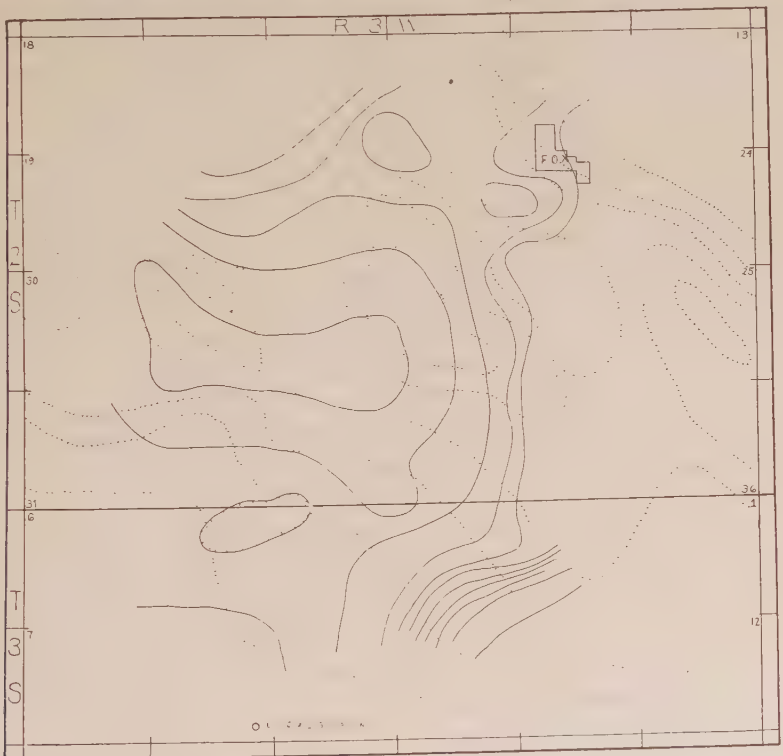

FIG. 13.—c. Isogams of Fig. 13b superimposed on a generalized regional subsurface structure contour map.

effects, great dependence on such faint indications of structure should not be made in reconnaissance work; the type of analysis that was made at Fox was warranted only by the very complete net of stations which cover the area with a much greater thoroughness than would be warranted in mapping most wildcat prospects. In the absence of geological indications of the structure, such faint indications will be better than nothing, but in a great many cases, the indications will prove false or the position of the structure will be faultily located.

A mistake that will have to be debited against the torsion balance, the writer, and insufficient experience, was made at Fox. An axis with

large gradient arrows on the southwest and small and abnormally oriented gradients on the northeast can be seen running northwest-southeast through the center of the south side of T. 2 S., 3 W. in Fig. 13a. The axis was interpreted as indicating the southeastward extension of the sharp structure of northwest Fox oil field and as a result of our interpretation the Amerada Petroleum Corpn. drilled a well in section 2 T. 3 S., R. 3 W. not far from the upper "25" of Fig. C, but the well proved to be extremely low structurally. But as can be seen from Figs. 13b or c, the detailed study and analysis of the gravity situation at Fox, which was not made until about two years later, showed the torsion balance results as indicating that the axis was plunging rapidly southeastward and that the well was located structurally low.

Use of Torsion Balance in Mining

In mining, only a slight attempt has been or is being made to use the torsion balance. Some work was done in the lead and zinc districts of Missouri, but apparently was not very successful. It is being tried out at present in the Michigan iron-ore district. The Colorado School of Mines and the U. S. Bureau of Mines are coöperating in some experiments in the use of the torsion balance in mining problems in Colorado. The torsion balance has been used in the Gulf Coast to map the extent of the cap rock in some of the salt domes where the cap rock carries commercial deposits of sulfur.

The Torsion Balance in Mexico

The results of the very extensive torsion balance work in Mexico by several different oil companies are known only vaguely to the writer. El Aguila Co. has been using torsion balances for four years in detailing the salt domes of the Isthmus of Tehuantepec. Several accounts have come through of exact verification by the drill of predictions made from torsion balance results in regard to the flanks of the salt core. In the "Golden Lane," the torsion balance is said to have done some very interesting work, the details of which are not known. In the Panuco district, the torsion balance is said to have shown that a trend of production which unexplainably lay at an angle to the strike of the surface structure coincided with the crest of a buried ridge in the basement complex. A brilliant piece of work is reported in the location of faults in the Panuco district.

The Future of the Torsion Balance

A distinct parallelism holds between the relation of the gravimetric surveys to geologic structure and the relation of structural geologic surveys to the occurrence of oil. As there is a tendency to a parallelism

between the occurrence of oil and of certain types of structural situations, geologic structure may be used as an indirect method of finding oil. As there is similarly a tendency to a parallelism between geologic structure and the anomalous distribution of density, gravimetric surveys with the torsion balance may be used as an indirect method of mapping geologic structure. As with oil geology, the torsion balance can handle certain types of situations brilliantly, does fairly good work in many others, and can do nothing at all in some cases. The torsion balance, like all the other geophysical instruments, is not a panacea for hunting geologic structure and can not replace geology, where good geologic work is possible; it should be used to supplement geology, in areas where only a small amount of structural work can be done, or to fill in the structure in areas where no structural work is possible. The success of the torsion balance in locating and defining oil structures will be just about the same as the success of geology in finding oil.

Use of the Balance in Oil Work

In the use of the torsion balance in oil work, four facts should be remembered: (1) that what the torsion balance maps is differences in density; (2) that the number of stations necessary to map a structure with a certain degree of accuracy is independent of the size of the structure; (3) that surficial and topographic irregularities of which the effects can not be calculated limit torsion balance surveys; and (4) that the interpretation of geologic structure from gravimetric surveys is just as complicated a subject as the interpretation of the probable occurrence of oil from a structural geologic survey.

The torsion balance may be used either for reconnaissance or detail mapping but in planning a program of work the possibilities and limitations of the method, as well as the importance of working some particular area, must be kept in mind. The method will give the most brilliant results where it is used in mapping structures that have produced considerable and sharp contrasts in density—for example, salt domes, granite ridges, faults in which massive limestone or anhydrite are faulted against sands and clays—and it will produce those results with a wider spacing of stations on large structures than on small. Where prolific production is obtained from structures that do not produce much of a density contrast and that are not reflected in the surface geology, it may be well worth while to attempt to get some clue to the structure through torsion balance surveys, but the results will not be so brilliant or so reliable as the preceding cases; and it will require an extensive survey with a closely spaced set of stations and an analysis of results of a much higher grade of technique and experience in interpretation. The practicability of torsion balance work is limited by surficial irregularities of

mass in the upper subsoil and by rugged topographic relief, but if the necessities warrant the expense, an area of very considerable surficial irregularities of mass can be worked by occupying an enormously increased number of stations very closely spaced and by using a least-square adjustment to smooth out the irregularities. If the necessities warrant, an area of fairly rugged relief often could be worked by making an extensive survey of the topography and of the densities of the formations and then calculating the effects of the topography on the gradient and curvature. Except in mountainous or semi-mountainous regions, the effects of topographic irregularity are not so serious as is somewhat commonly believed, if the gravity anomalies due to the structure to be mapped are at least moderately large in size.

The present status of the torsion balance method for most oil companies is about the same as if the companies had to depend on young civil engineers for their geology. In a region of moderate relief where an easily identifiable limestone keybed can be walked out around the hillsides, a young civil engineer would be able to make an accurate plane-table map of the surface structure. If he had a hazy idea of the anticlinal theory of the occurrence of oil, he would be able to make a fairly good guess where to drill on well defined anticlines and domes, but his naïve interpretation of more complicated structural situations would be badly in error. Torsion balance observations are rather simple affairs. The visual types of instruments are simpler to operate than an explorer's alidade and, with the simple tabular computation forms, the calculations are reduced to a routine of elementary addition, subtraction, and multiplication. A bright young man without technical education can be trained to be an efficient torsion balance operator (for the Süss type of balance) more easily than he can be trained to be an efficient plane-table instrumentman. Such an observer normally will be able to bring an accurate map of the gradient and curvature of an area, but neither he nor a geologist who has not studied the interpretation of torsion balance results will be able to make an accurate interpretation of his results. They would be able to make a fairly good, approximate qualitative interpretation of the survey of some salt domes and some buried granite ridges. In a survey of the Healdton field, they would recognize the presence of some large, sharp structure but would interpret its form and position incorrectly. In surveys of Fox-Graham or of Garber, they would not recognize the presence of the structures.

Qualifications of an Interpreter of Torsion Balance Results

The difficulty is that the indirect relation between geologic structure and the distribution of density is more intricate and complex than the indirect relation between geologic structure and the accumulation of oil.

A torsion balance survey for practical purposes is an indirect method of mapping the distribution of density in the subsurface, and mapping the distribution of density in the subsurface is only an indirect method of mapping structure. Therefore the interpreter of the torsion balance surveys must know: (1) the mathematical technique of interpreting bodies from the distributions they produce of gradient and curvature values; (2) the relationship between geologic structure and the distribution of density; and (3) the probabilities, possibilities, and impossibilities in the way of occurrence of geological conditions—and, not least, he should have a level-headed sense of the limitations and imperfections of the method.

To get the most out of torsion balance surveys, the interpreter should be a good mathematician and geologist, should have had a chance to study the mathematics of interpretation and should have had a broad practical experience with the method. There are very few men who have had sufficient opportunity to study the mathematics of the interpretation and who have had sufficiently broad experience to qualify as competent interpreters. It is, and will be, easy to get men who are able and competent observers and who possess a fair but superficial knowledge of interpretation. A great danger to the full realization of the possibilities of the method is that the company employing such a man as an interpreter may fail to recognize his limitations. Another danger is that a company that realizes the necessity of training itself and its interpreter by theoretical study and practical experience will become overimpatient and over-enthusiastic and may force him to practical utilization of the method before he is competent to use it. A company that has had much experience with geology has become educated to the fact that geology is not a "sure-shot" panacea for finding oil, and is satisfied to allow geology a considerable number of failures if it turns up a fair number of successes; but there is a good possibility that the same company will condemn the torsion balance after a very few failures, without regard to whether the failures were due to the method or to the incompetence or inexperience of the interpreter.

Conflict between the gravimetric and the geological interpretations of the same structural situation will inevitably arise in the course of the torsion balance surveys. In some cases, the conflict will be due to the incomplete concordance of structure and the distribution of density, but in many cases the geologic interpretation may not be completely accurate. In the absence of better data over wide areas, the geologist has to depend on those from widely scattered wells and often on the lithologic log to log correlation of drillers' logs not checked by geologists or paleontologists. Although such correlation may be necessary and useful, it holds grave possibilites of error, and although generalization from the scattered wells also may be necessary and useful, it should be recognized and remembered

as a first and very rough approximation to the truth. Because he is forced continually to use more or less inaccurate and inadequate data to make conclusions that must be acted on as if accurate, the geologist tends to become callous, and to forget the probably considerable degree of inaccuracy of his conclusions, and, in judging the merits of a conflict between the geological and the torsion balance interpretations of a structural situation, he will tend to overvalue the validity of the geologic interpretation and undervalue the validity of the torsion balance interpretation. For example, if the surface evidence of the Healdton (Okla.) structure were effectually concealed beneath a mantle of Trinity sand, and if the only wells drilled in the area were a deep test some two miles northeast of the northeast edge of the field and another deep test some four miles southwest of the southwest edge of the field, most geologists familiar with the area, but unfamiliar with the torsion balance, would condemn the torsion balance indication of a big structure out in the middle of a very deep Red Bed syncline as an impossibility and a bad "bust" on the part of the torsion balance method or the geophysicist. If therefore the torsion balance gives a definite indication of some type of structure widely divergent from that indicated by geology, the probable accuracy of the geologic interpretation must be reëvaluated. The conflict between the geologic and torsion balance interpretation of some structural situation in many cases will be analogous to the proverbial conflict in the description of an elephant by three blind men; one of whom felt the trunk, the second the side of the elephant, and the third a tusk.

Combination of Geophysical Instruments Desirable

Although the torsion balance is superior to the other types of geophysical instruments in doing certain types of work in certain types of geologic situations, it is inferior to some of the geophysical instruments for other purposes or in other types of situations, and in many cases it is preferable to combine several geophysical methods. In the salt-dome area of the Texas-Louisiana Gulf Coast, the mirage elastic earth-wave method is superior to the torsion balance method for reconnaissance for salt domes, on account of greater speed and ability to cover difficultly leasable or passable acreage, lower cost and equal accuracy in detecting the shallower domes and greater accuracy in detecting the deeper domes; but the torsion balance is superior for detailing the shallower domes and possibly certain types of the deeper domes. If the structural deformation is slight or if it does not cause much deformation of the distribution of density, and if a moderately thick hard bed lies at moderate depth under a cover of much softer beds, the reflection elastic earth-wave method may be far superior in accuracy, speed, and cost. Although the results of magnetometer surveys can not be depended on to the same extent

as those of torsion balance surveys, the magnetometer does give indication of certain types of structure in a great many places, and as it is faster and less costly to run than the torsion balance, it can be used advantageously on reconnaissance to discover areas to test with the torsion balance.

Usefulness of Torsion Balance in Pure Science

In connection with the pure-science problems of geology, the future seems to offer possibilities of usefulness for the torsion balance. The reconnaissance survey already made across the buried "granite" ridges of the Mid-Continent indicate that regional reconnaissance of such sedimentary basins will reveal the structure and the structural trend lines of the basement and will allow buried mountain systems to be traced out into and across such basins. The prolongation of the Appalachian mountain system after it plunges out of sight under the sedimentary cover of the Gulf Coastal Plain comes up frequently in the broader philosophic discussions of the orogenesis of the American continent and allied problems. Although the geologic map of Alabama does not seem to indicate that the mountains may swerve from a course that will carry them straight out under the Coastal Plain into the Gulf of Mexico, it is not uncommon to find the argument that they bend westward to connect with the late Paleozoic mountains of Arkansas and Oklahoma. The torsion balance would probably be able to give some very brilliant results in proving once and for all what becomes of the Appalachian Mountains under the Coastal Plain of Alabama, and whether they dip gently under the Coastal Plain sediments out into the Gulf of Mexico or are abruptly faulted off.

In the discussions of isostasy and the questions whether or not certain areas or structural units are isostatically compensated or supported by the rigidity of the crust, arguments are used that in part depend on gravity anomalies observed at rather scattered and in many cases isolated pendulum stations. From those stations it is not possible to determine the depth of the masses that are producing the anomalies, but it has been suspected that many of them are relatively shallow. Many gravity anomalies have been mapped in regional reconnaissance with the torsion balance that are relatively local in area, that are demonstrably due to relatively shallow anomalies of mass, and that are of the same magnitude as the anomalies observed at the pendulum stations. Surveys made by the torsion balance in conjunction with the pendulum observations will allow a determination of the approximate order of depth or depth of the greater per cent. of the anomalies of relatively shallower mass and will make it possible to determine whether or not the anomaly observed at a pendulum station is due to a local cause or a regional one.

The batholith is one of the livest and most discussed problems of petrology, and the petrologists are keenly interested in extrapolating

from the surface data down into the interior of the batholith to determine its extent and composition. If a batholith is in a region that is not too complicated structurally or lithologically or where the surface relief is not too great—for example, Dartmoor, in England—there seems to be a reasonable chance that a careful torsion balance survey would reveal some very interesting data regarding the form, depth and composition of the batholith. The relative proportions of limestone and igneous rock in a coral island are reported to have been studied by a Japanese geologist by means of a torsion balance survey. The glacial geologist who is interested in mapping pre-glacial drainage systems should be able to get some brilliant results with the torsion balance, especially in areas where the bed rock is metamorphic, igneous, or limestone. For a wide range of problems in which the relations of rocks of abnormally high or low density are involved, the torsion balance is an instrument for looking down into the earth's crust, as it were, as it is possible now only in a scattered and very limited way by shafts, drill holes, and canyons.

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DISCUSSION

D. C. Barton.—The field surveys which I have been privileged to use as illustrations for this paper were done for Mr. DeGolyer's companies and were released through his favor.

W. J. Millard, Tulsa, Okla.—Has any oil been found by the torsion balance in West Texas?

D. C. Barton.—No new oil field has been discovered on the basis of torsion balance work. It has had some part in controlling the policy of certain companies in playing the extension of known oil fields. A number of "T. B." structures have been drilled and dry holes obtained. Of those known to the writer, one apparently was a structural high, one is a broad general high but locally low and probably is a deep-seated structural high, one seems normal although the control is poor, and one was considered structurally low but is now suspected by some geologists of being structurally high.

E. L. DeGolyer, New York, N. Y.—A large gravity structure, generally referred to, I believe, as the Crosby County Structure, has been found in West Texas. This structure is perfectly symmetrical with the arrows all pointing toward a common center, very much like the Nash salt-dome pattern in Dr. Barton's paper (Fig. 9). One of the peculiar things about it, from the standpoint of the gravity high, is that it is so large. The map that I have seen shows that this symmetrical inward-pointing arrangement of the gravity arrows extends over an area at least 30 miles in diameter. The thing looks, strictly from the standpoint of the arrangement of the arrows, a good deal like a salt dome. Of course nobody had any idea that it was a salt dome because it is entirely too large. I believe that subsequent drilling has not thrown any light on the mystery of what causes this particular and very peculiar gravity arrangement.

I would like to ask Dr. Barton if he has any ideas on the subject yet?

D. C. Barton.—It seems to me that it must be a big batholith. Mathematically it could be accounted for by a large structural high of limestone rising to within 2500 ft. of the surface and sloping off rather gently to a depth of 3 miles at a distance of about 20 miles from the crest. But there is a great haziness in our calculations of unknown structure. We can get a series of pictures of possible structure that are all equally probable mathematically. In general, they may have a minimum depth or a maximum depth. In this case, we could not bring the structure within 2500 ft. of the surface by any reasonable assumption, but the top of it might be 5000 ft., just as easily. (The Gulf Prod. Co. has drilled a test to below 5000 ft. without finding anything but the normal stratigraphic section.)

The anomaly at that dome is one of the biggest known to Major Bowie, of the Coast and Geodetic Survey, in United States. Within the area which we mapped, there was an anomaly of 0.3 dynes. The total anomaly must be on the order of 0.5 dynes, because where we picked up the gradient it was at its maximum and the gradient profile was flat, showing that the very large gradients were still extending for a very considerable distance westward and eastward.

My guess is that it is a very big batholith of some sort, because it is too symmetrical to be accounted for by erosion.

The Coast and Geodetic Survey intended to make some gravity determinations on the structure but was afraid that the geologists would think it an encroachment on their territory. I am sorry that the Coast and Geodetic Survey has not gotten absolute measurements on the magnitude of the anomaly.

F. E. Wright, Washington, D. C.—Coming from an activity entirely outside of the oil or mining industry, the Carnegie Institution of Washington, I wish to express

appreciation to Dr. Barton and also to Mr. DeGolyer's company for allowing information of this sort to become public, because we are all working toward the gathering of knowledge on these more or less obscure things, and the fact that they are being published and are becoming more general property will certainly in the long run tend to the advantage not only of the particular company itself but of all who are interested in geophysics.

S. F. KELLY, New York, N. Y.—What success has the torsion balance had in the region of Laredo, Texas, and in the southern part of the Mississippi Valley?

D. C. BARTON.—I have not had much experience in the Laredo district. In the Eagle Pass district we ran a profile across the Chittim anticline, and also did some mapping on part of the anticline. The results showed the presence of the anticline but did not give us quite as good information as we had from the geology. We have used a torsion balance in the back bottoms of the lower Mississippi Valley and I expected that it would behave horribly. It showed very little disturbance from the effects of the alluvium. We used it on a prospect on the Atchafalaya River and found a rather consistent regional gradient extending over a considerable area. We also used it at the Bayou Boullion salt dome, where the only place that the balance could be run was on the top of the natural levee of the Atchafalaya, and although we were run out by high water before we had taken all the desired stations, we obtained what seemed to be a fair determination of the edges of the dome at the Atchafalaya.

A. C. LANE, Tufts College, Mass.—Years ago Brooks[8] did some work on magnetism in which he found differences corresponding to the height of the instrument. When it comes to getting the depth, I should imagine that if the Eötvös balance were put above the ground and at the ground, there would be a difference. Has anything been done in that direction?

D. C. BARTON.—That will give the depth of boulders, perhaps, close under the instrument but will not give a criterion for the determination of the depth of the masses in which the geologist ordinarily is interested. The gradient and differential curvature are second derivatives and their variation in a vertical direction is a third derivative of the potential function. Being a higher order derivative, the vertical variation of the gradient and differential curvature is negligible except when the attracting mass is very close to the instrument.

On page 493 is a chart which shows the different sized prisms necessary to produce a gradient of 1 E at the origin. The chart is supposed to be a vertical section; each rectangle represents the cross-section of a prism of such dimensions that it produces a gradient of 1 E at the origin; the depths are plotted logarithmically; the length of the prisms varies with depth according to some chosen law. If the horizontal scale is taken at 100 ft. per unit, it can be seen that a vertical shift of the structure 10 or 20 ft. would produce a negligible effect, but if the scale were taken as 10 ft. per unit, a vertical shift of 10 ft. would make considerable difference in the effect of any mass within the zone of the chart.

Charts of this type give a very pretty method of calculating what sort of anomalies will be produced by a given mass or what sort of masses will produce a given gravity anomaly. The theoretical sections which I showed were calculated on charts of this type. You draw the cross-section on tracing paper and superimpose on the chart and add up the squares. If the anomaly is known, you keep molding the cross-section until the calculated results fit the observed values.

W. J. MILLARD.—There is in Venezuela an area in the States of Monagas, Anzoategui, Guarico and others where the surface rocks are flat-lying Quaternary sediments.

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[8] Geol. Survey of Michigan (1873) **1**, Pt. 1, Plate 16 and text.

The area lies between the Orinoco River on the south and the foothills of the coast range on the north. The distance from the foothills to the Orinoco River is about 90 miles. (See Fig. 14.)

The Quaternary consists of reddish ferruginous conglomeratic sandstones, sandy clays and sandstone. In Guarico and Anzoategui are found several ferruginous conglomerates from 3 to 5 ft. thick. The average thickness of the Quaternary is about 400 ft. thick. Near the foothills it appears to thicken considerably.

Beneath the Quaternary are from 5000 to 12,000 ft. of Tertiary sediments which consist largely of sands, sandy clays and clays. The lower 5000 to 6000 ft. is oil-bearing in the near-by island of Trinidad. Before the Quaternary was laid down the Tertiary was fairly well dissected by erosion and probably a fairly strong unconformity exists, so that old Tertiary valleys may be filled with flat-lying Quaternary.

Below the Tertiary sediments are 3000 ft. of black Cretaceous shale followed by about 800 ft. of Cretaceous limestone.

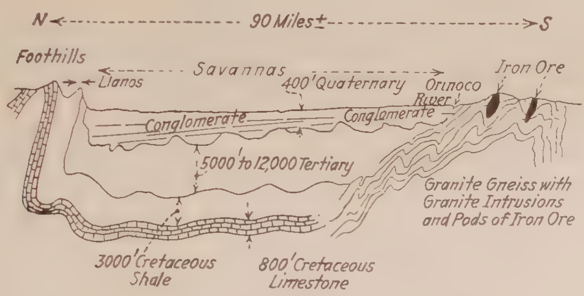

Fig. 14.

The Tertiary and Cretaceous have been deformed so that many long parallel folds exist which probably contain oil. The width of the structures is about 3 miles. The folding in the area under discussion is concealed by the flat-lying Quaternary.

There are several factors which would probably affect geophysical instruments:

1. Hills of Cretaceous shale and limestone at the north.

2. Large bodies of iron ore on the south side of the Orinoco, particularly south of the state of Monagas.

3. Layers of flat-lying ferruginous conglomerate in the Quaternary.

4. Rough Pre-Quaternary topography.

5. Depth to a body of rock such as the Cretaceous shale which differs considerably in specific gravity is from 6000 to 12,000 ft.; the depth of the Cretaceous limestone would be from 9000 to 15,000 ft. except where an extraordinary "high" existed. For example, at Pedernales in the Delta Amacuro the Cretaceous limestone is probably within 3000 or 4000 ft. of the surface.

6. Bodies of magnetic sands in the old Tertiary valleys.

With the above conditions would not the geophysical instruments reveal a picture of the Pre-Quaternary structure rather than the geological structure in the Tertiary? If such was the case, a "high" might be selected over a Tertiary hill in which 7000 ft. of Upper Miocene non-oil-producing beds exist.

D. C. Barton.—Theoretically, it is often hard work to diagnose a case like that. The proof of the pudding is in the eating. If I remember correctly, the torsion balances are working all right down there, aren't they?

W. J. Millard.—The only reports I have had were that it made very little difference in gradient.

D. C. Barton.—On that section the best way to find out is to try it. If the structure down below is big enough, then it probably would. It is a question of whether the deep-seated structure is on a large enough scale so that you can iron out the irregularities above. Those irregularities above will very possibly mask the smaller structure on the basement, but if there were a structure down there that was 15 or 20 miles across with a big, broad ridge coming up, if they were 3 miles wide I should imagine they would not. You can analyze apart the two sets of anomalies fairly readily, as I showed you on the Fox-Graham and on the Arbuckle, and in Cooke County there is another big mass on the other side of the basin; the two have a profile which looks something like that. You can analyze it into the two structures. You can separate Fox-Graham out with fair success; you can separate Healdton out with beautiful success, but if, in the case you have sketched, the underlying structure is only about 3 miles, there is a considerable possibility that the surface irregularities are of the same magnitude and that therefore the two sets mask themselves successfully.

E. L. DeGolyer, New York, N. Y.—As Dr. Barton has said, a great many of these things do not work as well as they should theoretically. As I get the results of gravity work, most of the time they are something like the algebraic sum of the number of different effects on the balances; the answer may be 7, but it may be 7 as the result of a plus 4 and a plus 3, or as the result of a plus 10 and a minus 3. A great deal of advancement is being made in the mathematical side of interpretation; some advancement will probably be made in the future in the actual technique of observation, in the physical work, but it seems to me that what is true for all geophysical work as I know it is particularly true for gravity work. We need to make a great many observations, a great many special cases. In other words, we need a background of experience by which to judge such results as we may get in the future.

When we took up this salt-dome work—and I believe that Dr. Barton brought in, for one of the companies with which I am associated, the first torsion balance, or one of the first torsion balances in the United States—we decided that the thing to do, because then it was entirely experimental, was to go out on a known salt dome, make a survey, and see whether or not we got results that appeared to be solid enough to be dependable, if they were secured as the result of exploratory work.

We first went to Spindletop and got brilliant results. The pattern of arrows is very similar to that which Dr. Barton has shown you for Nash. We were tremendously pleased and felt prepared to go out immediately and comb the unexplored areas of the coast in an endeavor to find other salt domes. We felt that the method was a very exact and a very reliable one, but caution suggested that it might not be a bad idea to try another dome or two. None of the succeeding results, as I remember them, were ever quite so good, and various sorts of complications began to be introduced. We found that all salt domes did not show exactly the same picture; in fact, I am not sure that we ever again got quite such brilliant results as those at Spindletop, except perhaps at Nash.

Our experience was that a great many of the domes seemed to lie in the twilight zone. The expression was more or less imperfect, and when we went into the field in an endeavor to find new domes, we tried a number of these imperfectly expressed gravity highs, with almost continuous failure. Finally we were almost disgusted with the method and about to abandon it altogether, when we ran across the Nash prospect. This was so striking and so outstanding in its indication of the salt dome that we felt prepared to make one further effort to find a dome, and the result was success.

It seems to me that in all the geophysical methods what we need as much as anything else is contribution in the way of actual surveys to some general fund of knowledge, so that we can advance the matter of interpretation.

Dr. Max Mason,[9] referred to geophysical methods as the attempt of the earth to tell us something about its constitution, and said that it does not necessarily speak English; it may not even speak Chinese, and at the moment we have the job of learning several new languages as well as advancing the technical side of the art.

D. C. Barton.—There is one reason for some of the troubles with results in the torsion balance, which also holds true with the other methods. It is more or less as if we surface geologists, when we first learned to do surface geology for oil, had been pitchforked into the red beds of Oklahoma; we would have made an awful mess of surface geology as a method of finding oil. Even nowadays oil geologists prefer some better place. It is easy to work in Kansas, where there are very pretty key beds, but the oil geologists did grow up working first on the easiest prospects. There is a tendency to use the torsion balance and physical methods where we want to find the oil, and not in the places where the torsion balance works particularly well. I think, of course, there was a commercial warrant, for instance for the use of the balance at Seminole. We did a great deal of work there but it is a horrible place for a torsion balance man; it is the place where I got some of my gray hairs trying to make out what the torsion balance was saying. I do not think we should put all our geophysical work in any place like that. There are places where the torsion balance works brilliantly.

F. W. Lee, Washington, D. C. (written discussion).—We all appreciate Dr. Barton's paper, as well as his lucid explanation of the practical interpretation of the evaluation of torsion balance measurements. It marks a distinct step in arriving at the ultimate possibilities in interpreting geologic structure by this method. Whereas most methods rely chiefly upon singularities of observations for classification of results, Dr. Barton has shown how to evaluate continuity of geologic structure on the indications of the balance. His method of approach may form a basis for a more refined magnetic and electrical conductivity interpretation heretofore not strongly emphasized or sufficiently delineated. We sincerely hope that Dr. Barton will find time to give us further details concerning his methods.

E. Lancaster-Jones, London, England (written discussion).—All who have endeavored—with problematical success—to express the essentially mathematical basic principles of the action of the Eötvös gravity balance in comparatively non-mathematical language for the benefit of mining engineers and geologists will be grateful to Dr. Barton for the exposition of these principles given in the earlier paragraphs of his paper.

It is, however, for the later portions of the paper that the experienced investigator in balance work will acknowledge a lasting debt to the author. Here we find, within the necessarily limited restrictions of the paper, a penetrating analysis of the efficiency of the torsion balance as an aid to the determination of geological structures. Although the experience of the author of the paper has been principally confined to one main field of activity of the instrument—the oil-producing region of the Gulf Coastal plain—his conclusions will, in the main, be endorsed by investigators in far different localities. He rightly stresses the function of the gravity survey as an auxiliary to any other method of revealing buried structures, and condemns the constantly recurring tendency to regard the instrument as a species of divinely inspired oracle.

In Dr. Barton's examples of actual surveys the gradual evolution of the technique of gravity surveying as applied to the location of oil-producing structures can be traced, and a very valuable lesson derived for future employment of the method in untried fields. The lesson, stated briefly, is that only by means of detailed surveys

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[9] See page 10.

conducted with due precautions by experienced observers and exhaustively analyzed with all the skill available can correct deductions be made as to the efficacy of the method in any particular project. Sketchy surveys and empirically derived interpretations may have brilliant successes in isolated instances, but must lead in the long run to disappointment and distrust. Gravity surveying for geological and economic purposes must be put upon the same basis as for geodetic ends.

The writer would suggest that Dr. Barton's references to the labor involved in quantitative analyses of surveys in difficult areas are unduly severe. It is true that, where several disturbing features unite to produce a complex resultant field of gravity at the surface, the interpretation is necessarily arduous and demands a high degree of expert skill. Nevertheless such analysis is done as a matter of course in all precise scientific work, for example in geodetic surveying, astronomy, etc., and there is no possible excuse for scamping it in gravity surveying.

Another criticism which the writer ventures to make is to the effect that Dr. Barton's paper implies that we have now attained to a technique of gravity surveying with the Eötvös balance which approximates to finality. The writer's opinion is, however, that we are still only in the preliminary stages of evolving that technique. The instrument itself is being constantly modified to suit individual cases, and the Schweydar Z-beam type is by no means the last word. Everything points to the rapid evolution of a series of instruments, some designed for geodetic purposes, many others of different type for mineral locating. We shall ultimately have our reconnaissance instruments, our standard instruments and our precision and detail types, just as we have with theodolites. Similarly, the technique of operation and interpretation will evolve into distinct branches. Above all we must anticipate an increasing collaboration of geophysical instruments and methods of varying types with a new type of geological investigation. At present the physical characteristics of strata are practically unknown, but our knowledge is being rapidly extended.

The present is not yet the time to decide whether and to what extent gravity surveying for geological and economic purposes is efficient, but every account of actual surveys like those given in this paper, analyzed by an author of the experience and ability of Dr. Barton, cannot fail to exercise a profound influence on future development. Is it too much to expect that all investigators who have been similarly connected with gravity surveying in mineral-producing areas will follow the author's example, and give to an eager world the results of their work and the conclusions they derive? Only when papers such as these, supplemented wherever commercial interests permit by full details of the surveys, have been multiplied to cover all the principal work done with the torsion balance, will it be possible to say to what extent in the past the balance has proved a valuable aid to the geologist. But not even then shall we be able to forecast its future possibilities. Its records are permanent and practically immutable. The interpretation which, for lack of sufficient data, appears to fail today may be expanded into success tomorrow when further data have been collected and analyzed. The gravitational field can be confidently expected to retain its local characteristics indefinitely, so far as subterranean structures are involved. Gradually those characteristics will be revealed as surveys are expanded and detail added. The resources of interpretation are as yet scarcely tapped, for the simple reason that very little of the actual surveying done has been published in sufficient detail for outside investigators to make constructive criticism of the interpretation. When it is realized that for some years past over one hundred torsion balances have been engaged in intensive surveys, probably involving tens of thousands of determinations at separate stations, of which not one per cent. has been published in anything remotely resembling the manner in which Eötvös himself gave his data to the world, it will be evident that any estimate of efficiency based purely on one individual's experiences can be accepted only as a beginning.

The present paper is a remarkably good beginning and should serve as a shining example. The writer heartily congratulates its author upon what will in future be generally accepted as an epoch-making summary of recent torsion balance work.

H. SHAW, London, England (written discussion).—I congratulate Dr. Barton on his lucid exposition of the principles and operation of the torsion balance. His novel method of presentation has the advantage of giving a simple explanation of several features which are not readily understood, and which are still more difficult to expound, in a manner that will render them intelligible to the average student.

One is reminded of the remarkable advances that have been made during the last few years in the gravitational method of surveying, and more particularly in the methods of interpretation, for whereas a decade ago, the location of a salt dome, fault, or anticline, by means of the torsion balance would have been regarded as a great achievement, however approximate the interpretation might have been, yet today, not only must the salt dome be located, but it is regarded as part of the duty of the geophysicist to indicate the depth below the surface and the thickness of the cap rock, while the confines of the dome are also required to be known within reasonable limits. The accuracy with which this can be done under favorable conditions may be gathered from the figures given by Dr. Barton of his own interpretation at the Hoskins Mound salt dome.

The location of faults and ore deposits have also been attended with similar advances, and it is now possible, under favorable conditions, to give a quantitative as well as a qualitative interpretation of the gravitational results.

In discussing the relation of subsurface bodies to the gravity effects at the surface, Dr. Barton states, quite correctly, that any distribution of gradient and curvature values at the surface can be produced by only one body, definite in form, relative density and position. Our problem is to determine these characteristics of the body, from an examination of the gravity values at the surface. If the gradient and differential curvature values are known at every point on the surface, it is possible to arrive at a correct and complete interpretation of the problem, but in practice it is obviously impossible to determine these values at every point, and we have to be content with a comparatively inadequate knowledge of the distribution of the gradient and differential curvature based on the determination at a definite number of stations. The approximation of our hypothetical distribution to the true distribution will depend on the number of points at which the determinations have been made, and the care with which these points have been selected.

One of the fundamental assumptions in torsion balance work is that the gradient and differential curvature values should not vary too rapidly between neighboring stations, and if variations of this kind do actually occur between adjacent stations, it is evident that they cannot be recognized, and therefore cannot be taken into consideration, so that the resulting interpretation will necessarily be in error.

The closer we make our station network, the more nearly will our knowledge of the gravity distribution approximate to the theoretical ideal, and the greater will be the reliability of our interpretation. The advantages of this method of working have been adequately demonstrated by the recent work in Mexico referred to by Dr. Barton, when faults have been located with precision by siting stations at intervals of 20 m. and even less.

From an economic standpoint, however, it is desirable to restrict the number of stations, so that the close network portion of the survey should be confined to the areas in which it is absolutely essential. The experienced geophysicist will be able to select these areas with discretion, and to locate his stations so as to provide the desired information in the most efficient manner.

On the other hand, it is dangerous to place too great a reliance on the indications of a series of scattered stations, and corroboration should most certainly be obtained from an additional number of carefully placed stations before arriving at any important conclusions. One ventures to suggest that if these precautions had been taken in some of the cases referred to by Dr. Barton, the number of recorded failures would have been reduced appreciably.

W. Schweydar, Potsdam, Germany (written discussion).—The measurements of the horizontal variation of gravity with the torsion balance have become very important in recent years. Especially in the United States and Mexico, great districts have been surveyed and much information about the usefulness of this method has been gathered. However, many geologists still have little knowledge of its application, because the results have not been published.

Therefore Dr. Barton's paper will be welcomed, especially as it gives such a clear and detailed explanation of the method and its application. He explains the theory of the balance and of the method in an original manner, without employing higher mathematics and also outlines the field work. The important correction of the measurements in respect of the irregularities of the terrane is only generally described, but the description gives a useful idea of the influence of the several forms of the terrane masses. Apart from the latter influence, the instrument is deflected by every geological anomaly, under the earth surface. Mr. Barton divides these anomalies into five orders of magnitude. The first four orders comprise the usually deeply situated great geological formations; the fifth order, produced by the small surficial irregularities of mass in the subsoil, rather close to the instrument, hinder the accuracy of the conclusion to the deeper anomalies. Mr. Barton deals thoroughly with this important question and I join him in his method of trying to eliminate these fifth-order influences. His diagrams of the variation of the gradients and differential curvatures across (*a*) vertical fault, (*b*) 30° fault, (*c*) symmetrical infinite (anticline) ridge, (*d*) asymmetrical infinite (anticline) ridge, (*e*) ridge of finite extension, clarify the discussion of the measurements.

Mr. Barton explains the well known fact that the gravimetrically determined top of an asymmetrical anticline does not coincide with the true top. It is possible to tell in which direction the gravimetrically determined top is shifted. Also, in some cases it is possible, by an approximative method, to estimate the amount of the shifting.

Mr. Barton shows that in some cases the depths and extension of the buried formation can be calculated. It is known that in the case of an ore bed the depths can be rather exactly calculated. This quantitive interpretation is possible only if the average form of the buried body is known to be of simple construction. From a mathematical point of view, the observed distribution of the gradients etc. can be explained by many anomalies of density but this number is reduced by the geological probabilities. If the geology of the district is now known, or only vaguely known, the calculation of the depths is possible only in rare cases. The easiest problem is a buried vertical fault; also an anticline of a simple form can be quantitatively interpreted. In the more difficult cases Mr. Barton recommends a method also used by myself, that of calculating a hypothetical body which explains the measurements. In many cases such a body can give to the geologist a good basis for his study, and in geologically unknown countries, the balance gives, in most cases, by a qualitative analysis a valuable exploration of faults, anticlines and beds. I agree with Mr. Barton that very often a refined analysis is necessary. One has to distinguish between conformably and unconformably folded strata. If the deeply situated layers are folded with rather steep flanks, their influence on the balance can be very considerable. We have examples of this from Mexico, Texas and Oklahoma. If the particular layer in which the geologist is interested is conformably folded in relation to the other layers, then there is little difficulty in finding its top, but if it is unconformably folded, this top can

in some cases be found only by a very refined analysis made by a skilled geophysicist. Such refined analysis is necessary in South Oklahoma and also in West Texas. Mr. Barton gives an example of the Fox oil field. He used for his method the isogams, but I think that the form of the gradient curves gives a better indication.

Mr. Barton also gives a very interesting general view of the successes of the torsion balance in the United States and I agree with him that this balance is an excellent method for the reconnaissance and detail mapping work in connection with large buried structures and certain deposits. For some problems, of course, I agree with him that the seismological method is more practical. I am sure that in the future no serious geological institution will fail to use these two methods.

D. C. Barton (written discussion).—Supplementing Mr. DeGolyer's description of our work, I would say that the Southern Division, Amerada Petroleum Corpn., imported two balances during September or October, 1922. The Roxana Petroleum Corpn. imported two balances in November or December, 1922, and took its first station on Dec. 26, 1922. The first station with the Amerada instruments was taken by A. Gilmour and myself on Dec. 29, 1922, but as Mr. Gilmour and one instrument were immediately transferred to Mexico, we did not get effectively to work until March, 1923. About one-third of the domes we mapped gave results of approximately the same order of brilliance.

Of the salt domes mapped by us, only Markham gave as indistinct results as those indistinct gravity highs drilled by the Rycade Oil Corpn. Each of those highs had some surface indication that almost would have warranted drilling: oil shows at Beasley, H_2S water of the right type at Hunting Bayou, and water that seemed more probably a deep-seated than surface water at Lake Arthur. We knew that our results at those localities were not guaranteeing the presence of a dome, but with the presence of the surface indications, we hoped that we had domes, which, like Markham, gave very indistinct results.

Replying to Mr. Lancaster-Jones, I would say that the exigencies of commercial work are different from those of scientific work and place different limits to the practicable amounts of arduous adjustment and analysis undertaken. Also, there are two tendencies prevalent: to expect the geophysicist to make his interpretations without sufficient time or clerical assistance for the necessary adjustments of observations and analysis of the results; and to overemphasize the difficulties and impracticability of the torsion balance in certain complex situations, especially those of considerable relief. In emphasizing the arduousness of analysis to counteract the former tendency, the writer did not mean to support the latter tendency. Such analysis is necessary in commercial work as well as in scientific; the necessity of it will be recognized more and more, yet there will be limits to the commercial practicability of tedious and time-consuming analysis.

Mr. Shaw quotes the author as saying that any distribution of gradient and curvature values at the surface can be produced by only one body, definite in form, relative density and position. The writer would not now make that statement with quite the same definiteness. It may be true mathematically, but within the limits of error of field observation in most areas and with the usual indefiniteness of the knowledge of the specific gravity relations in the subsurface, a family of bodies can be obtained which will give the same anomaly. A partial discussion of the question was taken up in a paper by the author in the August symposium of the A. I. M. E.[10]

F. W. Lee, Washington, D. C.—The Bureau of Mines has gradually recognized the importance of geophysical work and is endeavoring to establish some of the results in an experimental way, and we do appreciate the paper by Dr. Barton. It

[10] D. C. Barton: Calculations in the Interpretation of Observations with the Eötvös Torsion Balance. See page 480.

is obvious that this work cannot be done by an amateur; you might say that a real professional is needed to carry on gravimetric work. We have tried gravimetric work on a piece of property in Colorado in which we endeavored to check the various methods against each other, but we have not so far had any official record of that checking. It would be well to check the seismic method against the gravimetric method in cases where the underground contour is largely a matter of speculation.

The Russians have done much work with gravimetric and seismic methods, and recently a considerable amount of work on X-ray penetration, a translation of which we intend to issue as an information circular.

S. J. Jennings, New York, N. Y.—There is one point that I think would be of great interest to those who are responsible for the financial part of the program of geophysical prospecting; that is the relative cost of the two gravitational methods. The last speaker said that one method should be checked by some other. If two methods of geophysical prospecting are to be used, is not the expense likely to be as great as that of drilling a few holes in favorable places indicated by the surface geology?

D. C. Barton.—The cost of doing torsion balance work is greater when done by a consulting geophysical company than when done by administration. In all Mr. DeGolyer's work of which I had charge, it cost us, including depreciation and everything, $1000 to $1200 a month per single torsion balance. That is based on two stations per day. It costs a little more per station to take three stations, and yet a little more to take four stations per day.

S. J. Jennings.—Do I understand that two stations a day, which would be 50 stations a month, would cost about $1200?

D. C. Barton.—Yes. The standard consultant rates run about $5500 a month with a two-instrument party. The figures which I gave cover the cost to a company that does its own work.

W. J. Millard, Tulsa, Okla.—In reconnaissance work, how much would it cost per square mile?

D. C. Barton.—In northwestern Cooke County, 6 linear miles of traverse per day can be covered in reconnaissance which will show the major features. The traverses need to be only some four miles apart. Two to three months' work with a single torsion balance will give a fairly detailed reconnaissance of the county. In the detailing of the smaller structures in the area or the detailing of most structures in which the mining geologist is interested, it may be necessary to put a large number of stations in a small area. The cost of very detailed work of that type will be less per instrument per month than in oil work, as one crew can look after several instruments and as moving is much simpler.

E. L. DeGolyer.—I have had a good deal of experience in meeting the cost of geophysical work in the monthly accounts, and my finding is that it is extremely expensive. The only question, I think, from a practical standpoint, is whether it can be made worth the expense. There is some evidence, or the beginning of some evidence, for the Gulf Coast. So far as geophysical work is concerned, the results on the Gulf Coast of the United States are the one great outstanding achievement. The torsion balances have shown some ability to find outlying salt domes; the seismic or sonic methods have shown very considerable ability in that direction.

I am sorry we have no paper on the seismic or sonic method, but I suspect the thing is so mixed up with the patent situation that it is rather difficult to get a good paper. The seismic method or sonic method practically replaced the torsion balance, which was first in the field as an instrument to be used in the reconnaissance of wide

areas for salt domes. Much more ground can be covered, and it can be covered more rapidly. I believe our own people, when asked how much ground they can cover in a search for salt domes, usually say 100,000 acres per month. They are really disappointed if their figure is not above 200,000. In some cases, where the conditions were ideal, as much as 40,000 acres has been covered in a day for a short time.

The seismic method is very exact. Recently, as a result of a large survey for salt domes, we had found at least two prospects which we were told were certainly salt domes. Both were out in the water, one of them about two miles from shore and the other about three miles from shore. We drove piling, built rigs and drilled wells. One of these wells has not yet found a salt dome, but apparently the location of the well has a great deal to do with that.

However, the last time I was in Houston, neither of the wells had yet found any salt-dome material, although both had passed the depth at which the physicists thought salt would be found. So I gathered together the mystery men on this subject to go over the results. I brought in a man who had had a great deal of experience with the method but who was not at all acquainted with the results on these particular cases. After going over what are called the fans and looking at the leads, which are the speeding up, the saving in time interval between the shot and its reception—the leads give the first indication of the existence of salt—and after going over the profiles which had been shot across these domes, from which the velocities of the surrounding rock and of the dome material itself can be deduced, this man simply said, "Well, there are two salt domes there, that is all there is to it. In the profiles, there are velocities in excess of 5000 m. per sec. and there is nothing on the Gulf Coast that will give that velocity except salt. Certainly there are salt domes there."

That was very reassuring. It is always reassuring to talk to a technical man who is certain of his results. This man said that one well was in a bad location but the other one was all right, and he believed we would get salt. Since he was so confident about the second one it seemed to me that here was a good chance for the method to prove itself. In that case it did. This particular project impressed itself tremendously on me.

At the time the salt dome was found to be of economic importance as a locus of oil fields, when Captain Lucas drilled his first successful well, Spindletop, in 1901, a great many of the salt domes were known. They expressed themselves topographically distinctly, and in the succeeding two or three years the existence of most of these domes was proved by the drill. At the end of a period of two or three years following the original discovery perhaps 46 domes had been drilled. In other words, by about 1905 practically all of these salt domes were known. In the 20 years that followed, a great amount of drilling was done on the Gulf Coast, following out meager indications such as little sulfur-water wells or springs, or a slight topographic irregularity. A tremendous amount of drilling was done; millions of dollars were spent in searching for new salt domes, and the total result of all of that tremendous expenditure was just exactly five additional domes. I should think that the cost of the failures, the dry holes, ran up literally into the millions during those 20 years.

The geophysical methods came in. The Nash dome, in 1922, was the first to be found by geophysical method. The sonic or seismic method was taken up immediately and in the three or four years immediately following its introduction probably 30 new domes were found, so that the record as to ability to prospect was exceedingly favorable.

Bearing this in mind and having this man so extremely confident about the dome out under the lake three miles from shore, it was a source of great satisfaction to me, a day or two later, to get a wire from the man in charge of the operation saying that 20 or 30 ft. below the point of the well where we had been rechecking our data, it had actually gone into salt.

The Gulf Coast has been an extremely favorable area for the use of all of these methods because the structures stand out so distinctly; that is to say, the country rock is on a comparatively level line and comparatively homogeneous and the great salt domes that are thrust into it are comparatively isolated features and stand up like lone peaks on a plain.

With regard to Mr. Lee's remark about checking these methods, I think that we shall probably get to the point where we shall use one method to tell us certain things and another method to give us additional data, and that really we will be able to make very considerable progress in digging out structures, so that at the end of our geophysical work and before drilling has commenced, we shall have reached the point where usually we are now after we have drilled three or four or five dry holes. I think, however, that what Mr. Lee had in mind particularly was a sort of experimental period of checking the methods against each other rather than the necessity of using one to check another for commercial work. If one had a pattern of the type that Dr. Barton showed for the Nash dome (Fig. 9) it would be a waste of time to spend money using any other method to go in and confirm the results. Likewise, if a person gets a perfectly definite salt-dome result by the use of the seismic or sonic method, it would be a waste of time and a waste of money to check that by another method.

D. C. Barton.—In connection with the statistics on finding domes, I think it is interesting to note that from 1905 to 1924 some five or six domes were found in the Gulf Coast area by geology and by accident. The company of which Mr. DeGolyer was talking admits the discovery of two domes. We happen to know accurately that it has two more domes, the discovery of which it will not admit.

One of the interesting things in connection with this is the way one sometimes uses the other fellow's dope. There happened to be two G. R. C. groups, one working for Mr. DeGolyer's company and the other for a rival company. The other group picked up the salt dome at the same time that Mr. DeGolyer's group picked it up; so we happen to know that the salt dome is there. We cannot find out whether or not Mr. DeGolyer's company actually has found the fourth dome, although there is a rumor that it has discovered domes five and six. The seismic work of that company has been going on for six months and must cost about $35,000 a month. Four, and perhaps six domes, have been discovered at that cost.

[The record of the Louisiana Land and Exploration Co. referred to above has now, June 1, 1928, eight and probably nine discoveries of salt domes previously unsuspected—Calcasieu Lake, Cypremort Point, Kings Lake, Caillou Lake, Pelto Lake, Caillou Island, Lake Barre domes, one dome six miles north of the Pelto Lake dome, and one additional dome which is supposed to have been discovered. The salt has been drilled into on the Cypremort Point dome. One well was drilled on the flank of the Calcasieu Lake dome. The other domes are untested.]

Mr. DeGolyer spoke of 48,000 acres a day as their maximum. Here is one of their actual records: One week they averaged 10 shots a day, which is good shooting, and on one day they made 18 shots and covered about 50,000 acres. I did not happen to know how many acres they covered that week, but if they covered about 50,000 acres with 18 shots, and averaged 10 shots a day for the whole week, they must have covered about 150,000 acres for that week. I know the chief, and know that he is a man who would not shoot for acreage with a sacrifice of accuracy. That was an old record, made by one G. R. C. group of Mr. DeGolyer's consulting company a year and a half ago, when they were not going as well as they are at the present time.

S. F. Kelly, New York, N. Y.—Mr. DeGolyer said that this work was quite expensive, but did not tell us how much it would cost to get the same amount of information of equal value by drilling or geological work. In other words, has anyone any idea of the comparative cost of geophysical and straight geological work?

E. L. DeGolyer.—In the Gulf Coast area, the geological work will not give the information, so that puts it out of the question. There is only one other method that does give the information and that is by drilling wells, which would be tremendously expensive, because in order to prove the ground positively or negatively it would probably be necessary to drill a pattern of wells about 1½ mile apart. From the standpoint of the shooting reconnaissance, we believe we can do it for 11 to 15 c. per acre, according to how the land lies and the ease of communication, accessibility, and all that sort of thing.

D. C. Barton.—I have understood that ordinarily it figures 4 to 8 c. per acre with the seismograph, and 15 c. with the torsion balance for reconnaissance for salt domes for the operation of the geophysical survey but not for the blocking of acreage. For detail work with the torsion balance, for a standard-size dome with a diameter of about 1 mile, about 60 stations will be necessary; that is, about six weeks' work. The cost would be $2000 to $4500 for about 1500 acres.

R. C. Wells, Washington, D. C.—It is pretty generally recognized, I think, from Darton's Geological Survey Report, that there is a very large body of salt covering the Panhandle district, part of New Mexico and extending up to Kansas, but when he outlined that body of salt there were areas as large as counties where he could not draw the line exactly. Would not one of these methods be efficient for drawing the limits of that salt area more exactly?

D. C. Barton.—The torsion balance works well on most granite ridges. There are a number of these under cover from which we have not yet heard. The circular anomaly of which Mr. DeGolyer spoke is on the east edge of the salt basin. In our work, we picked up what I am satisfied are two granite ridges, but they have not yet been drilled.

R. C. Wells.—You are working along the edge of the salt basin more or less?

D. C. Barton.—Not necessarily.

S. H. Hamilton, Philadelphia, Pa.—I have the cost of seismographic work done on a small section in Venezuela. It amounts approximately to 16 c. an acre.

E. L. DeGolyer.—That type of work is susceptible to exactly the same variations that Dr. Barton mentioned with regard to other work. It varies with the method of working and the object sought. In the Gulf Coast region, I believe they are trying to make their shots about five miles long, and of course it makes a great difference how many receiving stations are used.

D. C. Barton (written discussion).—Research since the February meetings has shown definitely that for the purposes of practical field work, the statement on page 440 does not hold true. Studying rather long anticlines with a symmetrical cross-section, we have found that a rather complex family of forms can produce the same observed gradient profile, within the accuracy of field observation. Within certain limits, the depths to the top of the anomalous mass, depth to the bottom of the anomalous mass, the slope of the flanks of the mass, and its specific gravity may vary without affecting the gradient profile within the accuracy of field observation. Even if the specific gravity is constant, there is a considerable possible range of the depths and of the slope of the flanks. For a preliminary report on one phase of the possible variation of the anomalous mass producing a given gradient profile see the author's paper, Calculations in the Interpretation of Observations with the Eötvös Torsion Balance.[11]

Drilling since February has confirmed the presence of the salt dome near Dewalt (mentioned on page 447) and disclosed the presence of a promising oil field.

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[11] See page 480.

Calculations in the Interpretation of Observations with the Eötvös Torsion Balance

By Donald C. Barton,* Houston, Texas

(Boston Meeting, August, 1928)

Success in the use of the Eötvös torsion balance method of mapping geologic structure depends largely on the accuracy in the interpretation of the observed results. Skill in that interpretation depends (1) on the knowledge of the types of the variation of gravity, the gradient of gravity horizontally, and of the differential curvature produced by bodies of different sizes, shapes, dimensions, and densities; (2) on the knowledge of the possible types of structural bodies which might produce the observed anomalies; (3) on the empirical knowledge gained by experience not only in the geological interpretation of torsion balance surveys, but of interpretations of inadequate geological data; and (4) on a knowledge of the possible and probable types of geologic structures in the area of the survey.

The knowledge of the anomaly which a known body will produce and conversely the knowledge of the body or bodies which will produce a known anomaly may be gained by empirical experience and by mathematical study. The knowledge gained by empirical experience is fragmentary and inexact. The geology of most structures actually is known very superficially. The surface geology may be known accurately. If the structure is petroliferous, the subsurface geology of the oil field will be known down to 3000, 4000, 5000 ft. or in a few oil fields down to 7500 ft., but the deeper geology will be unknown and the metamorphic, igneous, or massive limestone basement below the reach of the drill may contrast sharply in density with the overlying sediments.

Away from the oil field, the subsurface geology is known only from scattered drill holes, which do not go to the basement. A fair but not necessarily accurate knowledge of the geology of the nonproductive area can be obtained by correlation of the wells but the degree of its accuracy in general is proportional to the density of the occurrence of the wells. The geology below the bottom of the wells however is unknown.

An anomaly mapped by the torsion balance is the sum of the effects of the contrasts in mass of its whole geologic surroundings. If the dimensions of a body of abnormal density are large enough, depth or distance makes no difference in the effects on the gradient and differential curvature; and as a matter of fact, in many places, the basement and irregulari-

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ties of density within the basement profoundly affect the torsion balance from a depth of several miles. What is known of the total geologic setting producing a given torsion balance anomaly is only a small part of the whole. The geologic setting of such an anomaly furthermore, in most all places, is complicated and empirically it is difficult to distinguish clearly the effect of one portion from the effect of the rest. The laws of variation of the gradient and differential curvature therefore can not be deduced from empirical knowledge with sufficient accuracy to be of much practical value. That empirical knowledge, however, is of importance in checking and orienting the practical application of the deductions from mathematical analysis.

Examples of gradient and differential curvature profiles are given in the literature for a few bodies of simple geometrical shape and homogeneous density in a medium homogeneous in density. Except for spheres, most of those examples are for bodies infinite at right angles to the section and bounded by two horizontal plane faces and one or two plane vertical or inclined faces. Such profiles are sufficient to give a student of interpretation a rough qualitative conception of the gradient and differential curvature produced by only a few simple types of bodies.

Gradient and Differential Curvature

The main formulas[1] by which a student of interpretation can calculate for himself the gradient and differential curvature produced by different types of bodies in different types of situations are:

(*A*) For an infinite horizontal slab (Fig. 1*A*), bounded by[2]

$$x = x_1 \text{ and } +\infty, \quad y = \pm\infty, \quad z = z_1 \text{ and } z_2 \ (z_1 < z_2)$$

$$U_{xz} = K\delta \log_e \frac{x_1^2 + z_2^2}{x_1^2 + z_1^2} \tag{1}$$

[1] For a compilation of the more usable formulas see E. Lancaster-Jones: The Computation of Eötvös Gravity Effects (see page 505). Many of the formulas are given in a very condensed notation which is rather forbidding to the uninitiated but which will become clear after a little study. Also,

W. H. Fordham: Oilfinding by Geophysical Methods. *Jnl.* Inst. Petr. Tech. (1925) **11,** 448.

Karl Mader: Ein Beispiel der gravimetrischen Tiefenforschung im Wiener Becken mit der Drehwage von Eötvös. *Österr Monatsschr. f. öffentl. Baudienst u. d. Berg. u. Hüttenwesen* (1924) **5,** 121–126.

J. A. A. Mekel: Theorie von het Tektonisch-Gravimetrisch Onderzoek. Delft, 1928.

[2] U_{xz}, U_{yz}, U_{xy_1} and U_Δ are condensed notations, respectively, for $\frac{\partial^2 U}{\partial x \partial z}$, $\frac{\partial^2 U}{\partial y dz}$, $\frac{\partial^2 U}{\partial x dy}$, and $\left(\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}\right)$ where the z axis is vertical and the x and y axes are horizontal and at right angles. In field work, by convention the $+x$ axis is chosen as north and the $+y$ axis as east. In calculations, the x axis is usually chosen perpendicular to any linear structure.

$$U_{\Delta} = -K\delta 2\left(\text{arc tan }\frac{z_2}{x_1} - \text{arc tan }\frac{z_1}{x_1}\right) \tag{2}$$

$$U_{yz} \text{ and } U_{xy} = 0$$

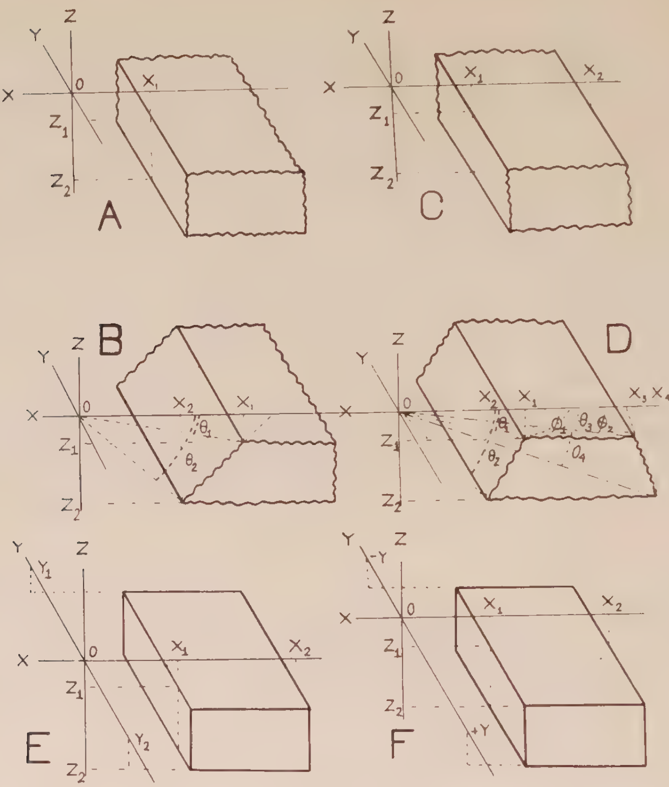

FIG. 1.—DIAGRAMMATIC SKETCHES OF BLOCKS AND SLABS CORRESPONDING TO FORMULAS (1) TO (14).

(*B*) For a similar slab Fig. 1*B*, bounded by an inclined face instead of a vertical face as in Fig. 1*A*:[3]

$$U_{xz} = K\delta\left(\sin^2\phi \log_e \frac{x_2^2 + z_2^2}{x_1^2 + z_1^2} - (\theta_2 - \theta_1)\sin 2\phi\right) \tag{3}$$

$$U_{\Delta} = -K\delta\left(\frac{1}{2}\sin 2\phi \log_e \frac{x_2^2 + z_2^2}{x_1^2 + z_1^2} + 2(\theta_2 - \theta_1)\sin^2\phi\right) \tag{4}$$

$$U_{yz} = 0 \quad U_{xy} = 0$$

(*C*) For an infinite horizontal prism with vertical faces (Fig. 1*C*) bounded by $x = x_1$ and x_2, $y = \pm\infty$, $z = z_1$ and z_2

$$U_{xz} = K\delta \log_e \left(\frac{x_1^2 + z_2^2}{x_1^2 + z_1^2} \cdot \frac{x_2^2 + z_1^2}{x_2^2 + z_2^2}\right) \tag{5}$$

[3] W. H. Fordham or E. Lancaster Jones: *Op. cit.*

$$U_\Delta = -K\delta 2\left(\text{arc tan}\frac{z_2}{x_1} + \text{arc tan}\frac{z_1}{x_2} - \text{arc tan}\frac{z_1}{x_1} - \text{arc tan}\frac{z_2}{x_2}\right) \tag{6}$$

$$U_{yz} = 0 \quad U_{xy} = 0$$

(*D*) For an infinite horizontal prism with inclined faces (Fig. 1*D*),

$$U_{xz} = K\delta\left(\sin^2\phi_1 \log_e\frac{x_2^2 + z_2^2}{x_1^2 + z_1^2} - \sin^2\phi_2 \log_e\frac{x_4^2 + z_2^2}{x_3^2 + z_1^2}\right.$$
$$\left. -(\theta_2 - \theta_1)\sin 2\phi_1 + (\theta_4 - \theta_3)\sin 2\phi_2 \right) \tag{7}$$

$$U_\Delta = -K\delta\left(\frac{1}{2}\sin 2\phi_1 \log_e\frac{x_2^2 + z_2^2}{x_1^2 + z_1^2} - \frac{1}{2}\sin 2\phi_2 \log_e\frac{x_4^2 + z_2^2}{x_3^2 + z_1^2}\right.$$
$$\left. +2(\theta_2 - \theta_1)\sin^2\phi_1 - 2(\theta_4 - \theta_3)\sin^2\phi_2\right) \tag{8}$$

$$U_{yz} = 0 \quad U_{xy} = 0$$

(*E*) For a finite rectangular prism with vertical and horizontal faces parallel to the axes and symmetrically disposed in regard to the plane then the xz axes (Fig. 1*E*), bounded by $x = x_1$ and x_2, $y = y_1 = -y_2$, $z = z_1$ and z_2

$$U_{xz} = K\delta \log_e\left(\frac{\sqrt{x_1^2 + y^2 + z_1^2} + y}{\sqrt{x_1^2 + y^2 + z_1^2} - y} \cdot \frac{\sqrt{x_1^2 + y^2 + z_2^2} - y}{\sqrt{x_1^2 + y^2 + z_2^2} + y} \cdot \right.$$
$$\left. \frac{\sqrt{x_2^2 + y^2 + z_1^2} - y}{\sqrt{x_2^2 + y^2 + z_1^2} + y} \cdot \frac{\sqrt{x_2^2 + y^2 + z_2^2} + y}{\sqrt{x_2^2 + y^2 + z_2^2} - y}\right) \tag{9}$$

$$U_\Delta = 2K\delta\left(\text{arc tan}\frac{yz_2}{x_2\sqrt{x_2^2 + y^2 + z_2^2}}\right.$$
$$- \text{arc tan}\frac{x_2z_2}{y\sqrt{x_2^2 + y^2 + z_2^2}} - \text{arc tan}\frac{yz_1}{x_2\sqrt{x_2^2 + y^2 + z_1^2}}$$
$$+ \text{arc tan}\frac{x_2z_1}{y\sqrt{x_2^2 + y^2 + z_1^2}} + \text{arc tan}\frac{yz_1}{x_1\sqrt{x_1^2 + y^2 + z_1^2}}$$
$$- \text{arc tan}\frac{x_1z_1}{y\sqrt{x_1^2 + y^2 + z_1^2}} - \text{arc tan}\frac{yz_2}{x_1\sqrt{x_1^2 + y^2 + z_2^2}}$$
$$\left. + \text{arc tan}\frac{x_1z_2}{y\sqrt{x_1^2 + y^2 + z_2^2}}\right) \tag{10}$$

$$U_{xy} = 0 \text{ and } U_{yz} = 0 \tag{11}$$

(*F*) For the case where the block is unsymmetrically disposed in regard to the plane through the XZ axes, *i. e.*, where $y = y_1$ and y_2 (Fig. 1*F*),

$$U_{xz} = K\delta \log_e\left[\frac{\sqrt{x_1^2 + y_1^2 + z_1^2} + y_1}{\sqrt{x_1^2 + y_2^2 + z_1^2} + y_2} \cdot \frac{\sqrt{x_1^2 + y_2^2 + z_2^2} + y_2}{\sqrt{x_1^2 + y_1^2 + z_2^2} + y_1} \cdot \right.$$
$$\left. \frac{\sqrt{x_2^2 + y_2^2 + z_1^2} + y_2}{\sqrt{x_2^2 + y_1^2 + z_1^2} + y_1} \cdot \frac{\sqrt{x_2^2 + y_1^2 + z_2^2} + y_1}{\sqrt{x_2^2 + y_2^2 + z_2^2} + y_2}\right] \tag{12}$$

$$U_{yz} = \text{interchange } y \text{ for } x \text{ and } x \text{ for } y \text{ in (12)} \tag{12a}$$

$$U_{xy} = K\delta \log_e \left[\frac{\sqrt{x_1^2 + y_1^2 + z_1^2} + z_1}{\sqrt{x_1^2 + y_1^2 + z_2^2} + z_2} \cdot \frac{\sqrt{x_1^2 + y_2^2 + z_2^2} + z_2}{\sqrt{x_1^2 + y_2^2 + z_1^2} + z_1} \cdot \frac{\sqrt{x_2^2 + y_1^2 + z_2^2} + z_2}{\sqrt{x_2^2 + y_1^2 + z_1^2} + z_1} \cdot \frac{\sqrt{x_2^2 + y_2^2 + z_1^2} + z_1}{\sqrt{x_2^2 + y_2^2 + z_2^2} + z_2} \right] \tag{13}$$

$$\begin{aligned} U_\Delta = -K\delta\Bigg(&\arc\tan \frac{y_2 z_2}{x_2\sqrt{x_2^2 + y_2^2 + z_2^2}} - \arc\tan \frac{x_2 z_2}{y_2\sqrt{x_2^2 + y_2^2 + z_2^2}} \\ &+ \arc\tan \frac{y_1 z_1}{x_2\sqrt{x_2^2 + y_1^2 + z_1^2}} - \arc\tan \frac{x_2 z_1}{y_1\sqrt{x_2^2 + y_1^2 + z_1^2}} \\ &+ \arc\tan \frac{y_2 z_1}{x_1\sqrt{x_1^2 + y_2^2 + z_1^2}} - \arc\tan \frac{x_1 z_1}{y_2\sqrt{x_1^2 + y_2^2 + z_1^2}} \\ &+ \arc\tan \frac{y_1 z_2}{x_1\sqrt{x_1^2 + y_1^2 + z_2^2}} - \arc\tan \frac{x_1 z_2}{y_1\sqrt{x_1^2 + y_1^2 + z_2^2}} \\ &- \arc\tan \frac{y_1 z_1}{x_1\sqrt{x_1^2 + y_1^2 + z_1^2}} + \arc\tan \frac{x_1 z_1}{y_1\sqrt{x_1^2 + y_1^2 + z_1^2}} \\ &- \arc\tan \frac{y_2 z_2}{x_1\sqrt{x_1^2 + y_2^2 + z_2^2}} + \arc\tan \frac{x_1 z_2}{y_2\sqrt{x_1^2 + y_2^2 + z_2^2}} \\ &- \arc\tan \frac{y_1 z_2}{x_2\sqrt{x_2^2 + y_1^2 + z_2^2}} + \arc\tan \frac{x_2 z_2}{y_1\sqrt{x_2^2 + y_1^2 + z_2^2}} \\ &- \arc\tan \frac{y_2 z_1}{x_2\sqrt{x_2^2 + y_2^2 + z_1^2}} + \arc\tan \frac{x_2 z_1}{y_2\sqrt{x_2^2 + y_2^2 + z_1^2}} \Bigg) \end{aligned} \tag{14}$$

In the formulas K is the gravity constant and is 66.7×10^{-9}; δ is the specific gravity of the block, or is the difference between the specific gravities of the block and of the surrounding medium, *i. e.*, $\delta = \delta_1 - \delta_2$, where δ_1 is the specific gravity of the block and δ_2 is the specific gravity of the medium in which the block lies; the logarithms used in the formulas are the natural logarithms; $\log_e A = 2.3026 \log_{10} A$ and in practice, it is usually easiest to combine the factor 2.3026 with 66.7×10^{-9} and use the coefficient 153.65×10^{-9} with common logarithms; the angles are measured in radians.

Simplification of Actual Geologic Bodies

Most geological bodies are not satisfactorily approximated by such simple geometrical bodies as those to which formulas (1) to (14) correspond and for the purposes of the mathematical calculation of the gradient and of the differential curvature effects, it is necessary to split most geologic bodies into a series of simple bodies for which formulas are available and are not too complicated. The gradient and differential curvature effects are calculated for each of the simple bodies and then

summed to get the respective effect for the whole body. If the ridge of Fig. 2 is infinite at right angles to the plane of the section, it may be represented for example by the four prisms of Fig. 2*A* or by the seven rectangular prisms of Fig. 2*B* and the gradient and differential curvature effects can be calculated for each prism of Fig. 2*A* by formulas (7) and (8), respectively. If the ridge is finite at right angles to the plane of the section, it is necessary to use the rectangular finite prisms of Fig. 2*B*.

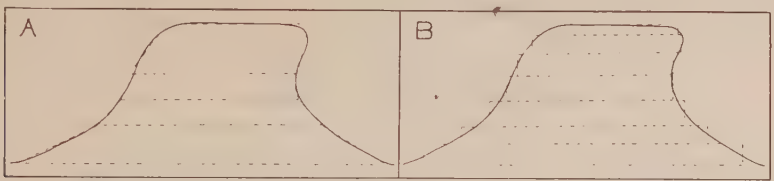

Fig. 2.—Diagrammatic cross-section of a structural ridge showing approximation by series of mathematically simpler blocks.

The gradient and differential curvature at any point can be calculated by formulas (12), (12*a*), (13), and (14) or along a horizontal axis of symmetry by formulas (9) and (10).

The number of times that such a formula as (5), (6), (7), (8) has to be calculated to obtain the gradient or differential curvature profile for a geologic body is very considerable. Each calculation of formula (5) for example gives the gradient for a single rectangular block at a single station. The gradient and differential curvature profiles for bodies of the general cross-section shown in Fig. 2 are of the general form shown in Fig. 3. The minimum number of points necessary to determine the U_{xz} curve in Fig. 3 with rough accuracy is five and with good accuracy is nine points. For the U_Δ curve, seven points are the minimum and 13 or more are preferable. To obtain the gradient profile for the infinite ridge of Fig. 2*A* formula (7) must be calculated at least 20 and preferably 36 times; for the ridge as in Fig. 2*B*, formula (9) would have to be calculated at least 35 times and preferably 63 times. To obtain the differential curvature for the same two cases, a minimum of 28 (preferably 52) calculations of formula (8) and a minimum of 49 (preferably 91) calculations of formula (10) would have to be made. If the line of section in the case of Fig. 2*B* were not a plane of symmetry for the block, formula (12) would be used in place of (9), (14) in place of (10), and as many calculations of the U_{yz} formulas similar to (12) would have to be made as for (12), and as many calculations of (13) as for (14). As the available tables of square

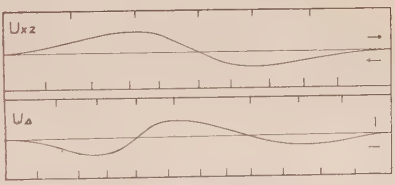

Fig. 3.—Generic type of gradient and differential curvature profiles produced by structural ridges.

roots are not carried out far enough, the 35 (preferably 63) calculations of formula (12) would require 280 (504) operations of square root on an eight-place calculating machine.

The calculation of these formulas is more tedious than difficult but yet it is dangerous to entrust them to a calculator who has not had considerable mathematics or experience in calculation. If the calculation is carried on constantly for a considerable period of time, considerable facility is acquired, but to a geophysicist or a geologist who wishes to use such formulas as (7) to (14) only rarely, they are quite formidable.

Interpretation of Torsion Balance Anomalies by Inspection

The reverse case, the determination of the form, dimensions and depth of the unknown body producing an observed distribution of gradient and differential curvature values is the primary interest of the geophysicist. Considerable facility in qualitative interpretation can be acquired by him from experience and from a thorough study of the types of gradient and differential curvature anomalies produced by the different types of possible structural situations. From inspection of the results of a torsion balance survey of a rather simple structural situation, a well-trained and experienced geophysicist may be able to say that the gradient and differential curvature anomalies are the effects of a such and such shaped body at slight, moderate, or great depth. But structural situations that are quite different to the commercial geologist give torsion balance effects that are similar. The exact position of the crest of a structure is important to the oil geologist, but if the structure is asymmetric, the position of the crest of the structure is difficult to determine by inspection of the torsion balance results with sufficient accuracy for many purposes of the oil geologist. If the anomaly due to this structure is obscured by anomalies of lesser or greater size, the determination of the position of the structure by simple inspection of the results is yet more difficult. Semiquantitative calculation of the probable structure which is giving rise to the observed torsion balance anomalies is no panacea for accurate interpretation, but it does bring out many relations that are not evident in a simple inspection of the torsion balance results; it throws out as impossible or improbable certain of the alternative structural possibilities that may seem equally probable under simple inspection of the results; it will determine more accurately the position of the crest of the structure; it, in short, reduces considerably the uncertainty in the interpretation and is necessary to intelligent interpretations.

Trial and Error Calculation

The calculation of such cross-sections consists of trial and error calculations of a body that will give gradient and differential curvature effects

as closely as possible the same as those observed. From his knowledge of the gradient and differential curvature produced by different types of bodies, the geophysicist sketches a tentative cross-section of the structure as indicated to him by the observed results of the torsion balance survey. The calculator then splits the structure into blocks as in the case represented by Fig. 2, *A* and *B*, and calculates the gradient and differential curvature block by block. If, as rather usual, the calculated gradient and differential curvature profiles diverge considerably from the respective observed ones, it is necessary to add and subtract blocks from the cross-section until a close agreement is obtained between the calculated and observed profiles. The complete calculations have to be gone through for each block that is added or subtracted. It is very difficult furthermore to visualize quantitatively the effect of a given small mass simultaneously at several different stations, and it is difficult therefore to visualize how much mass should be whittled away or added at particular points in the section. Such calculations are entirely feasible but are extremely tedious. The quantitative calculations in regard to Hoskins Mound previously mentioned[4] were made by using formula (9) in the manner described. The calculation of the six radial sections at Hoskins Mound required about three months of steady calculation. The tediousness of such work therefore precludes much practical use of such calculation of cross-sections.

Short-cut Determination of Dimensions and Depths of Simple Bodies

A few short-cut methods have been suggested for the determination of the shape, depth and dimensions of simple bodies, but in general they are more of mathematical and theoretical interest than of practical importance. The most thoroughgoing study of such methods is that by Karl Jung.[5] By use mainly of the abscissas of the numerical maxima and minima of the gradient and differential curvature profiles, he has devised formulas and graphical methods for the recognition of certain simple bodies and the determination of their dimensions and depths. But in practice, the abscissa of the point of numerically maximum gradient or differential curvature can seldom be determined with sufficient accuracy to allow the use of Jung's formulas or graphical methods; in actual surveys the numerical maxima are obscured by greater and lesser order anomalies and except for bodies coming relatively close to the surface, the gradient and differential curvature profiles are very flat

[4] See page 448.

[5] Karl Jung: Die Bestimmung von Lage und Ausdehnung einfacher Massenformen unter Verwendung von Gradient und Krümmungs grösse. *Ztsch. f. Geophysik* (1927) **3**, 257.

curves at the points of numerical maxima. With the exception of the sphere, all of the bodies covered by Jung's formulas and graphical methods are infinite at right angles to the vertical plane of the section, have a cross-section of simple geometrical shape, and are of homogeneous density in a medium of homogeneous density. Most geologic structures, however, can not be treated satisfactorily as infinite in cross-section and as simple in cross-section as the bodies used by Jung; and as of homogeneous density in a homogeneous medium. Jung's paper covers practically all the work that has been published on such formulas and gives the references to the particular papers in which the suggestions are made. If formulas of this type are expressed qualitatively, they may have considerable applicability in getting a qualitative and very rough conception of the approximate form, depth and dimensions of bodies, but even semiquantitatively they must be used only with very great caution.

Graphical Methods

Graphical methods can be applied to these calculations with an enormous saving of time and effort and an enormous increase in simplicity and power. The following method was devised during our calculation of the cap rock at Bryan Heights, and in its earliest form reduced the time consumed in the calculation in the ratio of 8 hr. to 15 min.; that is, by about 97 per cent. The method is fast, simple and flexible. It handles the cases of finite bodies as easily as it does those of similar infinite bodies, the cases of bodies of tolerably irregular cross-section with nearly the same facility as those of similar bodies of geometrically simple cross-section, and moderately complex density situations with only a slight increase in the time and ease required for the simple case.

A characteristic set of the graphs is shown diagrammatically in Fig. 4. Each graph represents a vertical section along a line of symmetry. Each of the rectangles into which each graph is divided represents the vertical cross-section of a prism at right angles to the plane of the section. The respective horizontal length of each prism on the two sides of the plane of the section are equal and are determined by some given law for each graph; the half length of each prism in Fig. 4*A*, *B* is infinite, in Fig. 4*C*, *D* 1.5 times the depth of the base of the prism, and in Fig. 4*E*, *F* 0.5 times the depth of the base of the prism. The cross-section of each prism is so calculated that the prism produces a gradient (or differential curvature) of 1 Eötvös unit (in some cases 0.5, 2, 5, or 10*E*) at the origin with an assumed specific gravity of 1.0.

The construction of the graph exemplified by Fig. 4 is based on formulas (5) and (6) for the infinite case and formulas (9) and (10) for the finite cases. For technical ease in the construction of the graphs, to facilitate ease in the use of the graphs, and to make the accuracy of the

graph as nearly as possible the same at all depths, the intervals between the horizontal planes forming the upper and lower surfaces of the prisms are taken so that $z_{n+1} = \frac{10}{9} z_n$, where z_n and z_{n+1} are the depths to any

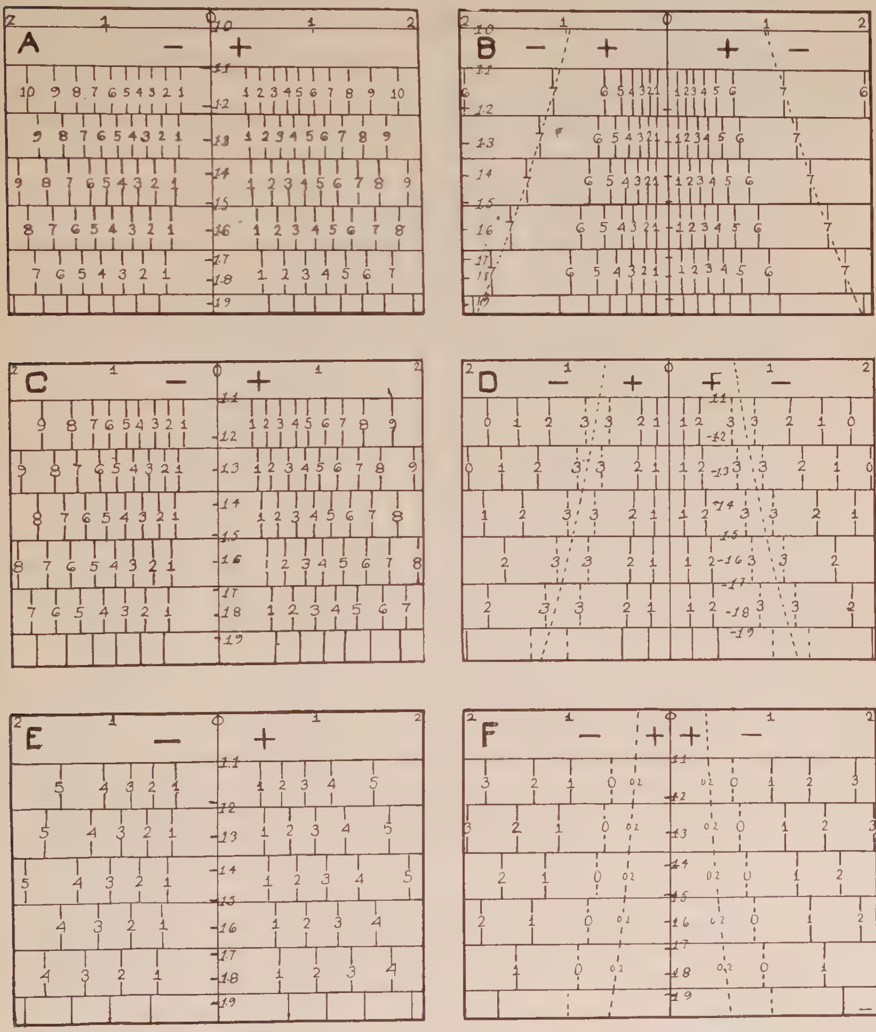

FIG. 4.—EXAMPLES OF GRAPHIC CHARTS FOR CALCULATION OF GRADIENT AND DIFFERENTIAL CURVATURE PROFILES OR OF STRUCTURAL SECTIONS FROM OBSERVED PROFILES. *A*, *C*, AND *E* ARE GRADIENT, AND *B*, *D*, *F* DIFFERENTIAL CURVATURE CHARTS.

two consecutive planes. If log z is plotted instead of z, the interval on the graph between the planes is a constant. The graphs are constructed either with the depths plotted numerically or logarithmically. The graphs with the depths plotted numerically have the advantage that

the structure is not distorted; the semilogarithmic charts have the advantage that a very much greater vertical range is possible on each chart and that it is possible to handle bodies coming much nearer to the surface. The charts of Fig. 4 are semilogarithmic and the scale divisions with slanting numerals give the depths in terms of the horizontal unit of distance.

Specific Gravity	2.25	Red	2.40	Blue	2.80	Black		
Depth	+	−	+	−	+	−	+	−
5	5.2	6.2	2.2	3.1	1.1	5.1		
	3.1				1.2	2.7		
					3.2			
6	1.1	9.1	5.9	2.1	7.1	4.2		
	1.8							
	2.0							
7	6.9	9.1	5.7	1.9	7.3	4.6		
8	2.2	5.8	6.0					
	2.3	5.3		2.4	5.5	6.1		
	5.4							
9	8.8	10.1	8.5	3.3	6.8	7.1		
10	7.0	8.1	12.7	5.0	7.0	5.3		
						5.2		
11	6.3	8.1	12.5	7.0	5.9	14.2		
	2.7							
12	12.0	14.5	13.3	7.7	5.4	12.3		
+Sums.........	66.8	76.3	66.8	32.5	50.5	66.8		
Totals........	−9.5		+34.3		−16.3			
×Specific gravity	−21		+82		−46			

Station: D5 Total = +15

FIG. 5.—SUGGESTED FORM FOR SCRATCH PAGE FOR USE WITH CHARTS.

There is no particular unit of measurement to these graphs. The depths on each chart are in terms of the horizontal unit and the latter may be taken as representing inches, feet, miles, centimeters, meters or kilometers. Most commonly it is convenient to use one graph unit as equal to 500, 1000, 2000, 3000 or 4000 ft. The scale should be chosen as large as possible to have the whole structure come within the dimensions of the chart.

To use one of the graphs to determine the gradient (or the differential curvature) that would be produced by a given structure, the cross-section of the structure is drawn on tracing paper and superimposed on the

graph. The number of squares covered by the structure are then counted and their algebraic sum times the difference between the specific gravity of the body and the surrounding medium gives the gradient (or U_Δ or U_{xy}) at the origin. If the structure involves a complex distribution of specific gravity, the area of each specific gravity in the section is lightly tinted a different color and in counting the squares it is convenient to used a scratch pad divided like Fig. 5. The horizontal zones on the chart are scanned one at a time and the respective number of plus and of minus squares of each specific gravity is counted and recorded on the scratch pad. The algebraic sum of the product of the algebraic sums of the columns and the respective specific gravities is the gradient (or U_Δ or U_{xy}, as the case may be) at the origin. To obtain the gradient (or U_Δ or U_{xy}) at another point, the tracing paper is shifted horizontally until the point is on the zero line of the graph and the depth lines on the tracing paper coincide with the corresponding depth lines of the graphs. The operation of counting up the squares is then repeated.

Tentative Picturization of Structures Producing Gradient

To calculate the probable or possible structures which would produce a given gradient (or differential curvature) profile, the geophysicist draws on tracing paper a tentative picture of the probable structure, as he interprets it from the torsion balance data. The value of the gradient (or U_Δ or U_{xy}) is then counted up according to the method of the preceding paragraph for a critical set of stations. Three stations, taken at the points of reversal and of the algebraic maximum and minimum usually are worked first to see if the tentative picture of the structure gives a calculated gradient profile moderately like the observed profile; if the congruence is reasonably close, the gradient (or the differential curvature) is then calculated for the five (preferably nine) points mentioned in a previous paragraph.

The tentative cross-section of the structure may be drawn once on the tracing paper and the latter may then be shifted horizontally in reference to the graph for the several points. But visualization of the situation is facilitated and the calculations expedited by tracing the several shifted positions of the structure in different colors on a single piece of tracing paper. In the molding and remolding of the structure, it is possible to visualize almost at a glance how a small change in the structure will affect the gradient (or differential curvature). The small change that seems plausible is sketched on a small piece of tracing paper which is then superimposed on the several positions of the structure and the effect of the change can be determined for those points in five minutes or less, whereas the mathematical calculation of the change would take several to many hours.

This method of simultaneous superposition of positions is practicable only for rather simple structural situations and for not more than five stations. Such a superposition of position is illustrated by Fig. 6. Curve I represents the tentative cross-section of the structure superimposed on the graph in the position to calculate the gradient at O, the point of its reversal. The gradient is desired also at its points of maximum and minimum, the points A' and B'. Curve II represents curve I shifted to the right until the point A'' is on the zero line of the graph, and curve III represents curve I shifted to the left until B''' is on the zero line of the graph. By inspection, it is possible to see that the small change in the structure at D will produce an effect of +1.6E for station O, −1.9E at station A, and +0.6E at station B. If that small change in the structure does not produce the desired effects, a slightly different change in the structure can be sketched in and its effects rapidly evaluated.

The differential curvature is obtained by similar use of the U_Δ charts. The U_{xz} is equally a U_{yz} chart, but from the nature of the assumed symmetry of the situation $U_{xy} = 0$, and if the chart is assumed to be a U_{xz} chart rather than a U_{yz} chart, $U_{yz} = 0$.

Selection of the Proper Chart

Specifications for the choice of the correct chart to be used need to be worked out for each particular case in the more accurate application of the method. The gradient is slightly sensitive to differences in the length of the structure at right angles to the line of the profile and the differential curvature is very sensitive to such differences. For accurate results therefore a graph must be used in which the length of the prisms approximates with the length of the structure. The sensitivity of the gradient and of the differential curvature to differences in length of a structure is an inverse function of the length in terms of the depth and therefore more care must be taken in making the assumptions in the case of the short structures than in the case of the long structures. If the length of the structure is very considerable, it may be assumed to be infinite.

The graphs of Fig. 4 are constructed on the assumption that the ratio of depth to length of the constituent prisms is constant for each graph. For graph Fig. 4C the formula is depth to top: depth to bottom: length (of each prism) = 9:10:10; for Fig. 4B, 9:10:30; and for Fig. 4A, 9:10:∞. These formulas give the law of variation of the length of anticlinal structures, for in an anticline the length increases with the depth. In a syncline however the length decreases with the depth, and the use of the graphs of Fig. 4 would give decidedly erroneous results. In many salt domes and volcanic stocks or necks the length is relatively constant and does not vary with the depth. Many structures can not be represented by a single formula for the depth: length ratio. In structures of an

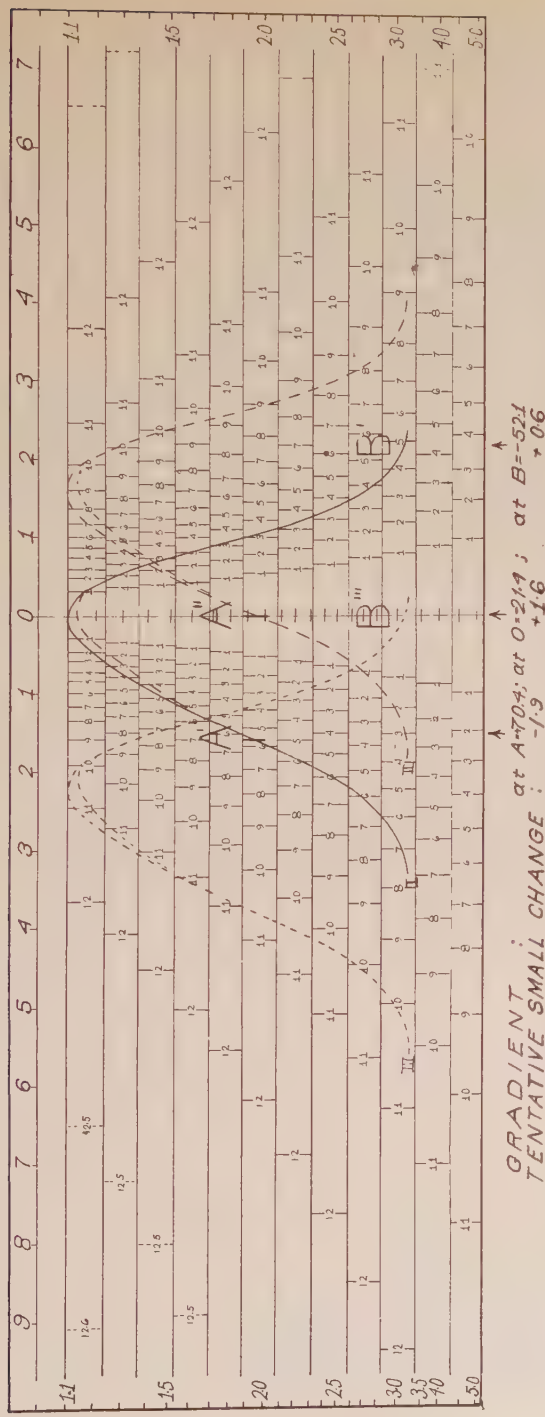

FIG. 6.—ILLUSTRATION OF THE SIMULTANEOUS MULTIPLE POSITION USE OF THE CHARTS.

elliptical or circular plan, the length of the structure at any given depth is much greater through the central portion of the structure than in the front or back portions.

Standard Series of Graphs

A standard series of graphs can be drawn up. The most convenient series is that based on the formula depth to top: depth to bottom: length (of each prism) $= a:b:c$, where the ratio $a:b$ is retained constant and c is varied Graphs constructed according to this formula technically are much the easiest to calculate and correspond to anticlines. The writer's standard series consists of graphs with the following ratios: 9:10:10-(20, 30, 40, 50, 80, 100, and ∞). Graphs with any other formula, as for example, for a trough where the length decreases with the depth, in most cases can be constructed from the standard series of graphs by taking the prisms for one depth from one, for another depth from another, and so on. If for example, the length and depth of a trough are as follows:

Depth, Miles	Length, Miles	Ratio
0	4	∞
0.5	3	10:60
0.75	2	10:26
1.0	1	10:10

The graph for this case would be constructed by taking the prisms from 0 to 0.2 miles from the 9:10:∞ graph for the corresponding depth on the new chart, for 0.4 miles from the 9:10:80 graph, for 0.6 miles from the 9:10:50 graph, for 0.7 miles from the 9:10:30 graph, for 0.8 miles from the 9:10:20 graph, and for 1 mile from the 9:10:10 graph. The prisms in the intervening depths would be interpolated.

Determination of Depth to Length Formula

The depth to length formula may have to be calculated from the torsion balance results. If the structure producing the observed torsion balance anomaly is unknown, inspection of the results of the torsion balance survey may indicate that the graph with such and such a depth to length formula may be used, but in many cases, it may be preferable to calculate the approximate longitudinal cross-section of the structure and thereby determine the depth to length formula; that is, if the east-west cross-section is to be calculated for a structure elongated in a north-south direction, a preliminary calculation will be made of the north-south cross-section and from it the depth to length formula can be determined for the calculation of the east-west cross-section.

The general scheme of the method may be illustrated by application to the hypothetical gravity high of Fig. 7. The transverse section along

the shorter axis of symmetry, aa', can be calculated the most easily and most accurately of all the possible sections. From the data of the cross-section obtained and from the symmetry shown by the torsion balance survey, the whole structure can be sketched in with a fair degree of approximate accuracy. If desirable, the longitudinal cross-section can be calculated but for most general purposes that is not necessary.

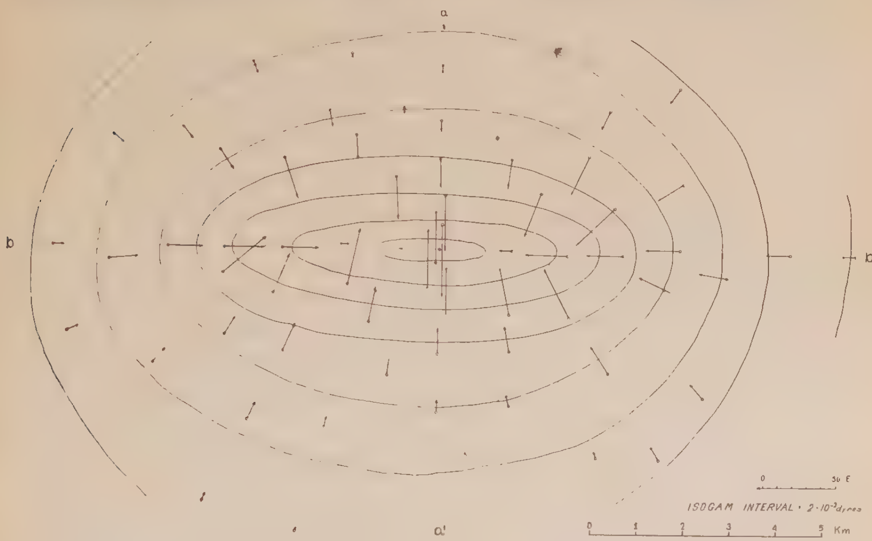

FIG. 7.—HYPOTHETICAL GRADIENT ARROW AND ISOGRAM MAP.

As the depth to length along bb' formula of the structure is not known, it is advisable to make a preliminary calculation of the section along bb'. From inspection of the map of Fig. 7, the graph with a depth to length ratio of 10:30 is chosen as giving a first approximation to the relative dimensions of the structure along aa'. From inspection of the map, a

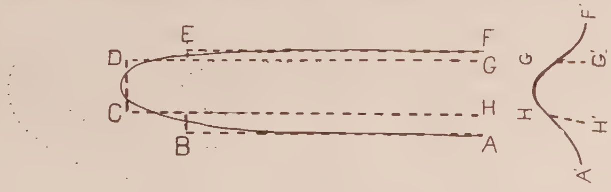

FIG. 8.—WORKING SKETCH FOR THE USE OF THE LENGTH TO DEPTH RATIOS IN CALCULATING THE CROSS-SECTION.

tentative outline for the structure is drawn, and is superimposed on graph 9:10:30; the squares are counted up, etc., and the structure is remolded until the solid line structure is obtained, which gives a fairly good fit of calculated to observed gradients. The length of the structure along bb' at any depth can be seen to be approximately 10 times the depth.

The same depth to length formula will not hold both for the central vertical portion of the structure, *HCDG*, Fig. 8, and for the two lateral portions, *ABH* and *GEF*. The formula 9:10:100 determined by the preceding calculations will hold for the central portion *HCDG*, but by estimation, it seems probable that a formula of 9:10:40 will approximate with reasonable accuracy the average ratio of length to depth in the lateral zones *ABH* and *GEF*.

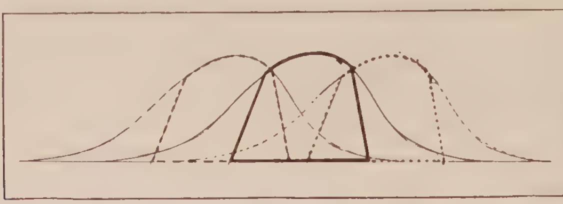

FIG. 9.—WORKING SKETCH TO BE DRAWN ON TRACING PAPER SHOWING THREE POSITIONS OF THE TENTATIVE STRUCTURE TO BE USED IN THE CALCULATIONS WITH GRAPH 9:10:100 FOR U_Δ AND 9:10:∞ FOR U_{xy}.

The graphs used will be: for the gradient, the 9:10:100 graph for the central portion and the 9:10:40 graph for the flank portions, but the 9:10:∞ graph may be used in place of the 9:10:100 graph with no essential loss of accuracy. For the differential curvature, the 9:10:100 and the 9:10:40 graphs will be used respectively for the central and the flank portions of the structure.

A tentative picture of the cross-section of the structure along *aa′* is sketched on tracing paper in three and then later five positions with a different color for each position. This sketch is then traced on another piece of tracing paper. On the first, the central portion of position is outlined in heavy pencil (Fig. 9), and on the second, the flank portions. The first sheet is superimposed on graph 9:10:100 (or for the gradient 9:10:∞) and the second on graph 9:10:40.

Revamping of Tentative Picture to Fit Observed Values

The usual procedure is gone through, of counting the squares and of molding and remolding the structure until a reasonable fit is obtained

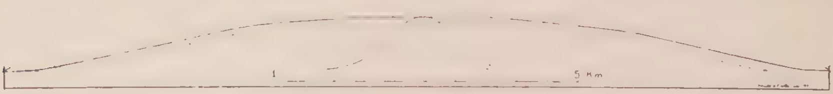

FIG. 10.—CALCULATED LONGITUDINAL AND TRANSVERSE SECTIONS OBTAINED BY THE GRAPHIC METHOD OF CALCULATION FOR THE STRUCTURE PRODUCING THE OBSERVED ANOMALY OF FIG. 7.

between the calculated and the observed values. The structure which seems to be the most probable of the various ones tried is represented in Fig. 10. The preliminary section which was calculated along *bb′* is revamped to agree with the section which has just been calculated for *aa′*; the central points of the two sections must coincide; if the depth to

the top of the anomalous mass along bb' has to be increased, the relief of the section is increased and its length slightly decreased, or vice versa if the depth to the top of the anomalous mass has to be decreased. The original outline of the structure along the bb' section is shown by the dotted line on Fig. 10, and the revamped outline by the line above. The structure contours of Fig. 11 are then drawn on the basis of the elliptical symmetry of the isogams and on the basis of the depths taken from the

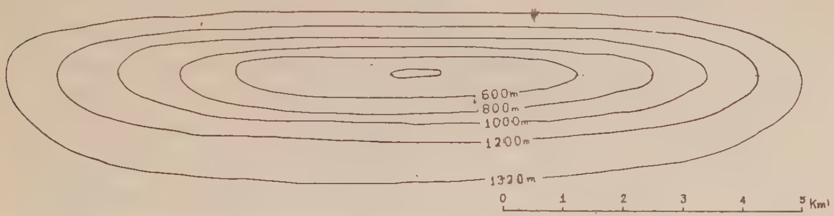

FIG. 11.—CALCULATED STRUCTURE CONTOUR MAP OBTAINED FOR THE STRUCTURE PRODUCING THE OBSERVED ANOMALY OF FIG. 7.

two structural sections. Unless the structure has been tied into a well which has gone to the anomalous heavy (or light) mass, the actual value of the structural contours is somewhat indeterminate.

Flexibility of Graphic Method

The flexibility of this graphic method in handling structures that are complicated both in their geometric form and in their density relations should already have been evident. The handling of a complicated situation is seldom, if ever, as easy as the handling of simple situations, and the use of these to calculate the gradient or the differential curvature for structures with complicated cross-sections or density relations is not as easy and rapid as for simple structures and simple density relations, but it does not take much longer to count the squares within a figure of irregular than within a figure of simple geometric shape; and only a little additional trouble and time are necessary to count the squares by different densities and multiply by the respective specific gravities. Complicated density relations can be handled easily only if there is little variation of density at right angles to the plane of the section. If the length to depth formula of one of the standard graphs corresponds sufficiently closely to the length to depth formula of the structure, the calculations for a structure of finite length are exactly as simple as for a body of infinite length. If one of the standard graphs does not correspond with sufficient closeness, a composite graph can be constructed from the standard series of graphs according to any desired depth to length formula at only a slight expenditure of time and energy; the construction of the graph largely is merely a matter of simple drafting. The use of the composite graph is the same as the use of any of the standard graphs.

The grade of skill and ability to use these graphs is much lower than that required for the calculation of such formulas as (1) to (14). When the scheme of the work has been laid out by the geophysicist, the manual labor of counting up the squares is rather simple, and about the only requisite for the calculator is common sense.

Limitations

A word of caution is necessary on the limitations to the use of such calculations. In a few especially favorable situations it is possible to make quantitative calculations that will have a high degree of accuracy; for Hoskins Mound salt dome, quantitative calculations made by Mr. Gilmour, then assistant to the writer, have been proved by subsequent drilling to be in error by only 0.6 to 7 per cent.; the actual depths found by subsequent drilling against the predicted depths were: predicted 900 ft. (274.5 m.), actual 840 ft. (256.2 m.); predicted 1000 ft. (304.8 m.), actual 947 ft. (288.8 m.); predicted 1250 ft. (381.3 m.); actual 1222 ft. (372.7 m.); predicted 1575 ft. (480.4 m.), actual 1565 ft. (477.3 m.). The depth, thickness and character of the cap were known for one quadrant of the dome; it was possible to determine the specific gravity of the cap rock from the numerous cases available, the specific gravity of the salt and of the surrounding sediments could be assumed within narrow limits; the problem, therefore, was a fairly simple one.

At the Bryan Mound salt dome, similar calculations made under the writer's direction were unsatisfactory. The two domes are only some 15 miles apart, the Bryan Mound dome is on the coast and Hoskins Mound dome only a few miles back from it; they are very similar in their size, form, depth, and amount of cap rock, and from the geological situation, they would be expected to have almost identical gravity anomalies, but as a matter of fact, the mean maximum value of the gradients due to Hoskins Mound dome were about 42E and those due to the Bryan Mound dome about 12E. The Hoskins Mound dome rises slightly closer to the surface, has slightly more cap rock, and has a slightly greater difference between the specific gravity of the cap rock and of the surrounding sediments.

The conditions in the Gulf Coast, however, are exceptionally favorable. The salt dome is an exaggerated and clean-cut type of structure rising through sediments without strong variation in specific gravity down to great depths. The basement apparently is at enormous depth. There is practically no topographic effect on the gradient and differential curvature and the upper subsoil and the soil are relatively free from locally disturbing irregularities of mass. In most areas, little is known definitely about the mass or masses mainly responsible for the anomalies observed.

Effect of Density Difference on Calculations

The density relations in the subsurface rarely are known with any high degree of accuracy. The density difference in a great many cases can only be assumed to be somewhat between 0.25 and 0.35, or between 0.30 and 0.40, or 0.35 and 0.45, as the case may be. But a difference of 0.10 in the density difference will cause a considerable difference in the results of the calculations. If the density is known, a series of somewhat similar structures with steeper or gentler relief and at a greater or less depth may give the same gradient or differential curvature profile within the accuracy of torsion balance observations. If the depth to the crest

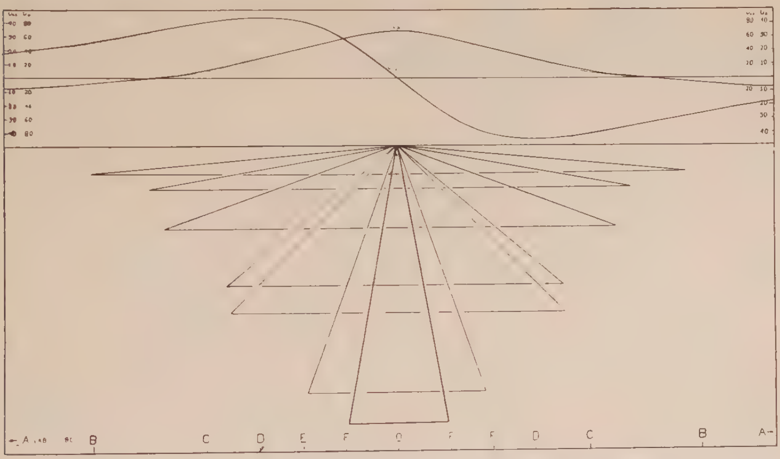

Fig. 12.—Triangular cross-sections of a series of prisms, all of which produce the same gradient and differential curvature profile—within the limits of error of field observation, and with a certain specific gravity for each block.

is known, and the density difference is unknown, a series of somewhat similar structures with a relief varying with the assumed density difference may all produce the same gradient profile or the same differential curvature profile within the accuracy of the field observations.

The danger and uncertainty of calculation of the form of an unknown structure with unknown density relations from an observed variation of the gradient or of the differential curvature are illustrated by Fig. 12. All of the various infinite triangular prisms of Fig. 12 give gradient profiles that for practical purposes are identical, if a certain assumption is made in regard to the density difference of the prism. The deviations of calculated from the mean value of the gradient are given in Table 1. The last column gives the specific gravity difference which it was necessary to assume in order to get the gradient profile of that particular prism to fit the mean curve. These specific gravities may be multiplied or divided

all by the same number and the gradient profiles will still coincide in the same degree as before, although their amplitude will be multiplied or divided correspondingly. As the specific gravity difference practically never exceeds 0.5 and commonly is between 0.2 and 0.4, and as field determinations of the gradient seldom are accurate within 1.5E and often not within 2.5E, the deviations of the gradient of the triangular prisms of Fig. 12 everywhere are less than the probable error of the field observations. That means that even if the depth to the crest of such a structure is known and if the analysis is made only from the variation of the gradient, there is a series of structures, all of which are equally probable. The geological limits to the range of the density difference will place limits to the range of the structures of that series. If the depth to the crest of the structure and the density difference are both known, this study shows that in general there will be two structures of equal degree of probability, the one broad and shallow and the other deep and narrow.

TABLE 1.—*Deviation of Gradient Produced by the Prisms of Fig. 12 from the Mean Gradient for Density Differences Chosen to Give Minimum Deviation of the Gradient from that Mean*

Station	*A*	*B*	*C*	*D*	*E*	*F*	*O*	
Mean Gradient	10.3	24.7	40.6	44.4	41.7	30.6	0	
Dip of Side of Prism	Deviation from the Mean Gradient							Specific Gravity for Prism
5°	3.0	0.3	1.9	0.5	3.6	2.3	0	2.44
10°	4.3	0.1	1.6	0.2	0.5	2.2	0	1.92
20°	0.8	0.7	0.4	0.0	0.2	0.6	0	1.20
40°	0.9	1.1	1.2	0.3	1.1	0.9	0	1.05
45°	0.7	0.3	1.1	0.1	0.2	1.4	0	1.00
70°	2.6	1.2	0.9	0.2	0.2	0.8	0	1.62
80°	3.3	1.3	0.7	0.1	0.4	1.2	0	2.85

The results of a similar study of the variation of the differential curvature produced by the triangular prisms of Fig. 12 are given in Table 2. The variation of the density difference necessary to produce the greatest similarity among the curves is not essentially different from that for the gradient. The deviation of the individual values from the mean is very much greater than for the gradient and in general is about equal to or less than the probable error of the field observations. The use of calculations of the structure from the differential curvature may throw out as improbable certain of the structures which are probable from the calculations from the gradient and thereby reduce the number of equally

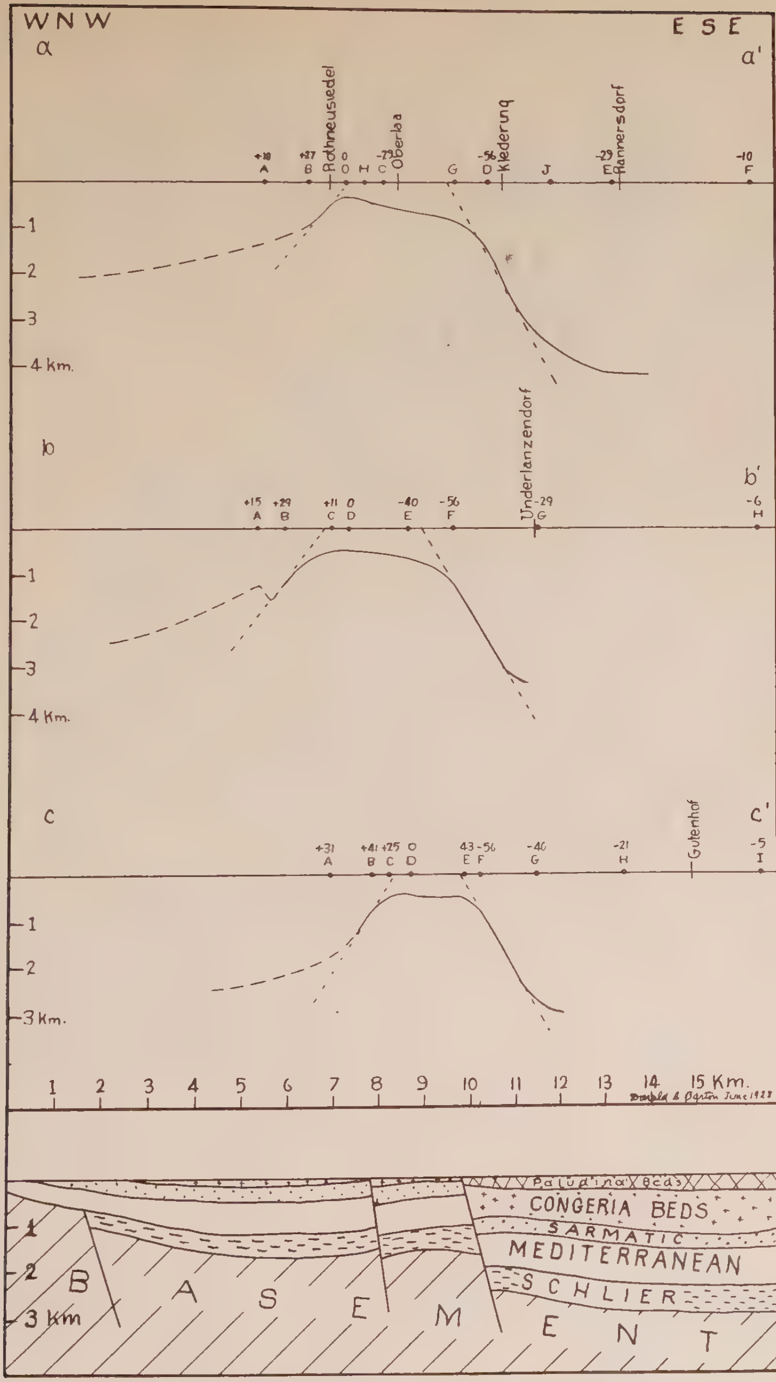

FIG. 13.—SECTIONS ACROSS THE LANZENDORF DOME NEAR VIENNA, AUSTRIA. THE UPPER THREE SECTIONS ARE CALCULATIONS FROM TORSION BALANCE OBSERVATIONS BY SCHUMANN. THE LOWER SECTION IS BASED ON GEOLOGY AND IS AFTER FRIEDL.

probable alternative structures which might have produced the observed torsion balance anomalies.[6]

TABLE 2.—*Deviation of Differential Curvature Produced by the Prisms of Fig. 12 from the Mean Differential Curvature for Density Differences Chosen to Give Minimum Deviation of the Differential Curvature from the Minimum*

Station	*A*	*B*	*C*	*E*	*O*	
Mean Differential Curvature	−8.2	−6.0	9.3	43.0	72.8	
Dip of Side of Prism	Deviation from the Mean Differential Curvature					Specific Gravity for Prism
5°	+1.9	+2.2	+4.7	0	− 3.8	2.34
10°	+2.0	+4.4	−1.4	0	−11.0	1.93
20°	+0.8	+2.9	−1.5	0	+ 2.6	1.20
40°	+0.1	+0.5	−3.1	0	+11.9	1.19
45°	+0.1	−0.6	−1.5	0	+ 2.2	1.00
70°	−1.4	−2.9	+1.2	0	+ 6.0	1.62
80°	−3.7	−6.0	+2.7	0	+ 3.4	3.00

The calculations in this study of a triangular prism producing essentially the same gradient and the same differential curvature were done graphically. The over-all time in making the calculations was 16 hr. Little research has been done of this type on account of the involved character of the mathematics. The need of such research is evident, if an attempt is to be made to interpret what structure is producing observed torsion balance results.

Graphical Analysis of Lanzendorf Dome

An example of the result of this graphical method of analysis of the unknown structure producing an observed anomaly is given in Figs. 13 and 14. The values of the gradient given by Schumann[7] for three profiles across the Lanzendorf dome near Vienna were given by the writer to his calculator with the instructions to calculate the probable structure producing them, but without any information in regard to the probable form of the structure. Using arbitrarily a density difference of 0.30, she obtained the three profiles of Fig. 13. With a different choice of the assumption in regard to the density difference, a flatter and less pronounced ridge could probably have been obtained. Geologically, the three profiles would seem to represent a very sharp anticlinal ridge of a horst. As the torsion balance survey did not extend far enough to the

[6] Research subsequent to the writing of this paper has shown that down to a certain limiting depth there are infinite series of mostly triangular prisms which will produce the same gradient profile.

[7] R. Schumann: Ergebnisse aus Drehwagemessungen im Wiener Becken. *Osterr. Berg. u. Hüttenmänn.* (1921–22) **69–70.**

west of the crest of the structure, the west flank is uncertain, but the east flank would seem definitely to be a steep, eastward facing scarp, probably a fault with a big downthrow to the east. The west flank is suggestive of a lesser fault with downthrow to the west.

The geology[8] of the Lanzendorf dome is given by the map and east-west cross-section. The east flank of the dome is the Leopoldsdorf fault with a throw of at least 500 m. and perhaps of 700 m. Near Leopoldsdorf the dip of the fault plane is 60° to 70°. A lesser fault, the Rothneusiedel

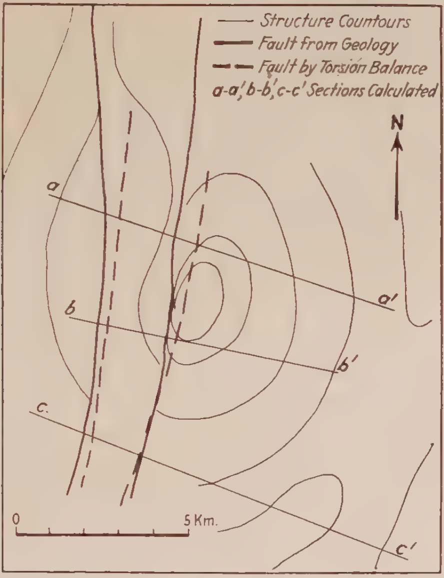

FIG. 14.—MAP OF THE LANZENDORF DOME SHOWING RELATIVE POSITION OF THE FAULTS BY GEOLOGY AND BY THE TORSION BALANCE; GEOLOGY AFTER FRIEDL.

fault, lies about 2 km. west of the Leopoldsdorf fault. The interpretation of the existence of the fault and its downthrow to the east is based on the elevation of the top of Sarmatie of +126 m. above mean sea level in the well at Rothneusiedel and +96 m. in a well about 4 km. to the north. From the data presented by Friedl, no reason is evident why the fault may not be west of the first well, east of the second well, and be downthrown on the west instead of the east. The throw of the fault is supposed to be only some tens of meters. The steep eastern scarp of torsion balance structure corresponds well to the Leopoldsdorf fault. If the plane of the east face of the torsion balance structure is prolonged to the surface, its surface trace coincides or is slightly east of the surface trace of the Leopoldsdorf. The plane of the suggested scarp on the west flank of the torsion balance structure may be prolonged to the surface with considerable uncertainty and its surface trace will lie parallel to and slightly east of the Rothneusiedel.

[8] Karl Friedl: Uber die Jüngsten Erdölforschungen im Wiener Becken. *Petroeum Ztsch.* (1927) **23,** 441.

The results of the torsion balance analysis suggest strongly a downthrow to the west rather than the downthrow to the east postulated by Friedl. The presence of the broad dome on the downthrown side of the Leopoldsdorf fault is strongly suggested by Schumann's isogams, although he does not mention the fact in his interpretation of them. The station interval on the profile across the dome was not close enough for the presence of the dome to be indicated in the calculations. The torsion balance survey of the Lanzendorf dome was a reconnaissance rather than a detailed survey and should be expected to show only the larger features of the structure. The station interval was not close enough to pick up the surface trace of the faults and the differential curvature, which is of great assistance in mapping faults, was not given. Nevertheless, the analysis of the general form of the structure producing the observed gradient anomaly gives a very definite suggestion of the structure. If the geology of the area were not workable, that suggestion would be an important contribution to the knowledge of the structural geology of the area.

Terrane Correction

The terrane correction for certain special situations may be calculated graphically by the charts, but a chart of slightly different design is more particularly adapted to such calculations.[9] The present type of chart may be used when the station is on a plane slope or is in a long straight mine gallery, or to calculate the extra terrane correction for a railroad embankment, a canal or similar linear features. In such use of the charts, it must be remembered that origin of the chart must be taken as coinciding with the center of gravity of the torsion balance. If the topography rises above the center of gravity of the torsion balance, the charts must be reversed and the sign of the effect changed algebraically.

Conclusion

The conclusion should not be drawn that in general it is hardly worth while to calculate the cross-section of the unknown structure producing observed torsion balance anomalies. Such calculations are not a panacea for the determination of the structure. The results are subject to great indefiniteness, nevertheless they definitely limit the probably possible types of structure to be considered. Such calculations are necessary both to an understanding of the general theory of interpretation and to the interpretation of definite situations. A few simple types of situations can be evaluated roughly by an experienced geophysicist without calculations, but many simple situations and nearly all complex situations can be evaluated by an experienced geophysicist only by calculations. The graphical method of calculation outlined in this paper makes such calculations simple enough to be practicable.

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[9] D. C. Barton: Graphical Terrane Correction for Gravity Gradient. U. S. Bur. Mines *Tech. Paper* 444 in press.

Computation of Eötvös Gravity Effects

By E. Lancaster-Jones,* London, England

(New York Meeting, February, 1928)

The gravity magnitudes obtained by means of observations with the Eötvös balance in the field are necessarily resultant or total effects due to all abnormalities of mass distribution, including even the effects due to the ellipticity and rotational motion of the whole earth.

Since the object of the survey is to isolate the effects which arise solely from some particular subterranean feature, and to deduce therefrom the configuration and extent of this feature, the first step is to eliminate from the observed effects all those due to "disturbing" (*i. e.*, unwanted) features, whether above or below the surface of the ground.

The disturbing features have different characteristics, according to their situation relative to the center of the balance and to the horizontal plane through the foot of the balance, or station point; this diversity was recognized by Eötvös, who divided the total effects into so-called Normal, Terrain, Topographical, Cartographical and Subterranean effects.

This classification has been adhered to by most subsequent investigators, but the writer prefers to employ the term "three-dimensional," which really indicates the essential character of the class of effect known as "Subterranean," at any rate from the standpoint of the computer of effects.

Before disturbing effects can be eliminated, the features must be measured up or surveyed, whether by topographical, mining, or boring methods, and then the effects of each feature calculated and subtracted from the total observed effects. After this the problem of interpreting the residual effects will itself necessitate the computation of the assumed feature, or even a series of trial features.

The task of computing gravity effects is therefore of great importance. In the early work, by Eötvös himself, the computation practically ceased when the terrain and topographical effects had been calculated, but later surveys have demanded an ever-increasing extension in the technique of computation. Certainly in surveys designed to locate minerals, particularly ores, the calculations have to be extended far beyond those of Eötvös, while his methods of computing both terrain

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and topographical effects have received considerable modification and improvement.

Various writers have developed isolated details of the computational program, but the results are scattered through many technical journals, almost always in foreign languages. Moreover, writers have generally concentrated upon the simplest types of computation and ignored the fact that actual masses which occur in nature are apt to be decidedly irregular. The present paper is intended to act as a survey of the field of gravitational computation. Full details are given, where it is deemed necessary, of the analysis involved, and the mathematics of this has been purposely simplified even at the expense of additional elaboration. In other cases, notably the terrain computations, it is felt that the method is too well known for repetition of the analysis to be necessary.

Elements of Mass Related to Gravitational Effects

The total gravitational effects of any mass can be obtained as an algebraical sum—or integral—of the individual effects of its constituent elements. These elements may be chosen at will in various ways. For example, we have point elements or particles of mass; line elements, or thin cylindrical elements, in various directions; and so on.

Point Element

The rest are derivable from the first, the point element, of which the Eötvös gravity effects are given by the formulas:

$$(1)\quad \begin{cases} \dfrac{\partial^2 U}{\partial y^2} - \dfrac{\partial^2 U}{\partial x^2} = 3\gamma\mu \dfrac{b^2 - a^2}{r^5} \\[2mm] \dfrac{\partial^2 U}{\partial x \partial y} = 3\gamma\mu \dfrac{ab}{r^5} \\[2mm] \dfrac{\partial^2 U}{\partial x \partial z} = 3\gamma\mu \dfrac{ac}{r^5} \\[2mm] \dfrac{\partial^2 U}{\partial y \partial z} = 3\gamma\mu \dfrac{bc}{r^5} \end{cases}$$

where $r = \sqrt{a^2 + b^2 + c^2}$ and (a, b, c) are the coordinates of the element of mass μ referred to rectangular axes Ox, Oy, Oz having their origin at O, the center of the balance.

γ = the gravitational constant.

If we refer the point element to a new set of axes of reference through the station point S, which has the coordinates (o, o, h) relative to the original axes, and if the second system of axes is a cylindrical set (ρ, α, ζ) given by the relations (see Fig. 1)

$a = \rho \cos \alpha;\ b = \rho \sin \alpha;\ c = h - \zeta$
so that ζ is positive vertically upwards,
whereas c is positive vertically downwards

the formulas become

$$
(2)\left\{\begin{aligned}
\frac{\partial^2 U}{\partial y^2} &= \frac{\partial^2 U}{\partial x^2} = -3\gamma\mu\frac{\rho^2 \cos 2\alpha}{r^5} \\
2\frac{\partial^2 U}{\partial x \partial y} &= 3\gamma\mu\frac{\rho^2 \sin 2\alpha}{r^5} \qquad\qquad r^2 = \rho^2 + (h - \zeta)^2 \\
\frac{\partial^2 U}{\partial x \partial z} &= 3\gamma\mu\frac{\rho(h - \zeta) \cos \alpha}{r^5} \\
\frac{\partial^2 U}{\partial y \partial z} &= 3\gamma\mu\frac{\rho(h - \zeta) \sin \alpha}{r^5}
\end{aligned}\right.
$$

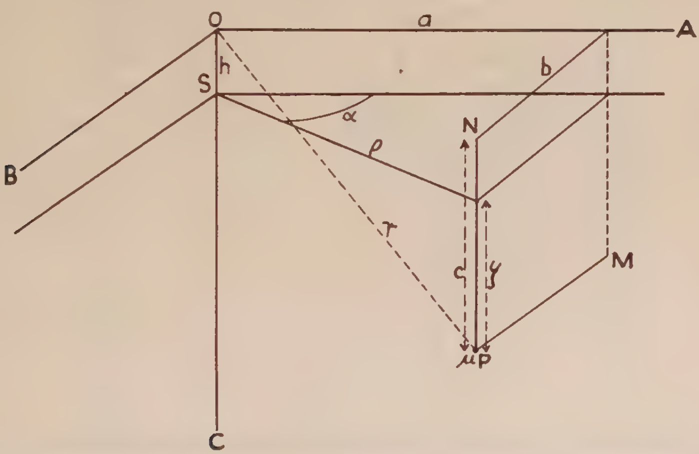

FIG. 1.—COORDINATES OF ATTRACTING PARTICLE REFERRED TO CENTER OF BALANCE O AND TO STATION POINT S.

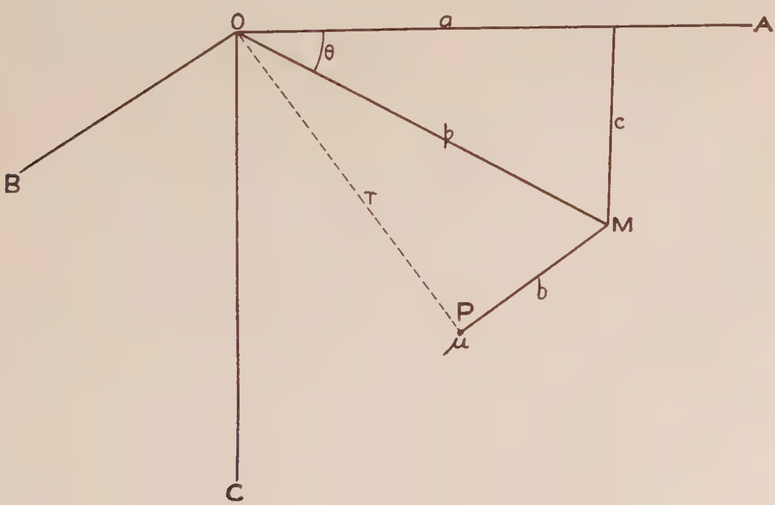

FIG. 2.—CYLINDRICAL COORDINATES REFERRED TO CENTER OF BALANCE O.

Similarly, if the point element is referred to cylindrical axes (ρ, α, c) through the balance center O, the axis of c being as in (1), the formulas are

$$(3)\begin{cases} \dfrac{\partial^2 U}{\partial y^2} - \dfrac{\partial^2 U}{\partial x^2} = -3\gamma\mu\dfrac{\rho^2\cos 2\alpha}{r^5} & \text{where } r^2 = \rho^2 + c^2 \\ 2\dfrac{\partial^2 U}{\partial x \partial y} = 3\gamma\mu\dfrac{\rho^2 \sin 2\alpha}{r^5} \\ \dfrac{\partial^2 U}{\partial x \partial z} = 3\gamma\mu\dfrac{\rho c\cos\alpha}{r^5} \\ \dfrac{\partial^2 U}{\partial y \partial z} = 3\gamma\mu\dfrac{\rho c\sin\alpha}{r^5} \end{cases}$$

While if we refer to cylindrical axes (p, θ, b) (see Fig. 2), we have

$$a = p\cos\theta;\ b = b;\ c = p\sin\theta;\ r^2 = p^2 + b^2$$

$\therefore$

$$(4)\begin{cases} \dfrac{\partial^2 U}{\partial y^2} - \dfrac{\partial^2 U}{\partial x^2} = 3\gamma\mu\dfrac{b^2 - p^2\cos^2\theta}{r^5} \\ \dfrac{\partial^2 U}{\partial x \partial y} = 3\gamma\mu\dfrac{bp\cos\theta}{r^5} \\ \dfrac{\partial^2 U}{\partial x \partial z} = \dfrac{3}{2}\gamma\mu\dfrac{p^2\sin 2\theta}{r^5} \\ \dfrac{\partial^2 U}{\partial y \partial z} = 3\gamma\mu\dfrac{bp\sin\theta}{r^5} \end{cases}$$

Line Element

If we have a series of point elements along a line, they may be regarded as forming a thin cylinder of uniform cross-section δ and density σ.

Suppose the elements μ are concentrated along the line PN (Fig. 1), we have the formulas

$$(5)\begin{cases} \dfrac{\partial^2 U}{\partial y^2} - \dfrac{\partial^2 U}{\partial x^2} = 3\gamma\sigma\delta(b^2 - a^2)\displaystyle\int_0^c \dfrac{dc}{r^5} = \gamma\sigma\delta\dfrac{(b^2 - a^2)}{(a^2 + b^2)^2}\left(\dfrac{3c}{r} - \dfrac{c^3}{r^3}\right) \\ \dfrac{\partial^2 U}{\partial x \partial y} = 3\gamma\sigma\delta ab\displaystyle\int_0^c \dfrac{dc}{r^5} = \gamma\sigma\delta\dfrac{ab}{(a^2 + b^2)^2}\left(\dfrac{3c}{r} - \dfrac{c^3}{r^3}\right) \\ \dfrac{\partial^2 U}{\partial x \partial z} = 3\gamma\sigma\delta a\displaystyle\int_0^c \dfrac{c\,dc}{r^5} = \gamma\sigma\delta a\left(\dfrac{1}{(a^2 + b^2)^{3/2}} - \dfrac{1}{r^3}\right) \\ \dfrac{\partial^2 U}{\partial y \partial z} = 3\gamma\sigma\delta b\displaystyle\int_0^c \dfrac{c\,dc}{r^5} = \gamma\sigma\delta b\left(\dfrac{1}{(a^2 + b^2)^{3/2}} - \dfrac{1}{r^3}\right) \end{cases}$$

Or, in cylindrical coordinates (ρ, α, c)

$$(6)\begin{cases} \dfrac{\partial^2 U}{\partial y^2} - \dfrac{\partial^2 U}{\partial x^2} = \dfrac{\gamma\sigma\delta}{\rho^2}\cos 2\alpha\left(\dfrac{3c}{r} - \dfrac{c^3}{r^3}\right) \\ 2\dfrac{\partial^2 U}{\partial x \partial y} = \dfrac{\gamma\sigma\delta}{\rho^2}\sin 2\alpha\left(\dfrac{3c}{r} - \dfrac{c^3}{r^3}\right) \\ \dfrac{\partial^2 U}{\partial x \partial z} = \dfrac{\gamma\sigma\delta}{\rho^2}\cos\alpha\left(1 - \dfrac{\rho^3}{r^3}\right) \\ \dfrac{\partial^2 U}{\partial y \partial z} = \dfrac{\gamma\sigma\delta}{\rho^2}\sin\alpha\left(1 - \dfrac{\rho^3}{r^3}\right) \end{cases}$$

As a special case, when the line element is infinitely long (*i. e.*, $c \to +\infty$) and $r \to +\infty$ with it, we have $\frac{c}{r} = +1; \frac{c^3}{r^3} = +1; \frac{\rho^3}{r^3} = 0$

$$\text{(6a)} \quad \begin{cases} \dfrac{\partial^2 U}{\partial y^2} - \dfrac{\partial^2 U}{\partial x^2} = 2\gamma\sigma\delta \cdot \dfrac{\cos 2\alpha}{\rho^2} \\ 2\dfrac{\partial^2 U}{\partial x \partial y} = 2\gamma\sigma\delta \cdot \dfrac{\sin 2\alpha}{\rho^2} \\ \dfrac{\partial^2 U}{\partial x \partial z} = \gamma\sigma\delta \cdot \dfrac{\cos \alpha}{\rho^2} \\ \dfrac{\partial^2 U}{\partial y \partial z} = \gamma\sigma\delta \cdot \dfrac{\sin \alpha}{\rho^2} \end{cases}$$

If the point elements are concentrated along the line PM, using the cylindrical coordinates (p, θ, b) we have

$$\text{(7)} \quad \begin{cases} \dfrac{\partial^2 U}{\partial y^2} - \dfrac{\partial^2 U}{\partial x^2} = 3\gamma\sigma\delta \displaystyle\int_0^b \dfrac{b^2 - p^2 \cos^2 \theta}{r^5} db = \\ \qquad\qquad -\dfrac{\gamma\sigma\delta}{p^2} \left\{ \cos^2 \theta \cdot \dfrac{3b}{r} - (1 + \cos^2 \theta) \dfrac{b^3}{r^3} \right\} \\ \dfrac{\partial^2 U}{\partial x \partial y} = 3\gamma\sigma\delta \cdot p \cos \theta \cdot \displaystyle\int_0^b \dfrac{b \cdot db}{r^5} = \dfrac{\gamma\sigma\delta}{p^2} \cos \theta \left\{ 1 - \dfrac{p^3}{r^3} \right\} \\ \dfrac{\partial^2 U}{\partial x \partial z} = \dfrac{3}{2} \gamma\sigma\delta \cdot \rho^2 \sin 2\theta \displaystyle\int_0^b \dfrac{db}{r^5} = \dfrac{\gamma\sigma\delta}{2p^2} \sin 2\theta \left\{ \dfrac{3b}{r} - \dfrac{b^3}{r^3} \right\} \\ \dfrac{\partial^2 U}{\partial y \partial z} = 3\gamma\sigma\delta p \sin \theta \displaystyle\int_0^b \dfrac{b \cdot db}{r^5} = \dfrac{\gamma\sigma\delta}{p^2} \sin \theta \left\{ 1 - \dfrac{p^3}{r^3} \right\}. \end{cases}$$

and for the special case of an infinitely long element, when $b = +\infty$

$$\text{(7a)} \quad \begin{cases} \dfrac{\partial^2 U}{\partial y^2} - \dfrac{\partial^2 U}{\partial x^2} = -\dfrac{\gamma\sigma\delta}{p^2} \cos 2\theta. \\ \dfrac{\partial^2 U}{\partial x \partial y} = \dfrac{\gamma\sigma\delta}{p^2} \cos \theta. \\ \dfrac{\partial^2 U}{\partial x \partial z} = \dfrac{\gamma\sigma\delta}{p^2} \sin 2\theta. \\ \dfrac{\partial^2 U}{\partial y \partial z} = \dfrac{\gamma\sigma\delta}{p^2} \sin \theta. \end{cases}$$

while for the special case of a doubly infinite line element, extending from $b = -\infty$ to $b = +\infty$

$$\text{(7b)} \quad \begin{cases} \dfrac{\partial^2 U}{\partial y^2} - \dfrac{\partial^2 U}{\partial x^2} = -\dfrac{2\gamma\sigma\delta}{p^2} \cos 2\theta \\ \dfrac{\partial^2 U}{\partial x \partial y} = 0 \\ \dfrac{\partial^2 U}{\partial x \partial z} = \dfrac{2\gamma\sigma\delta}{p^2} \sin 2\theta \\ \dfrac{\partial^2 U}{\partial y \partial z} = 0 \end{cases}$$

Terrain Features

The term "terrain region" is generally restricted to that area of the *natural* ground immediately surrounding the station point. It is important to appreciate that the standard terrain formulas, such as those given by Schweydar, are only applicable in this region provided certain conditions are fulfilled. The conditions, briefly stated, are as follows:

1. Within the region the ground surface is assumed to be gently undulating only, abrupt changes of altitude being absent.
2. It is assumed that the density of the ground is uniform at all points above the horizontal plane passing through the station point.
3. It is further assumed that, throughout the terrain region, the altitude of the ground at any point relative to the station point is small

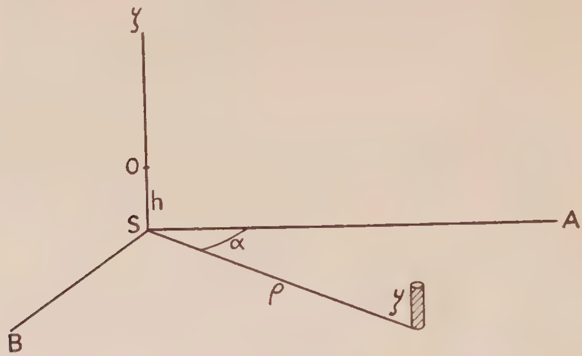

Fig. 3.—Illustrating effect of a small element of "terrain," referred to station point S.

compared with the horizontal distance between the station point and the ground point. In practice it will be sufficient if the former is not more than one-tenth of the latter.

On these assumptions, if we take reference axes (ρ, α, ζ) through the station point S (Fig. 3) and consider the effects of a small cylinder of ground, of cross-sectional area $\delta = \rho \cdot d\rho \cdot d\alpha$, density σ, and height h we have, throughout the cylinder

$$r^2 = \rho^2 + (h - \zeta)^2 = \rho^2 + h^2 - (2h\zeta - \zeta^2)$$

$$\therefore r = (\rho^2 + h^2)^{1/2}\left\{1 - \frac{2h\zeta - \zeta^2}{\rho^2 + h^2}\right\}^{1/2}$$

$$= (\rho^2 + h^2)^{1/2} \text{ to a first approximation}$$

so that r can be regarded as constant throughout the cylinder.

By substituting $\sigma \cdot \rho \cdot d\rho \cdot d\alpha \cdot d\zeta$ for μ in formula 2 and integrating between the limits $\zeta = 0$ and ζ we get for the total effect due to the whole cylinder

$$
(8) \begin{cases}
\dfrac{\partial^2 U}{\partial y^2} - \dfrac{\partial^2 U}{\partial x^2} = -3\gamma\sigma \cos 2\alpha \cdot d\alpha \dfrac{\rho^3 d\rho}{r^5} \cdot \zeta \\
2\dfrac{\partial^2 U}{\partial x \partial y} = 3\gamma\sigma \sin 2\alpha \cdot d\alpha \cdot \dfrac{\rho^3 d\rho}{r^5} \cdot \zeta \qquad r^2 = \rho^2 + h^2 \\
\dfrac{\partial^2 U}{\partial x \partial z} = 3\gamma\sigma \cos \alpha \cdot d\alpha \dfrac{\rho^2 d\rho}{r^5} \zeta\left(h - \dfrac{\zeta}{2}\right) \\
\dfrac{\partial^2 U}{\partial y \partial z} = 3\gamma\sigma \sin \alpha \;\; d\alpha \dfrac{\rho^2 d\rho}{r^5} \zeta\left(h - \dfrac{\zeta}{2}\right)
\end{cases}
$$

In order to obtain the effects due to all the ground in the terrain region, it would be necessary to integrate the right-hand sides of the above formulas with respect to α and ρ between certain limits. Since ζ is obviously a function both of ρ and α at any point, it is necessary to make some assumption regarding the functional relationship before the integration can proceed.

It will be of interest to consider the simple case where

$$\zeta = k\rho \cos \alpha$$

k being a constant, which corresponds to the assumption that the ground surface is a plane, having its lines of greatest slope in the direction $\alpha = 0$ and its inclination to the horizontal given by $\tan^{-1} k$.

Integrating now between the limits $\alpha = 0$ to 2π
we obtain $\rho = 0$ to R

$$
(9) \begin{cases}
\dfrac{\partial^2 U}{\partial y^2} - \dfrac{\partial^2 U}{\partial x^2} = 0 \quad \text{since} \displaystyle\int_0^{\pi} \cos 2\alpha \cdot \cos \alpha \cdot d\alpha = 0 \\
\dfrac{\partial^2 U}{\partial x \partial y} = 0 \quad \text{since} \displaystyle\int_0^{2\pi} \sin 2\alpha \cdot \cos \alpha \cdot d\alpha = 0 \\
\dfrac{\partial^2 U}{\partial y \partial z} = 0 \quad \text{since} \displaystyle\int_0^{2\pi} \sin \alpha \cdot \cos \alpha \cdot d\alpha = 0 \\
\text{while} \\
\dfrac{\partial^2 U}{\partial x \partial z} = 3\gamma\sigma k \displaystyle\int_0^R \frac{\rho^3 d\rho}{r^5} \int_0^{2\pi} \left(h \cos^2 \alpha \cdot d\alpha - \frac{\rho k}{2} \cos^3 \alpha \cdot d\alpha\right) \\
\qquad = 3\gamma\sigma\pi k h \displaystyle\int_0^R \frac{\rho^3 d\rho}{r^5} \quad \text{since} \int_0^{2\pi} \cos^3\alpha \cdot d\alpha = 0 \\
\qquad\qquad \text{and} \displaystyle\int_0^{2\pi} \cos^2\alpha \cdot d\alpha = \pi \\
\therefore \dfrac{\partial^2 U}{\partial x \partial z} = \gamma\sigma\pi k\left[2 - \dfrac{3h}{u} + \dfrac{h^3}{u^3}\right] \text{ where } u^2 = h^2 + R^2
\end{cases}
$$

From the above formulas, it is evident that if R is reasonably large compared with h, we can get a useful approximation by means of the simple formula,

$$\frac{\partial^2 U}{\partial x \partial z} = 2\pi\gamma\sigma k$$

which gives the maximum gravitational gradient corresponding to a large inclined-plane ground surface, and enables us to appreciate the effect of even a comparatively insignificant slope in the terrain. For, corresponding to an inclination of 1°, $k = \frac{\pi}{180}$ so that, assuming $\gamma = \frac{200}{3}$E and $\sigma = 1.8$ we have

$$\frac{\partial^2 U}{\partial x \partial z} = \frac{4}{3}\pi^2 \mathrm{E} = 13.2\mathrm{E}$$

which is quite a large quantity in torsion balance measurements.

Also, it is evident that approximately one-half of the total amount comes from the ground between $R = 0$ and $R = 2\sqrt{2}h$, for which region we have $u = 3h$ so that

$$\begin{aligned}\frac{\partial^2 U}{\partial x \partial z} &= \pi\gamma\sigma k\left(2 - 1 + \frac{1}{27}\right)\\ &= 1.037\pi\gamma\sigma k\end{aligned}$$

In the average case this would be the ground inside a circle of about 9-ft. radius while for $u = 10h$ (the ground inside a circle of about 30-ft. radius)

$$\begin{aligned}\frac{\partial^2 U}{\partial x \partial z} &= \pi\gamma\sigma k[2 - 0.3]\\ &= 1.7\pi\gamma\sigma k\end{aligned}$$

which is about 85 per cent. of the total amount.

It will be evident from these simple examples that ground which slopes so imperceptibly that the average eye would not detect it can yet give quite large effects, and that a great proportion of the terrain effect arises from the ground in the immediate vicinity of the station, say the first 20 to 30 ft.

In the case of the normal undulating terrain region, the method of computation actually adopted in practice is in its essentials that established by Eötvös himself, which has been subsequently subjected to detailed analysis and improvement by Schweydar. The method is too well known to render its description necessary here. It is quite a mechanical procedure and leaves little to the initiative of the operator in the field or the analyst in the office, except the determination of the limits to which it is desirable to carry the actual leveling in the field.

If there is a special feature within the usual terrain region—such as an embankment or deep ditch which violates the assumption upon which the standard terrain formulas are based; *i. e.*, the assumption that the ground undulates naturally—great caution must be exercised in applying the terrain formulas. The best procedure is to assume that, throughout the region occupied by the feature, the altitudes of whatever points

of the terrain network occur in the region, relative to the station point, are nil, and to calculate the effects due to the special feature separately by the methods used for three-dimensional features.

Cases of this nature occur quite frequently in England, where it is often necessary to site observing stations close to some artificial feature which violates, both in its configuration and density, the normal terrain conditions.

As regards the outer limits of the terrain region, this is usually considered to end where the radial distance ρ becomes so large compared with the altitude of the balance center h that the magnitude $r = \sqrt{\rho^2 + h^2}$ becomes practically equal to ρ.

In England, we regard $\rho = 100$ (feet) as representing this limit. Outside this circle, the topographical region commences.

Topographical Features

As in the case of the terrain region, it is assumed also for the topographical region that we are dealing with the natural undulations of ordinary ground, so that we can still neglect such quantities as ζ^2 and $h\zeta$, and also now h^2 itself, in comparison with ρ^2 for every vertical column of matter into which the ground above the horizontal of the station point can be considered to be divided. Since, however, the radial distances ρ in the topographical region are necessarily larger, abrupt features which would require separate consideration if occurring within the terrain region become relatively unimportant outside, and can usually be considered as merged into the natural ground.

Generally speaking, it has been found necessary to take into account topographical features in England, although the effects are very rarely comparable with those due to terrain irregularities.

The topographical formulas for a vertical cylindrical element of area $\rho \cdot d\rho \cdot d\alpha$ and height ζ are

$$(10)\quad \left\{\begin{aligned} \frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2} &= -3\gamma\sigma \cos 2\alpha \cdot d\alpha \cdot \frac{d\rho}{\rho^2} \cdot \zeta \\ 2\frac{\partial^2 U}{\partial x \partial y} &= 3\gamma\sigma \sin 2\alpha \cdot d\alpha \cdot \frac{d\rho}{\rho^2}\zeta \\ \frac{\partial^2 U}{\partial x \partial z} &= 3\gamma\sigma \cos \alpha \cdot d\alpha \cdot \frac{d\rho}{\rho^3} \cdot \zeta\left(h - \frac{\zeta}{2}\right) \\ \frac{\partial^2 U}{\partial y \partial z} &= 3\gamma\sigma \sin \alpha \cdot d\alpha \cdot \frac{d\rho}{\rho^3}\zeta\left(h - \frac{\zeta}{2}\right) \end{aligned}\right.$$

In order to evaluate the total effects for a standard topographical region (say between $\rho = 100$ and $\rho = 1000$) two methods are possible. The first method, used by Eötvös himself and further developed by

Schweydar, was in essence an extension of the method used for terrain features; *i. e.*, the employment of a standard network at each point of which ζ is measured by leveling. This method has, however, serious drawbacks in practice, since the type of leveling involved—*i. e.*, a "radial" type—is not very economical over large areas. Also, the computation is unnecessarily elaborate. A better method is the one outlined by Nikiforov and Numerov,[1] which is based on a division of the topographical region into bands of equal altitude—*i. e.*, into contour bands—and the computation of the effects due to each separate band in turn by means of an ingenious graticule of radial lines and circles, which can be drawn upon transparent cloth or celluloid and superposed upon the contoured plan of the region. This plan can be obtained in the most convenient manner suited to the survey region, and it has been found quite feasible in England to construct it from the actual terrain data for the network of stations, supplemented by a station to station traverse leveling, with occasional offsets into regions not covered by the terrain skeleton. Such a contoured plan, with contour lines at 5-ft. vertical intervals, forms in itself a valuable record for other purposes. It is possible at a glance to check approximately the terrain effects, and of course the plan has uses apart from the gravity survey.

The graticules (only two are necessary, one for curvature magnitudes and the other for gradients) are so constructed that the sectors bounded by adjacent radial lines and circles have equal "weight." For the curvature graticule:

$$\int_{\rho_n}^{\rho_{n+1}} \int_{\alpha_m}^{\alpha_{m+1}} \frac{d\rho}{\rho^2} \cos 2\alpha \cdot d\alpha = \text{constant}$$

for all values of n and m and in the gradient graticule and

$$\int_{\rho_n}^{\rho_{n+1}} \int_{\alpha_m}^{\alpha_{m+1}} \frac{d\rho}{\rho^3} \cos \alpha \cdot d\alpha = \text{constant}$$

for all values of n and m. Consequently it is only necessary to find by superposition how many sectors are comprised within a given contour band, and multiply this by the appropriate constant and altitude factor—*i. e.*, ζ for curvatures and $\zeta\left(h - \frac{\zeta}{2}\right)$ for gradients—to obtain the total effect due to the whole band. Without changing the setting of the graticule, each contour band can be "numbered" in this way with great ease, and the total effect due to topography calculated.

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[1] Nikiforov: Physical Basis of Gravitational Method of Geophysical Survey. *Bull.* Inst. of Practical Geophysics. U. R. S. S. Leningrad (1926, No. 2), 232.

B. Numerov: Graphische Methode zur Berucksichtigung des topographischen Einflüsses und des Einfüsses der unterirdischen Massen auf die gravimetrischen Beobachtungen. *Ztsch. f. Geophysik* (1924–25) **8**, 367.

The writer has found it preferable, however, to mark the center of each sector by a dot, and afterwards erase the graticule construction lines. Since this graticule method of computing topographical effects is not so well known as the former system, and, so far as the writer knows, the exact method of constructing the graticules has not hitherto been published, it will be of interest to give details of the method adopted.

The curvature graticule consists of a series of radial lines $\alpha_0, \alpha_1, \ldots \alpha_m \ldots$ so spaced that $\sin 2\alpha_{m+1} - \sin 2\alpha_m =$ constant θ for all values of m intersected by a series of concentric circles of radii $\rho_0, \rho_1 \ldots \rho_n \ldots$ so chosen that

$$\frac{1}{\rho_n} - \frac{1}{\rho_{n+1}} = \text{constant } \kappa \text{ for all values of } n.$$

Then

$$\int_{\rho_n}^{\rho_{n+1}} \frac{d\rho}{\rho^2} \int_{\alpha_m}^{\alpha_{m+1}} \cos 2\alpha \cdot d\alpha = \frac{1}{2}\left(\frac{1}{\rho_n} - \frac{1}{\rho_{n+1}}\right)(\sin 2\alpha_{m+1} - \sin 2\alpha_m)$$
$$= \frac{1}{2}\kappa\theta \text{ for all values of } m \text{ and } n; \textit{ i. e.}, \text{ for all}$$

segments of the graticule.

Also by a suitable selection of the constant θ the same graticule has the characteristic that

$$-\cos 2\alpha_m = \sin 2\left(\alpha_m - \frac{\pi}{4}\right)$$

so that $\int_{\rho_n}^{\rho_{n+1}} \frac{d\rho}{\rho^2} \int_{\alpha_m}^{\alpha_{m+1}} \sin 2\alpha \cdot d\alpha = \frac{1}{2}\kappa\theta$ for all values of m and n if α is measured from a new zero line 45° from the old one; *i. e.*, the same graticule, rotated through 45°, serves to evaluate both

$$\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2} \text{ and } \frac{\partial^2 U}{\partial x \partial y}$$

Similarly the gradient graticule has a series of radial lines such that

$$\sin \alpha_{m+1} - \sin \alpha_m = \text{constant } \phi$$

intersected by a series of circles such that

$$\frac{1}{\rho_n^2} - \frac{1}{\rho_{n+1}^2} = \text{constant } \lambda$$

whence

$$\int_{\rho_n}^{\rho_{n+1}} \frac{d\rho}{\rho^3} \int_{\alpha_m}^{\alpha_{m+1}} \cos \alpha \cdot d\alpha = \text{constant } \frac{1}{2}\lambda\phi \text{ for all segments;}$$

and, by rotation through 90°, the graticule also makes

$$\int_{\rho_n}^{\rho_{n+1}} \frac{d\rho}{\rho^3} \int_{\alpha_m}^{\alpha_{m+1}} \sin \alpha \cdot d\alpha = \text{the same constant } \frac{1}{2}\lambda\phi \text{ for all segments.}$$

The actual lines of the graticule can be constructed from tables, or obvious graphical methods can be employed. If it is desired to use the dot method of representation, it is better to construct the preliminary network of lines at half the actual interval desired, so that alternate intersection points can be dotted.

Cartographical Features

Strictly speaking there is no essential difference between topographical and cartographical features. The cartographical region may be said to embrace all the region outside a selected circle which limits the topographical area, and the dividing limit will vary in different countries. It is assumed that contoured maps already exist for the cartographical region, and as the scale of vertical intervals on public maps varies considerably, it will be necessary to extend the outer limit of the topographical region in cases where the public maps are small-scale ones, with rough contours at large vertical intervals. In well mapped countries, there are usually available large-scale topographical maps having contours every 50 ft., or 10 m.; in such regions, the limit for private mapping—*i. e.*, for the topographical region—may well be put as low as 1000 ft. This has been found quite sufficient in England, where, however, practically no use is made of curvature magnitudes, which require specially extensive corrections for topography as compared with gradients.

The actual procedure of computing the cartographical effects is exactly the same as for topographical features, the same or similar graticules being employed. The "constants" of the graticules will depend upon the scale of the map, if the same graticule is employed.

The actual altitudes ζ and h (if this latter is employed at all) must be expressed as fractions of the unit used in the graticule for the distances ρ.

Three-dimensional Features

We have seen that, for all terrain, topographical and cartographical features, the essential assumption has been that throughout the disturbing feature the third coordinate ζ is regarded as so relatively small that it can be extracted from the magnitude r during the integration with respect to ζ. In effect, this transforms a three-dimensional summation into a two-dimensional one. Moreover, the three coordinates ρ, α and ζ in this case become isolated from one another; *i. e.*, we have to integrate functions of the type

$$f_1(\rho) \times f_2(\alpha) \times f_3(\zeta)$$

not of the more complicated type $f(\rho, \alpha, \zeta)$ where each coordinate is interconnected with the others.

These analytical characteristics disappear as soon as the coordinate ζ becomes at all comparable with the others, and the problem of summation of elemental effects is rendered much more difficult. Two methods of attacking this problem have been employed in practice. The first consists in developing formulas for certain special types of feature, characterized by some properties which render the summation determinable by the standard processes of integral calculus. The properties usually found desirable are (a) geometrical regularity and simplicity of outline of the boundary surface of the feature, and (b) extension of the feature in some one or more directions to infinity.

The second procedure is immediately applicable to any feature, however irregular and is an extension of that employed for topography; it may be termed a "graphical integration" process. In the first method it is assumed that, to a reasonable approximation the feature can be regarded as bounded by a geometrically simple surface, which is true in the sense that most features, however complicated, actually can be divided up into simple bodies of the geometrical type. This process is, however, necessarily limited in application by considerations of time and labor. In the writer's opinion, there has been a tendency to ignore the limitations of the method and to carry the process of "regularization" of complex features to extremes, although it is not denied that the method can serve as a useful guide.

We shall therefore divide up the three-dimensional features into "regular," in the sense of geometrical simplicity, and "irregular" classes, and enumerate the principal solutions in each case. In every instance the density will be assumed to be uniform.

Regular Features

The Sphere

The spherical body is the simplest case of an attracting mass, since, as is well known, its effects are exactly equal to those of a particle of mass equal to that of the whole body, concentrated at the center of the sphere.

Consequently any of the formulas 1 to 4 for a point element may be used to express the effects due to a sphere of total mass μ.

It is, however, very rare that features which occur in practice can be regarded as even roughly spherical, or split up into components of the spherical type.

Figures Bounded by Plane Surfaces

For practical utility and comparative geometrical simplicity for integration purposes, we pass naturally to simple figures bounded by plane surfaces. Such figures have the merit that they can be fitted

together without spaces, and so serve to build up into forms of quite complicated outline. We immediately think of a brick, or rectangular block, a figure eminently suited to analysis and synthesis, and we shall next show how the formulas for the Eötvös effects of a wholly finite, rectangular block, having its faces parallel to the normal rectangular coordinate planes, can be obtained by standard integration. The formulas as regards the gradients were given in outline by the present writer and his colleague in 1923, but, so far as he is aware, the corresponding formulas for curvature effects have not hitherto been published.

Finite Rectangular Block.—Bounded by planes $a = a_1, a = a_2\ (a_2 > a_1)$, $b = b_1, b = b_2\ (b_2 > b_1)$, $c = c_1, c = c_2\ (c_2 > c_1)$.

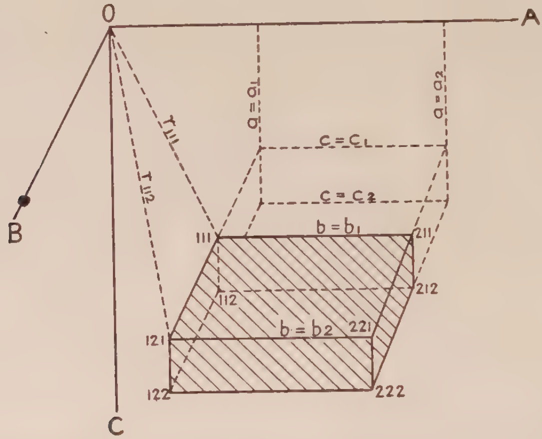

FIG. 4.—COORDINATE SYSTEM OF ATTRACTING RECTANGULAR BLOCK OF FINITE BOUNDARIES, REFERRED TO CENTER OF BALANCE O.

Using Formula (1), and replacing μ by $\sigma \cdot da \cdot db \cdot dc$ we have to integrate the expressions for point elements throughout the block volume V. To obtain $\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}$ we shall obtain $\frac{\partial^2 U}{\partial x^2}$ and $\frac{\partial^2 U}{\partial y^2}$ separately.

$$\frac{\partial^2 U}{\partial x^2} = \gamma\sigma \int_V \frac{3a^2 - r^2}{r^5} da \cdot db \cdot dc = \gamma\sigma \int_V \frac{\partial}{\partial a}\left(-\frac{a}{r^3}\right) da \cdot db \cdot dc$$

$$= \gamma\sigma \int_{c_1}^{c_2} dc \int_{b_1}^{b_2} \left[-\frac{a \cdot db}{r^3}\right]_{a=a_1}^{a=a_2}$$

$$= \gamma\sigma \int_{c_1}^{c_2} -dc \left[\left[\frac{a}{a^2 + c^2} \cdot \frac{b}{r}\right]\right]_{a_1, b_1}^{a_2, b_2}$$

$$= -\gamma\sigma \left[\left[\left[\tan^{-1} \frac{bc}{ar}\right]\right]\right]_{a_1, b_1, c_1}^{a_2, b_2, c_2}$$

Similarly

$$\frac{\partial^2 U}{\partial y^2} = \gamma\sigma \int_V \frac{\partial}{\partial b}\left(-\frac{b}{r^3}\right) da \cdot db \cdot dc$$

$$= -\gamma\sigma \left[\left[\left[\tan^{-1}\frac{ac}{br}\right]\right]\right]_{a_1,b_1,c_1}^{a_2,b_2,c_2}$$

$$\therefore \frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2} = \gamma\sigma\left[\left[\left[\theta - \phi\right]\right]\right]_{111}^{222} \text{ where } \theta = \tan^{-1}\frac{bc}{ar}$$

$$\phi = \tan^{-1}\frac{ac}{br}$$

$$= \gamma\sigma[(\theta_{222} - \phi_{222}) + (\theta_{211} - \phi_{211}) + (\theta_{121} - \phi_{121}) + (\theta_{112} - \phi_{112}) - (\theta_{111} - \phi_{111}) - (\theta_{122} - \phi_{122}) - (\theta_{212} - \phi_{212}) - (\theta_{221} - \phi_{221})]$$

where the suffixes 111, 121, 122, etc. relate to the eight vertices of the block.

Also $\dfrac{\partial^2 U}{\partial x \partial y} = \gamma\sigma \displaystyle\int_V \frac{3ab}{r^5} da \cdot db \cdot dc = \gamma\sigma \int_V \frac{\partial}{\partial b}\left(\frac{\partial}{\partial a}\cdot\frac{1}{r}\right) da \cdot db \cdot dc$

$$= \gamma\sigma \int_{c_1}^{c_2} dc \int_{b_1}^{b_2} db \left[\frac{\partial}{\partial b}\left(\frac{1}{r}\right)\right]_{a_1}^{a_2}$$

$$= \gamma\sigma \int_{c_1}^{c_2} dc \left[\left[\frac{1}{r}\right]\right]_{a_1,b_1}^{a_2,b_2}$$

$$= \gamma\sigma\left[\left[\left[\log_e (c + r)\right]\right]\right]_{a_1,b_1,c_1}^{a_2,b_2,c_2}$$

$$= \gamma\sigma \log_e \frac{(c_2 + r_{222})(c_1 + r_{211})(c_1 + r_{121})(c_2 + r_{112})}{(c_1 + r_{111})(c_2 + r_{122})(c_2 + r_{212})(c_1 + r_{221})}$$

and $\dfrac{\partial^2 U}{\partial x \partial z}, \dfrac{\partial^2 U}{\partial y \partial z}$ are found by exactly analogous reasoning.

Hence we have, summing up, for a finite rectangular block:

$$(11) \left\{ \begin{aligned} \frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2} &= \gamma\sigma\left[\left[\left[\tan^{-1}\frac{bc}{ar} - \tan^{-1}\frac{ac}{br}\right]\right]\right]_{111}^{222} \\ \frac{\partial^2 U}{\partial x \partial y} &= \gamma\sigma\left[\left[\left[\log_e (c + r)\right]\right]\right]_{111}^{222} \\ \frac{\partial^2 U}{\partial x \partial z} &= \gamma\sigma\left[\left[\left[\log_e (b + r)\right]\right]\right]_{111}^{222} \\ \frac{\partial^2 U}{\partial y \partial z} &= \gamma\sigma\left[\left[\left[\log_e (a + r)\right]\right]\right]_{111}^{222} \end{aligned} \right.$$

where the expression $\left[\left[\left[f_n(a, b, c)\right]\right]\right]_{111}^{222}$ signifies

$$\left\{ \begin{aligned} &f_n(a_2, b_2, c_2) + f_n(a_2, b_1, c_1) + f_n(a_1, b_2, c_1) + f_n(a_1, b_1, c_2) \\ &-f_n(a_1, b_1, c_1) - f_n(a_1, b_2, c_2) - f_n(a_2, b_1, c_2) - f_n(a_2, b_2, c_1) \end{aligned} \right\}$$

The preceding formula for a finite block becomes considerably simplified if some of the faces of the block are sufficiently remote from the origin to be considered at infinity.

For example, if we can regard the face $b = b_1$ as being at negative infinity and $b = b_2$ as at positive infinity, we can assume that

$$\frac{b_2}{r_{222}} = \frac{b_2}{r_{122}} = \frac{b_2}{r_{121}} = \frac{b_2}{r_{221}} = +1$$

and

$$\frac{b_1}{r_{111}} = \frac{b_1}{r_{211}} = \frac{b_1}{r_{212}} = \frac{b_1}{r_{112}} = -1$$

whence

$$\theta_{222} = \tan^{-1}\frac{c_2}{a_2};\ \theta_{211} = -\tan^{-1}\frac{c_1}{a_2};\ \theta_{121} = \tan^{-1}\frac{c_1}{a_1};\ \theta_{112} = -\tan^{-1}\frac{c_2}{a_1}$$

$$-\theta_{111} = \tan^{-1}\frac{c_1}{a_1};\ -\theta_{122} = -\tan^{-1}\frac{c_2}{a_1};\ -\theta_{212} = \tan^{-1}\frac{c_2}{a_2};\ -\theta_{221} = -\tan^{-1}\frac{c_1}{a_2}$$

and all the terms $\phi = 0$
so that, for an

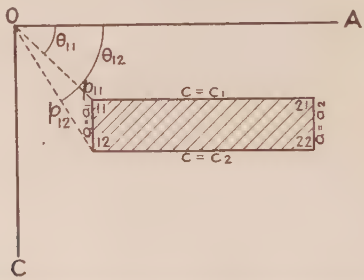

FIG. 5.—COORDINATE SYSTEM OF INFINITE BLOCK OF FINITE RECTANGULAR SECTION, REFERRED TO CENTER OF BALANCE O.

Infinite Rectangular Block ($b_1 = -\infty$, $b_2 = +\infty$) we have

$$\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2} = 2\gamma\sigma(\theta_{22} + \theta_{11} - \theta_{12} - \theta_{21})$$

where $\theta_{22} = \tan^{-1}\frac{c_2}{a_2}$, etc.

and it can be shown that under these conditions

(11a)
$$\frac{\partial^2 U}{\partial x \partial y} = 0$$

$$\frac{\partial^2 U}{\partial x \partial z} = 2\gamma\sigma \log_e \frac{p_{21} \cdot p_{12}}{p_{11} \cdot p_{22}} \qquad p_{22} = \sqrt{a_2^2 + c_2^2}, \text{ etc.}$$

$$\frac{\partial^2 U}{\partial y \partial z} = 0$$

If in addition the face $a = a_2$ is at positive infinity, the above reduces to

$$\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2} = 2\gamma\sigma(\theta_2 - \theta_1)$$

$$\frac{\partial^2 U}{\partial x \partial z} = 2\gamma\sigma \log_e \frac{p_2}{p_1}$$

Infinite Horizontal Block with Sloping Edge.—A very frequently employed regular feature is the one shown in Fig. 7, where the mass is assumed to be uniform in vertical cross-section, and to extend to infinity on both sides of the reference plane AOC parallel to the axis OB.

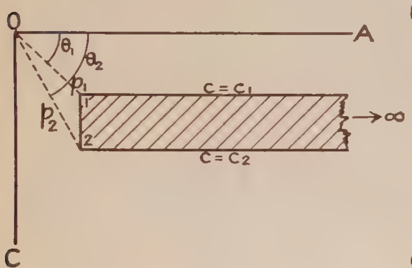

FIG. 6.—INFINITE BLOCK, RECTANGULAR SECTION, UNLIMITED TO RIGHT.

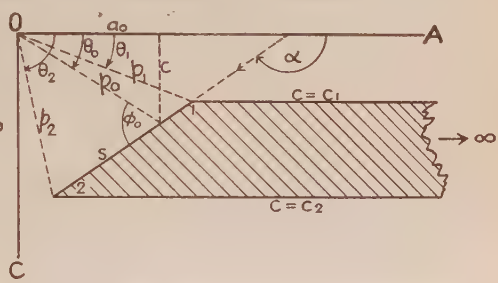

FIG. 7.—INFINITE BLOCK WITH SLOPING EDGE, UNLIMITED TO RIGHT.

In this case we can employ a modification of Formula (7*b*) for a thin, doubly infinite line element parallel to OB but using rectangular axes AOC. We have

$$\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2} = -2\gamma\sigma \int \frac{\cos 2\theta}{p^2}\delta = -2\gamma\sigma \int \frac{a^2 - c^2}{p^4} dadc$$

where $p^2 = a^2 + c^2$ and the integral is over the cross-section of the block.

$$= -2\gamma\sigma \int_{c_1}^{c_2} dc \int_{a_0}^{\infty} \frac{a^2 - c^2}{p^4} da \quad \text{where } a = a_0 \text{ on the sloping edge}$$

$$= -2\gamma\sigma \int_{c_1}^{c_2} \frac{dc \,.\, a_0}{p_0^2} \qquad \text{where } p_0^2 = a_0^2 + c^2$$

$$= -2\gamma\sigma \int_{c_1}^{c_2} \frac{dc}{p_0} \cos\theta_0 \qquad \text{where } \phi_0 = \alpha - \theta_0$$

$$= -2\gamma\sigma \int_{c_1}^{c_2} \frac{d\theta_0 \sin\alpha \cos\theta_0}{\sin\phi_0} \qquad p_0 d\theta_0 = ds \sin\phi_0, \quad dc = ds \sin\alpha$$

$$= 2\gamma\sigma \sin\alpha \int_{c_1}^{c_2} \frac{d\varphi_0 \cos(\alpha - \varphi_0)}{\sin\varphi_0}$$ and s is measured along the sloping edge

$$= 2\gamma\sigma \sin\alpha \left\{ \cos\alpha \int_1^2 \frac{d(\sin\varphi_0)}{\sin\varphi_0} + \sin\alpha \int_1^2 d\phi_0 \right\}$$

$$= 2\gamma\sigma \sin\alpha \left\{ \cos\alpha \log_e\left(\frac{\sin\phi_2}{\sin\phi_1}\right) + (\phi_2 - \phi_1)\sin\alpha \right\}$$

$$\left.\begin{aligned}
&\therefore \frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2} = -\gamma\sigma\left\{\sin 2\alpha \cdot \log_e \frac{p_2}{p_1} + 2\sin^2\alpha(\theta_2 - \theta_1)\right\}\\
&\text{Similarly}\\
&\qquad \frac{\partial^2 U}{\partial x \partial z} = +2\gamma\sigma \int \frac{2ac}{p^4}\, da\, dc = 2\gamma\sigma \int_{c_1}^{c_2} c\, dc \int_{a_0}^{\infty} \frac{2a\,da}{p^4}\\
&\qquad\qquad = 2\gamma\sigma \int_{c_1}^{c} \frac{c\, dc}{p_0^2} \text{ on the edge}\\
&\qquad\qquad = 2\gamma\sigma \int_1^2 \frac{d\theta_0 \sin\alpha \cdot \sin\theta_0}{\sin\varphi_0} = -2\gamma\sigma \int_1^2 \frac{d\varphi_0 \sin\alpha}{\sin\varphi_0} \times (\sin\alpha\cos\varphi_0 - \cos\alpha\sin\varphi_0)\\
&\qquad\qquad = -2\gamma\sigma\left(\sin^2\alpha \int_1^2 \frac{d(\sin\phi_0)}{\sin\varphi_0} - \sin\alpha\cos\alpha \int_1^2 d\phi_0\right)\\
&\therefore \frac{\partial^2 U}{\partial x \partial z} = \gamma\sigma\left\{2\sin^2\alpha \log_e \frac{p_2}{p_1} - \sin 2\alpha(\theta_2 - \theta_1\right\}
\end{aligned}\right\} \tag{12}$$

Infinite Cylinder of Polygonal Section.—Consider a mass in the form of a cylinder having its generating lines parallel to the reference axis OB, and continued to infinity in both directions.

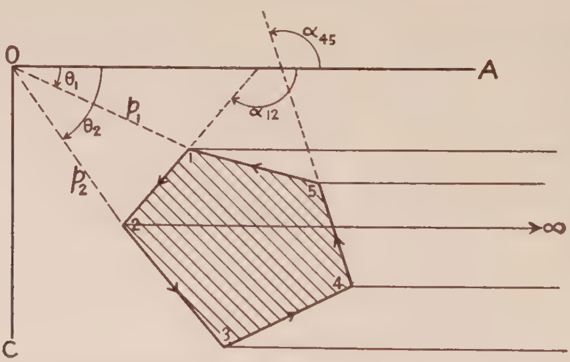

FIG. 8.—COORDINATE SYSTEM OF POLYGONAL SECTION OF INFINITE BLOCK.

Let the points 1, 2, 3, 4, 5 form the vertices of the section of the cylinder by the reference plane AOC.

If lines are drawn parallel to OA and extending to infinity, and if [12] denotes the trapezium formed by the line 12 and the corresponding lines through 1 and 2 parallel to OA and similarly for [23] [34] [45] and [51], it is evident that

$$\text{polygon } [12345] = [12] + [23] - [34] - [45] - [51]$$

Similarly the cylinder can be derived from the corresponding infinite blocks with sloping edges whose traces in the plane AOC are the lines 12, 23, 34, etc.

And it is evident, from Formula 12 that the Eötvös effects of the cylinder are given by the following formulas.

$$
(13)\begin{cases} \dfrac{\partial^2 U}{\partial y^2} - \dfrac{\partial^2 U}{\partial x^2} = -\gamma\sigma\left\{\sum \sin 2\alpha_{12} \log_e \dfrac{p_2}{p_1} + \sum 2 \sin^2 \alpha_{12}(\theta_2 - \theta_1)\right\} \\ \dfrac{\partial^2 U}{\partial x \partial z} = \gamma\sigma\left\{\sum 2 \sin^2 \alpha_{12} \log_e \dfrac{p_2}{p_1} - \sum \sin 2\alpha_{12}(\theta_2 - \theta_1)\right\} \end{cases}
$$

where the symbol $\sum$ denotes that the expressions are to be taken on each side of the polygon 1 2 3 4 5, in the sense indicated by the arrows.

Infinite Cylinder of Curvilinear Section.—It is evident that an infinite cylinder having its cross-section bounded by any closed curve can be approximately represented by a cylinder of the polygonal type, since the polygon can be drawn to correspond with the curve as closely as is desired.

Formulas 13 are then applicable to this general class of feature, which can be considered of great practical importance.

Assumptions of Infinite Extent in Regular Features

In computing the effects of an actual mass anomaly, when this is of the three-dimensional type and appears to be "cylindrical" in character—*i. e.*, there is reason to believe that it extends to a distance on each flank of the survey area, and preserves a uniformity in cross-section—it is quite customary to regard such an anomaly as actually of the infinite cylinder type. While this is doubtless justified in many cases, it is desirable to apply tests if the observations permit. There is no difficulty in formulating simple quantitative rules for such tests.

Referring to Formulas 7, which give the effects due to rods extending from the reference plane AOC parallel to the axis OB, it is evident that if b is large compared with p, we can get approximate formulas for a long rod. For we have

$$
r^2 = b^2 + p^2 = b^2\left(1 + \frac{p^2}{b^2}\right) = b^2(1 + \epsilon^2) \qquad \epsilon = \frac{p}{b}
$$

whence

$$
\frac{b}{r} = (1 + \epsilon^2)^{-1/2} = 1 - \frac{1}{2}\epsilon^2 + \frac{3}{8}\epsilon^4
$$

$$
\frac{b^3}{r^3} = (1 + \epsilon^2)^{-3/2} = 1 - \frac{3}{2}\epsilon^2 + \frac{15}{8}\epsilon^4
$$

whence

$$
\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2} = \gamma\frac{\sigma\delta}{p^2}\left\{(1 + \cos^2\theta)\left(1 - \frac{3}{2}\epsilon^2 + \frac{15}{8}\epsilon^4\right) - \cos^2\theta\left(3 - \frac{3}{2}\epsilon^2 + \frac{9}{8}\epsilon^4\right)\right\}
$$

$$
= \gamma\frac{\sigma\delta}{p^2}(1 - 2\cos^2\theta) - \frac{3}{2}\epsilon^2 \cdot \frac{\gamma\sigma\delta}{p^2} \text{ to a first approximation}
$$

$$
\frac{\partial^2 U}{\partial x \partial z} = \gamma\frac{\sigma\delta}{2p^2}\sin 2\theta\left\{3 - \frac{3}{2}\epsilon^2 + \frac{9}{8}\epsilon^4 - 1 + \frac{3}{2}\epsilon^2 - \frac{15}{8}\epsilon^4\right\}
$$

$$
= \frac{\gamma\sigma\delta}{p^2}\sin 2\theta - \frac{3}{8}\frac{\gamma\sigma\delta}{p^2}\epsilon^4 \sin 2\theta
$$

from which it is evident that the "surplus" as compared with the formulas for an infinitely long rod is for

$$\left\{\begin{aligned} \frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2} &= -\frac{3}{2}\epsilon^2 \cdot \frac{\gamma\sigma\delta}{p^2} \\ \frac{\partial^2 U}{\partial x \partial z} &= -\frac{3}{8}\epsilon^4 \frac{\gamma\sigma\delta}{p^2} \sin 2\theta \end{aligned}\right.$$

and, since there is no difficulty in approximately integrating these values over any section area (see later) we can obtain quantitative estimates of the errors introduced by the assumption that the cylinder is infinite.

It will be noted that the surplus for the curvature effect is proportional to ϵ^2 while that for the gradient is proportional to only ϵ^4, so that, as one would naturally expect, far greater precautions ought to be observed in assuming that the curvature effects of a long cylinder are the same quantitatively as those of the corresponding infinite cylinder, than need be taken as regards the gradient effects.

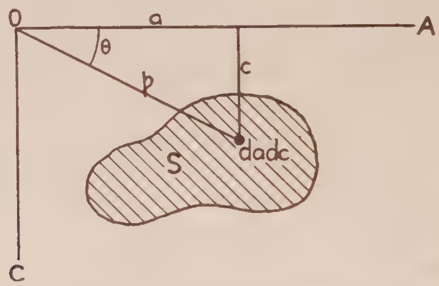

FIG. 9.—INFINITE CYLINDER OF IRREGULAR SECTION.

As approximate expressions for the surplus effects of the "long" cylinder compared with the "infinite" cylinder, we may evaluate the integrals.

$$\frac{3}{2}\gamma\sigma \int_s \frac{\epsilon^2}{p^2} da \cdot dc \text{ and } \frac{3}{8}\gamma\sigma \int_s \frac{\epsilon^4}{p^2} \sin 2\theta \cdot da \cdot dc$$

where S is the cross-sectional area of the cylinder, on the understanding that

$$\epsilon = \frac{\rho}{b} \text{ and } b \text{ is constant.}$$

We have, for the curvature effect:

$$\frac{3}{2}\gamma\sigma \int_s \frac{\epsilon^2}{p^2} da \cdot dc = \frac{3}{2}\frac{\gamma\sigma}{b^2} \int_s da \cdot dc = \frac{3}{2}\gamma\sigma \cdot \frac{S}{b^2}$$

and for the gradient:

$$\begin{aligned} \frac{3}{8}\gamma\sigma \int_{(s)} \frac{\epsilon^4}{p^2} \sin 2\theta \cdot da \cdot dc &= \frac{3}{4}\frac{\gamma\sigma}{b^4} \int_{(s)} ac \cdot da \cdot dc \\ &= \frac{3}{4}\gamma\sigma \frac{a\,c\,S}{b^4} \text{ approximately} \end{aligned}$$

where (a, c) are the coordinates of the center of gravity of the section S.

Irregular Features

Any three-dimensional feature, such as V in Fig. 10, can be regarded as lying within a vertical cylindrical surface, of which S is the trace on the horizontal reference plane AOB and S' is the line of contact of the cylinder and the surface of the body V. S' divides the whole surface of V into two parts, the upper U and the lower L. The body V can be regarded as formed by the subtraction of the cylinder, bounded above by the plane of reference AOB and below by the upper surface U, from the cylinder bounded above by the plane of reference AOB and below by the lower surface L.

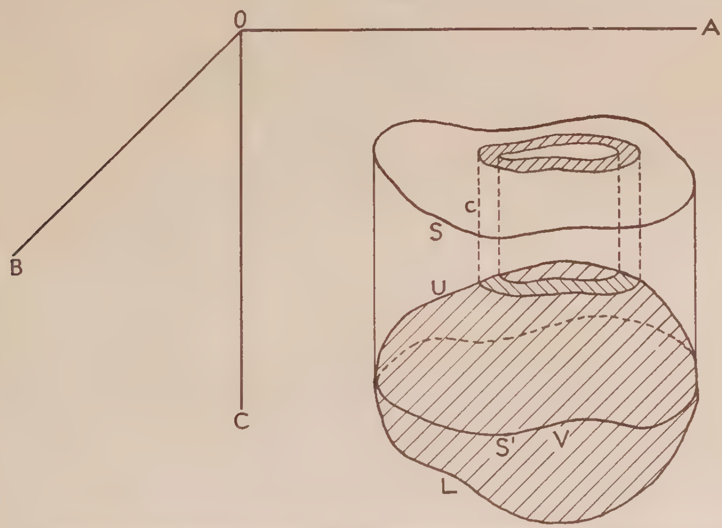

Fig. 10.—Diagram showing division of an irregular body into vertical, hollow cylinders.

Hence the problem of calculating the effects of V reduces to the problem of calculating the effects of both of these cylinders, which are of the same type, having one face in the reference plane. Consider the former cylinder SU.

The surface U can be divided up, by means of contour lines, and through each contour line cylinders can be drawn parallel to that through S. These cylinders divide up the cylinder SU into a series of cylindrical rings of constant length c. If, then, we can evaluate the gravity effects for such a cylindrical ring, we can sum all such similar effects and obtain the total effects for SU.

The problem of obtaining any one of the Eötvös effects for a cylindrical ring reduces to the problem of integrating, over the area of section of the ring, the effect of a thin vertical rod, the formulas which are given in (5) and (6), c being constant.

This is best performed graphically, after the manner in which topographical computations are made.

Divide up the area of the reference plane AOB into sectors by means of radial lines through O, $\alpha = \alpha_m$ and concentric circles $\rho = \rho_n$.

The integral of $\frac{d^2U}{dy^2} - \frac{d^2U}{dx^2}$ over the sector is (Formulas (6))

$$\gamma\sigma \int_{\alpha_m}^{\alpha_{m+1}} \cos 2\alpha \cdot d\alpha \int_{\rho_n}^{\rho_{n+1}} \frac{1}{\rho}\left(\frac{3c}{r} - \frac{c^3}{r^3}\right) d\rho \qquad \begin{aligned} \delta &= \rho \cdot d\rho \cdot d\alpha \\ r^2 &= \rho^2 + c^2;\ \tan\theta = \frac{c}{\rho} = k \end{aligned}$$

$$= \gamma\frac{\sigma}{2}(\sin 2\alpha_{m+1} - \sin 2\alpha_n)\left\{2\log_e \frac{r-c}{\rho} - \frac{c}{r}\right\}_{\rho_n}^{\rho_{n+1}}$$

$$\int \frac{c \cdot d\rho}{r\rho} = \log_e\left(\tan\frac{\theta}{2}\right)$$

$$= \gamma\frac{\sigma}{2}\theta f_n \qquad = \log_e(\sqrt{1+k^2} - k)$$

$$= \log_e \frac{r-c}{\rho}$$

$$\int \frac{c \cdot {}^3d\rho}{\rho r^3} = \cos\theta + \log_e\left(\tan\frac{\theta}{2}\right)$$

$$= \frac{c}{r} + \log_e \frac{r-c}{\rho}$$

$$= \frac{1}{\sqrt{1+k^2}} + \log_e(\sqrt{1+k^2} - k)$$

where, on the curvature topographical graticule θ is the constant (sin $2\alpha_{m+1}$ − sin $2\alpha_m$) and f_n is a constant depending only on ρ_n, ρ_{n+1} and c. Similarly, the integral of $\frac{d^2U}{dxdy}$ over the sector $(\alpha_m, \alpha_{m+1}, \rho_n, \rho_{n+1})$ is

$$\gamma\sigma \int_{\alpha_m}^{\alpha_{m+1}} \sin 2\alpha \cdot d\alpha \int_{\rho_n}^{\rho_{n+1}} \frac{1}{\rho}\left(\frac{3c}{r} - \frac{c^3}{r^3}\right) d\rho = \gamma\frac{\sigma}{2}\theta f_n$$

if the graticule is rotated through 45° as for topographical corrections.

The constants f_n depend on ρ_n, ρ_{n+1} and c, and therefore if the radii ρ_n, ρ_{n+1} etc., are fixed once for all, these constants can be calculated for all values of c, for each annular ring ρ_n to ρ_{n+1}. The computation of the effects for the cylindrical ring of constant height c thus reduces to the problem of superposing on it the graticule, counting the number of sectors from each annular ring of the graticule which fall within the annular section of the cylinder, multiplying each of these numbers by the appropriate constant $\gamma\frac{\sigma}{2}\theta f_n$ and summing the totals.

This process is repeated for each contour band until the resultant effect for the whole solid cylinder is obtained.

In the same way, for the gradient effects due to that part of the contour band of mean height c, which is within the sector $\alpha_m, \alpha_{m+1}, \rho_n, \rho_{n+1}$ we have

$$\begin{aligned}\frac{\partial^2 U}{\partial x \partial z} &= \gamma\sigma \int_{\alpha_m}^{\alpha_{m+1}} \cos \alpha \, d\alpha \int_{\rho_n}^{\rho_{n+1}} \frac{d\rho}{\rho}\left(1 - \frac{\rho^3}{r^3}\right) \\ &= \gamma\sigma(\sin \alpha_{m+1} - \sin \alpha_m) \left[\frac{\rho}{r} - \log_e\left(1 + \frac{r}{\rho}\right)\right]_{\rho_n}^{\rho_{n+1}} \\ &= \gamma\sigma\phi F_n\end{aligned}$$

where $\phi = \sin \alpha_{m+1} - \sin \alpha_m$ which is constant for the topographical gradient graticule and F_n is a constant for the annular ring ρ_n, ρ_{n+1} and the height c.

Similarly

$$\begin{aligned}\frac{\partial^2 U}{\partial y \partial z} &= \gamma\sigma(\cos \alpha_m - \cos \alpha_{m+1})F_n \\ &= \gamma\sigma\phi F_n\end{aligned}$$

for the same graticule, rotated through 90°. The constants F_n, can be calculated once for all for various values of ρ_n, ρ_{n+1} and c, and the gradient effects for each contour band, and so for the whole solid cylinder SU, found by superposing the graticule, counting the number of sectors of each annular ring for each contour band, and multiplying by the appropriate factors ϕ and F_n.

It should be noted that the constants θ and ϕ are the same throughout each graticule, but the constants f_n and F_n are only the same in any one annular ring for each particular height c of the contour band.

It would, of course, be possible to construct a series of graticules having the property that f_n was the same constant for all the annular rings for any given height c. There would then be one graticule for each altitude. Similarly for F_n. But this would necessitate a change of graticule for each contour band. It is preferable to keep to the standard topographical graticule, and use this in conjunction with tables of values of f_n and F_n.

The foregoing method of obtaining the effects arising from any feature, however irregular, has been developed by Nikiforov.[2] Numerov[2] has outlined a similar method, differing only in the employment of horizontal rod elements, parallel to the axis of b, instead of vertical elements parallel to the c axis.

Each of these methods is an extension of one originally used by Hutton,[3] in 1778, for computing plumb-line deflection effects due to local topography, and subsequently by Hayford[4] in 1909 for similar calculations.

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[2] Nikiforov and Numerov: *Op. cit.*

[3] C. Hutton: An Account of Calculations Made from the Survey and Measures Taken at Schiehallien. *Phil. Trans.* (1778) **48**, 689.

[4] J. F. Hayford: The Figure of the Earth and Isostasy from Measurements in the United States, 20 *et seq.* U. S. Coast and Geodetic Survey (1920).

This type of graphical computation can, of course, be applied to features of regular outline, and to infinitely long cylindrical features of irregular outline. In many cases, the graphical method of computation is more expeditious even for regular features than the analytical process.

Change of Axes

In connection with the computation of gravity effects, which have usually to be finally referred to specific, horizontal axes of reference, notably the magnetic north and east axes, it frequently arises that, for the purpose of computation, it may be more convenient to use other rectangular axes, particularly in the horizontal plane. In this case the magnitudes referred to the subsidiary axes can readily be transformed into the corresponding magnitudes referred to the standard axes by means of the following well known formulas of transformation.

If the standard system of reference axes $(0, xyz)$ is connected with the subsidiary system $(0, x'y'z')$ by the relations (direction cosines) given in the table

0	x	y	z
x'	l_1	l_2	l_3
y'	m_1	m_2	m_3
z'	n_1	n_2	n_3

whence

$$x = l_1x' + m_1y' + n_1z' \qquad x' = l_1x + l_2y + l_3z$$
$$y = l_2x' + m_2y' + n_2z' \qquad y' = m_1x + m_2y + m_3z$$
$$z = l_3x' + m_3y' + n_3z' \qquad z' = n_1x + n_2y + n_3z$$

we have

$$\frac{\partial U}{\partial x} = l_1\frac{\partial U}{\partial x'} + m_1\frac{\partial U}{\partial y'} + n_1\frac{\partial U}{\partial z'}$$
$$\frac{\partial U}{\partial y} = l_2\frac{\partial U}{\partial x'} + m_2\frac{\partial U}{\partial y'} + n_2\frac{\partial U}{\partial z'} \qquad \text{etc.}$$
$$\frac{\partial U}{\partial z} = l_3\frac{\partial U}{\partial x'} + m_3\frac{\partial U}{\partial y'} + n_3\frac{\partial U}{\partial z'}$$

and

$$\frac{\partial^2 U}{\partial x^2} = \left(l_1\frac{\partial}{\partial x'} + m_1\frac{\partial}{\partial y'} + n_1\frac{\partial}{\partial z'}\right)^2 U$$
$$\frac{\partial^2 U}{\partial x \partial y} = \left(l_1\frac{\partial}{\partial x'} + m_1\frac{\partial}{\partial y'} + n_1\frac{\partial}{\partial z'}\right)\left(l_2\frac{\partial}{\partial x'} + m_2\frac{\partial}{\partial y'} + n_2\frac{\partial}{\partial z'}\right)U, \text{ etc.}$$

In particular, if the z' axis coincides with the z axis, and the x' axis makes an angle θ with the x axis, we have

$$\begin{array}{lll} l_1 = \cos\theta, & l_2 = \sin\theta, & l_3 = 0 \\ m_1 = -\sin\theta, & m_2 = \cos\theta, & m_3 = 0 \\ n_1 = 0, & n_2 = 0, & n_3 = 1 \end{array}$$

whence we get the formulas

$$\left.\begin{aligned} \frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2} &= \left(\frac{\partial^2 U}{\partial y'^2} - \frac{\partial^2 U}{\partial x'^2}\right)\cos 2\theta + 2\,\frac{\partial^2 U}{\partial x'\partial y'}\sin 2\theta \\ 2\,\frac{\partial^2 U}{\partial x\partial y} &= -\left(\frac{\partial^2 U}{\partial y'^2} - \frac{\partial^2 U}{\partial x'^2}\right)\sin 2\theta + 2\,\frac{\partial^2 U}{\partial x'\partial y'}\cos 2\theta \\ \frac{\partial^2 U}{\partial x\partial z} &= \frac{\partial^2 U}{\partial x'\partial z'}\cos\theta - \frac{\partial^2 U}{\partial y'\partial z'}\sin\theta \\ \frac{\partial^2 U}{\partial y\partial z} &= \frac{\partial^2 U}{\partial x'\partial z'}\sin\theta + \frac{\partial^2 U}{\partial y'\partial z'}\cos\theta. \end{aligned}\right\} \quad (14)$$

Conclusion

From the foregoing review of the methods available at present for computing the Eötvös gravitational magnitudes arising from various types of mass distribution, it will be evident that this branch of gravity surveying has easily kept pace with the demands hitherto made upon it. It can also be assumed that future requirements will be met satisfactorily by the existing formulas, although doubtless these will be amplified from time to time to deal with special problems in the most economical manner.

It is, however, in the field of "interpretation" that there is most scope for development in computational technique. Even when the known disturbing effects have been eliminated, there remains the vital problem of differentiating the various features which usually combine to give the residual effects. Some of these features may, from an economic standpoint, be utterly valueless, but their effects may obscure those due to the valuable deposits, and obstruct the task of locating the latter.

Progress in interpretation technique is only to be expected by a systematic analysis of the effects due to various types of feature, gradually extended from the most simple to the more complex, from the single feature to the combination of features. Attention is being focussed upon this subject to an ever-increasing extent, and it is certain that the near future will witness a steady progress in this branch of analysis. It cannot be doubted that at present gravity surveying is in its infancy, and that the anticipated progress in instrumental development and field technique will be accompanied by a parallel advance in analytical methods, whereby the field of application of gravitational prospecting for minerals will be vastly extended beyond its present narrow limits.

[For discussion, see page 559.]

Gravity Surveying in Great Britain

By H. Shaw,* London, England

(New York Meeting, February, 1928)

It is now generally recognized that the gravitational method of geophysical surveying is a valuable aid in elucidating the geological structure of the subsoil and enables the practical geologist to deduce the presence of useful deposits.

The particular suitability of the torsion balance to the location of large and important subterranean structures has given rise to a wide application of this method by many of the important oil companies in their search for future supplies, and so intense have been these efforts that little has been heard of the method in other connections, with the exception of occasional references to its application in the location of hidden deposits of metallic ores, salt and lignite, and a number of accounts of its use in the elucidation of geological structures. Its success in the location of salt domes is well known, and establishes it as one of the most important of the geophysical methods, especially for the indirect location of oil.

In the British Isles, however, no large occurrence of oil is likely to be revealed, and here the practical application of the gravitational method has been restricted almost entirely to the search for ores. This type of survey possesses well marked characteristics, and presents a gravitational problem that has necessitated the development of a special and probably unusual plan of campaign, full details of which it is hoped will shortly be published. The problems and difficulties encountered in this work are of necessity fundamentally related to those met with in the search for oil by means of the torsion balance, but they also possess several special and distinctive features which undoubtedly differentiate them from the obstacles confronting the oil prospector, and which call for special consideration and treatment.

In certain of the various oil-producing areas of the world, and particularly in the Gulf Coast region, the geology may be regarded as comparatively simple, while the oil is usually associated with a large and well defined structure such as a salt dome or anticline, which is capable of giving gravitational indications sufficiently extensive to be detected by the torsion balance over a fairly wide area of the ground surface. It is customary, therefore, during the preliminary survey of an area for oil, to

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operate with stations at reasonably wide intervals, thus covering the ground at a fair speed. If any interesting indication is obtained, the network may be closed up locally, in order to get more detailed information in the vicinity of the source of disturbance.

The interpretation and delineation of gravitational anomalies produced by extensive structural features are usually comparatively simple, unless other extraneous disturbing factors are present. For example, in hilly or even undulating regions, the method of operation has to be modified considerably, but such visible irregularities of terrain and topography produce disturbing effects which may be eliminated with little difficulty by means of a more extended survey of the surface.

Direct Location of Minerals

In comparison with the size of a salt dome or anticline, orebodies usually occur in relatively small masses, or in veins, and give rise directly to gravitational effects on the surface which are purely local, and hence necessitate a much closer investigation to enable these indications to be examined and the deposit to be delineated.

Moreover, the very complex nature of the geology and topography in many cases introduces important difficulties due to a number of minor irregularities which are superimposed upon any general structure that may exist. If these geological and topographical irregularities are known, their resulting gravitational effects can readily be determined at as many points on the surface as is necessary, but the reverse process of disentangling from the resultant effects the various components of a complicated geological formation and so elucidating the complete geological structure is by no means easy, and is a science that is acquired only after considerable experience.

In some cases, and especially in virgin areas, it may be desirable first to locate certain structures that are favorable to the occurrence of the particular mineral in prospect, if this is not already known from geological considerations. In this way it is sometimes possible to confine the detailed survey to the most promising areas, which can then be covered with a close network survey.

The metallic ore deposit may be associated with a structural feature, such as a buried river bed or a fault, which can usually be located with little difficulty, but more frequently the ore is embedded in rock, and covered with a layer of overburden, so that the gravitational anomaly at the surface is a combination of the effects of these three important constituents, together with those of other less prominent ones.

Under these conditions the torsion balance may be expected to prove of most service in the location of masses of mineral, the specific gravity of which is appreciably higher or lower than that of the enclosing rock.

In cases where the specific gravity of the ore is practically the same as that of the surrounding country rock, only very large deposits of ore are at all likely to be located.

The gravity method, therefore, will give the best results where the rock is of high mineral content, and will be practically useless in cases where commercial success depends on the location of small quantities of very valuable mineral, although it may be possible, under certain circumstances, to locate such precious minerals indirectly; *e. g.*, masses of gold quartz enclosed in heavy basic igneous rocks.

Geological Conditions

In the particular area in which gravity surveying has been intensively employed in England, the main object has been the location of deposits of iron ore, which are usually found in the form of large and irregular masses. The geology of the area shows no pronounced structural feature of a magnitude sufficient to be detected by the torsion balance, but indicates that the area is highly faulted, and irregular.

The orebodies occur at depths between 200 ft. and 500 ft. embedded in carboniferous limestone, which is overlain by glacial drift to varying depths. The surface of the limestone is very undulating, while the overburden is of a heterogeneous nature and the ground surface also is fairly broken; all these irregularities have to be carefully located, interpreted and eliminated, in order to detect the mineral deposits.

In general, the gravitational effects due to any mineral mass are relatively small in comparison with the corrections which have to be introduced for the various irregularities, so that the corrections have to be computed as accurately as circumstances will permit, in order that the results may be reliable.

Faults

From a gravitational standpoint the numerous faults in this area fall naturally into two distinct classes:

1. A fault between limestone and conglomerate, the specific gravity difference of which is quite inappreciable. This type of fault is the more frequent and occurs throughout the area, and as the iron ore locally is known always to be associated with faults of this class, they are therefore of considerable economic importance. Such faults, unfortunately, on account of the very small difference of specific gravity, do not give effects within the limits of sensitivity of the balance, so that gravitationally they are of no assistance in the indirect location of the ore. In these circumstances one can detect the existence of an orebody only by its own gravitational effect, due to its greater specific gravity than the mother rock.

2. The other type of fault occurs between limestone and sandstone rocks, which have a specific gravity difference of 0.5, and consequently

such a fault may be readily detected by the balance, and its effect computed and eliminated.

Subdrift Topography

A feature of the greatest importance gravitationally is the surface of the limestone rock, immediately below the overburden, for it is evident that this subdrift topography contributes largely towards the resultant variations of the gravitational field, which can alone be distinguished at the surface of the ground. Assuming that this rock is of greater density than the overburden, which is usually the case, it follows that a gravity "high" will occur over prominences of the rock surface, while over a depression in the rock, there will be a defect of gravity. Over the inclined portions of the rock surface, therefore, we shall get gravitational gradients, directed towards the prominences of the subdrift rock, and away from the hollows. The greater these irregularities are, and the closer they approach to the surface of the ground, the greater will be their effect on the torsion balance, and the more they will complicate the problem of interpretation.

The computation of the gravitational effect of such an irregular subterranean anomaly presents many difficulties, but a method has now been evolved for evaluating at any point both the gradient and curvature effect. This method has proved to be applicable to the most complicated and irregular structures and has been applied to cases where the correction for subterranean topography has amounted to as much as 50 E, while the average correction over the whole area covered by the survey is about 20 E. In general, the error of determination of this correction does not exceed 2 E.

Overburden

So long as the overburden extends well beyond the area under examination, is homogeneous, and of uniform thickness, it will produce no effect on the torsion balance, so that it need not be taken into consideration. In practice, however, such a uniform and homogeneous overburden is exceptional, and it usually happens that the overburden has neither uniform specific gravity nor uniform thickness throughout the area, so that undesirable gravitational anomalies are at once introduced by these circumstances.

In the particular area under consideration, the overburden is glacial drift, and varies in thickness from a few feet to about 250 ft. This glacial drift also varies irregularly in density, from place to place, and consists of soil, sand, clay, and boulders of various sizes, with different combinations of these constituents, such as boulder clay, sand with boulders, etc. The specific gravity of such an overburden must vary in parts between fairly wide limits, according to the proportion of heavy

and light constituents it contains, and it is a matter of great difficulty to determine a representative value for the density under these conditions. The relatively high or low specific gravity of any portion of the overburden, on account of its proximity to the balance, may produce disturbing effects sufficiently large to obscure, or at any rate to distort, the effect of the main structure.

With an overburden of glacial drift, however, we have found in practice that some sections of the area produce remarkably consistent results and appear to indicate little interference from the heterogeneity of the overburden material, whereas in other sections of the same area, irregularities of a most pronounced nature are encountered, and it is only by obtaining observations at a large number of closely spaced stations that one can hope to eliminate the disturbing effect of these irregularities.

Topography

Eötvös, in his early work with the torsion balance, considered it essential to operate in flat and unbroken ground, and the useful employment of the instrument in undulating or hilly regions was thought to be impracticable. Even in later years the majority of the gravitational surveys have been conducted in level or practically level localities—*e. g.*, the Coastal Plains—although during recent years operations have been extended to more irregular areas. Flat localities certainly present ideal conditions for torsion balance operations, and enable surveys to be conducted with a minimum of effort, for the amount of leveling and computation necessary under these conditions is very small, so that stations can be completed relatively quickly.

In the mineral area referred to above, however, circumstances are entirely different, the surface of the ground being fairly irregular, with an occasional anomaly of outstanding importance. It is therefore essential that corrections for terrain and topography should be carefully and laboriously computed.

It has been the general practice to compute the terrain effect from leveling observations up to a radius of 100 ft., by the usual method, but at certain stations, where the slope of the ground surface was too great, a different procedure was adopted. An emplacement was dug in the hillside, of rectangular form, and about 10 ft. square, with the floor and sides carefully planed down. The instrument was then set up without difficulty in the middle of this area, and the necessary correction made for the emplacement. This procedure was found to serve satisfactorily in very hilly ground where it would otherwise have been difficult to erect the instrument and protecting hut. The cost of preparing an emplacement of this kind, however, is somewhat high, so that the method has been used only when absolutely necessary.

The topographical correction in hilly or broken ground requires the leveling to be extended to a radius of at least 1000 ft. from the station, which naturally is a much more laborious undertaking, and requires more than ten times as long as the terrain leveling. This topographical correction, in the case of the gradient, is usually found to be of far less importance than the terrain correction, but for curvature effects it is of supreme importance and must always be taken into consideration.

In the highly irregular portions of the area, very large gradient terrain corrections were obtained, so that the same reliance could not be placed upon the results as in more suitable areas, although in general they were remarkably consistent. The topographical effects were considerably smaller than the terrain, the maximum value obtained being about 16 E.

Other Conditions

Gravity field work in the United Kingdom, when compared with that in most other countries, must be regarded as less arduous both for the observer and the instrument, while the constant close proximity to supplies and spares of all kinds simplifies the organization of such a survey considerably, and doubtless tends to improve its efficiency. None of the difficulties of transport, which are so frequent and of such importance in many other countries, are encountered, while the climate is temperate with no rapid temperature fluctuations, so that survey work can be continued all the year round, without special precautions.

Practice in the United Kingdom

In order to overcome the effects of the relatively complex geological conditions and irregular topography of this area, disturbing effects which are not usually met with in normal torsion balance work must be carefully eliminated, and methods have been devised for their computation, and correction.

The complex conditions generally encountered in mineral areas, are unsuitable to the method of procedure employed in oil fields, and experience shows that different field methods are required to obtain sufficient information as to the subterranean mass distribution to enable the geophysicist and geologist to interpret the complicated arrangement of masses that usually exists, and to locate any valuable body of ore.

Such a task is possible only when a dense network of stations can be taken so as to obtain a faithful gravitational picture of the whole area, in which the minor irregularities are revealed, superimposed upon the major structure, and so rendered amenable to mathematical analysis.

The Quantities Measured

In the usual gravity survey the torsion balance serves to measure certain gravity values from which we are able to determine the magnitude

and direction of the maximum gradient of gravity, and also the magnitude and direction of the curvature vector.

Under simple conditions of terrain, topography and geological structure, which are very favorable to a gravity survey (such as the extensive plains of the Gulf Coast and in cases where any structural feature, for instance an anticlinal fold, is so large as to be regarded as infinite in the direction of its axis), both the gradient and curvature magnitudes are measured and employed, as both may contribute usefully towards the interpretation of the gravitational anomalies.

Owing probably to the increased labor involved the curvature value has not been used so widely as the gradient, even for locating simple structures, but under suitable conditions, it is capable of supplying exactly the same valuable information concerning the nature and extent of the subterranean anomalies as can be obtained from the gradient, and so it is able, under some circumstances, to furnish a valuable confirmation of the gradient results.

The curvature is occasionally preferred by some operators to the gradient, as in some cases it furnishes the desired information in a form which may be more convenient to the particular interpretation under consideration.

In general, however, the curvature is used less often than the gradient, as it is adversely affected in a greater degree by the various topographical and subterranean irregularities which are encountered in practice, so that, except in the most simple areas, it can serve only as an auxiliary and relatively unreliable means of assisting to elucidate the mass distribution beneath the surface of the ground.

Topographical Leveling Necessary

Any subterranean anomaly which is well below the ground, and situated at no great distance horizontally from the instrument, will have a greater effect upon the gradient than on the curvature, but any inconformity more remote from the instrument, and near the surface of the ground, will affect the curvature more than the gradient. When operating on an extensive flat plain, this is of little importance, but in all other areas it becomes a controlling factor, for in order to eliminate the effect of these irregularities on the curvature, the topographical leveling at each station has to be extended very considerably.

Leveling, for the elimination of the terrain effect, has to be done in any case whether the curvature is used or not, so that no additional labor of terrain leveling is involved in obtaining the curvature terrain correction, but in order to determine the topographical correction for curvature, with the same degree of accuracy as that for the gradient, the topographical leveling should be continued to a distance at least ten times that required for the gradient. Taking into consideration the

increased difficulty of topographical leveling over that of terrain leveling, this labor will probably be increased even in a greater ratio than the range, while the time required for such extensive leveling adds considerably to the expense of the survey and ultimately becomes prohibitive.

Some of the difficulties due to a heterogeneous overburden have already been mentioned, but it should be pointed out that the curvature is even more sensitive than the gradient to the various irregularities of the overburden, and in consequence greater errors are to be expected if the instrument is operated on this type of ground. Experiments which have been conducted by Konigsberger demonstrate this fact, and indicate that such localities are entirely unsuitable for the employment of curvature.

Effect of Subdrift Topography

Still further difficulties are encountered when we come to consider the subdrift topography. The usual computation methods, based on a summation of the effects produced by masses of regular form, were found to be totally inadequate for the purpose of determining this correction, and an improved method has been devised in which the actual shape of the surface, however irregular, is taken into account, enabling the resulting correction to be obtained to whatever degree of accuracy it is possible to determine the surface undulations. Just as in the case of the surface topography, the curvature is affected to a far greater distance than the gradient is, by the irregular nature of the rock surface, so that the necessary curvature correction must be computed to include undulations up to a relatively large distance from the station.

In the case of irregularities of the ground surface, it is possible by topographical leveling to obtain the necessary information to a very close approximation, if time and expense are not to be considered, but with the subdrift topography this is impossible, and reliable information is obtainable only from bore-hole data.

From the isogams much can be learned as to the general features of the subdrift topography, and it is possible, if the survey has been a very close one, to delineate these irregularities with reasonable precision, but the computation of these inequalities is an arduous task which requires considerable care and patience. As in the case of surface topography, the curvature correction for subdrift topography has to be made over a far more extensive area than is required by the gradient, so that the labor of calculation for the subterranean irregularities is enormously increased by the use of curvature.

Surveys for mineral deposits are usually confined to relatively small areas, while the task of determining the subdrift topography even in the actual vicinity of the deposit is so complex that it is desirable to restrict the area as much as possible, and it would certainly be impracticable to

extend it very considerably, for the sole purpose of enabling the subdrift curvature correction to be determined with greater accuracy.

An examination of the various corrections to be introduced in this type of work indicates that the ratio of the disturbing effect of these irregularities to the effect produced by any orebody is much greater for curvature quantities than for gradients, and hence the reliability of the resulting gradient value is likely to exceed that of the curvature.

In order to differentiate between the useful and useless constituents of any structure, it is essential, in the course of the interpretation, to assume some form of mass distribution, after which the gravitational effects of such an arrangement are calculated for the stations at which observations have been made, and the two results compared. By a process of synthesis, it then becomes possible to build up a mass arrangement that will give gravitational values agreeing closely with the observed values. In a complex region such as we are considering, this is a long and intricate task, even when the gradients alone are being used, but in the event of curvatures being included, the labor of testing any particular trial arrangement is increased at least tenfold, and the prospect of obtaining within a reasonable time an interpretation which is only approximately correct is greatly reduced.

Relative Value of Gradient and Curvature

It is evident, therefore, from the above considerations, that the relative value of the gradient and curvature magnitudes may be summarized as follows:

(1) Under suitable conditions, the gradient and curvature values are equally useful in furnishing information as to the problems concerning the distribution of mineral deposits within the earth's crust.

(2) The effect of topographical irregularities is greater in the case of curvature, necessitating on the average about ten times as much leveling as is required for the gradient alone. The amount of terrain leveling is unaffected by the use of curvature, but the work of calculating the corrections will be doubled.

(3) The effect of a heterogeneous overburden on curvature is so great that this method is entirely unsuitable for use in these circumstances.

(4) The curvature correction for subdrift topography, even if obtainable, would necessitate an extremely laborious computation at least ten times that for gradients.

(5) The relative magnitude of the disturbing effect to the ore effect is much greater for curvatures than for gradients, so that the reliability of the resulting value is much less.

(6) The labor involved in testing trial mass distributions is increased enormously if curvature is taken into consideration, and the process becomes unwieldy.

It is concluded, therefore, that the curvature magnitude is of little practical value for the purpose of disentangling the various constituents of a complex mass distribution with which mineral deposits are so commonly associated, and on account of the increased leveling and computation work involved it is preferable, in difficult regions, to eliminate the curvature magnitude from consideration altogether, and to concentrate rather upon securing the gradient value alone at a greater number of stations, thus contributing usefully to the data available for the purpose of interpretation.

Working with gradients alone, it has been found possible to operate, and to obtain reliable and consistent results, in areas that were quite impracticable for the employment of curvature, and it is probable that this experience will prove typical of the general application of the torsion balance to the location of valuable mineral deposits.

Location of Stations

The ability to locate and delineate any mass of ore, which is enclosed and hidden by irregular conditions such as have been indicated above, depends primarily on the possibility of determining each of the numerous disturbing factors accurately, and then of interpreting the remaining values.

The terrain and topographical corrections can usually be determined with an error of only 1 E, or at the most 2 E, except in the case of glacial drift, when trouble may be encountered owing to particularly local irregularities which appear to occur very locally.

After these corrections have been made we are left with the "subterranean effect" which is the combined effect of the "subdrift topography" and the "mineral body" together with that of other extraneous subsurface irregularities, and in order to separate these three quantities, it is above all necessary to have observations at very short intervals throughout the area. A close network arrangement of this kind is essential because the ore occurs in relatively small masses which have so small a gravitational effect that only slight distortions are indicated on the "subterranean isogams," and in order to detect these small distortions, the isogams must be computed and drawn out with great care from a large number of closely spaced stations. With such a close network of stations, we have found it possible to eliminate the effect of subdrift topography, and to locate quite minor features; to locate them not only in lateral extent, but also to ascertain their mean depth and thickness. We have also been able to eliminate the effects of these bodies when necessary, and to separate the anomalies due to mineralization, to locate and delimit the latter, and, generally speaking, to carry out the main functions of the survey.

This, however, has been rendered possible only by a very close order survey, in which the average distance between stations throughout the area is under 100 ft. A greater interval is taken in reconnaissance, but over the more important regions, the interval has frequently been reduced to 50 ft. or even less. Under these circumstances, sufficient information is furnished by the gradient without having recourse to the curvature, so that no special efforts have been made to employ the curvature, although the computations have been completed for a number of stations, for other reasons.

Further Practical Details

(1) *Transport.*—The close station network insures that, with a proper organization, the distance between consecutive stations is rarely greater than 200 or 300 yd., so that the method of transport is arranged accordingly. For long interstation shifts, often used in the preliminary search for large structures, some mechanical means of transport is practically essential, but in close survey work hand transport is the rule, as it is rarely economical to employ horse or mechanical transport. The original method of carrying the instrument by hand has proved quite convenient and efficient, while the Oertling method of clamping the instrument inside its hut and transporting them together on a trolley proves suitable in country which is not swampy or too broken; its advantage is that the station can be shifted in a high wind without the danger accompanying the dismantling and re-erecting of the canvas type of hut under these conditions.

(2) *Stability.*—Only on the most favorable ground will the torsion balance preserve its level if the feet of the instrument rest directly on the ground. Experience has shown that in types of terrain—rock, sand, shingle, meadowland, and elastic ground of the nature of peat—it is usually sufficient to stand the instrument on a solid triangular wooden framework, which has been well bedded down with its top flush with the ground surface. In very difficult ground—*e. g.*, swamp—this has been ineffective, and under such conditions the practice has been to support the instrument on three wooden pegs, driven well into the ground. Varying lengths of pegs, from 1.5 to 4.5 ft., have been used, according to circumstances, thus securing the required stability and rigidity and enabling the instrumental readings to be taken without difficulty.

(3) *Specific Gravity.*—In order that the various corrections may be calculated with the desired accuracy, it is essential that the specific gravity of the various disturbing features should be known very closely. Where this is impracticable, the results must be regarded as unreliable, unless confirmed by those at neighboring stations.

The geologist, generally, is able to give only a very rough indication of these values, so that it is necessary to determine them for oneself.

Published figures usually give limiting values which are so widely separated as to be practically useless to the geophysicist. The following examples, taken from a recent publication,[1] will indicate this point:

Rock	Specific Gravity	
	Limiting Values	Mean Value
Basalt	2.7–3.3	3.0
Limestone	2.3–3.0	2.7
Sandstone	1.8–2.8	2.3

The rock, when buried at some depth in the earth, is usually nearly or quite saturated with moisture, and it is the specific gravity of the rock in this condition that is required. With some rocks this value varies considerably from the specific gravity when dry, but in other cases practically no difference can be observed.

Accuracy of Results

The reluctance of the mining engineer and geologist to adopt the torsion balance has probably been caused more by the difficulties of interpretation than by those of obtaining instrumental observations. The instrumental development has always been well ahead of the methods of interpretation, and in spite of important recent advances of the latter, the closeness with which the various corrections can be computed and applied does not equal the standard of accuracy possessed by the instrument, except in favorable areas.

It is now generally accepted that the determinations of gradient made at any station with the torsion balance can be relied upon to within about 1 E, while on ground suitable for torsion balance work, the terrain effect and topographical effect due to surface irregularities can be computed with about the same degree of accuracy.

In more broken ground, however, where the terrain and topographical corrections are large, this standard cannot always be maintained, while unknown subterranean anomalies usually introduce errors into the observed gradient.

It is evident, therefore, that although it would be a comparatively easy matter to increase the sensitivity of the torsion balance even further, such an increase is quite unnecessary and even undesirable at present, as the various corrections cannot be determined with this increased precision. Obviously, it is of little use to increase the accuracy of observation much beyond that of the corrections, and the tendency at the present time is rather to reduce the sensitivity of the instrument, in

[1] R. Ambronn: Methoden der Angewandten Geophysik, 36, 1926.

order to bring it more into line with the reliability of the corrections. For commercial work, in undulating or broken ground, an instrument that could be relied upon to measure gradients to within 1.5 to 2 units would be as accurate as the reliability of the various correction determinations demand.

Future Developments

From a consideration of the methods we have found it necessary to adopt, in order to compute and eliminate the various irregularities of the complex formation involved, it is evident that, with the customary sparse location of balance stations adopted in the location of large structures, it would have been quite impossible to elucidate any information of value from a survey of this particular area. The detailed interpretation has been rendered practicable only by a close network of stations, by means of which it has been possible to delineate even small features, which would have escaped detection in a survey of the normal type.

It may be that the underground and surface conditions of this region are unduly complex, yet experience indicates that mineral areas are seldom of the flat and unbroken type, but that they generally have irregularities in some form or other. In order to determine and eliminate these undesirable features, and so be in a position to locate any mineral body that may be present, it will be necessary to employ a close order survey, the density of which will be governed by the complexity of the conditions.

Irregular features, both surface and subterranean, are particularly obstructional to the use of curvature. They increase the labor of topographical leveling considerably and render the computation work difficult, and little useful information is obtainable from curvature under these conditions. It is highly probable, therefore, that in the gravitational location of metallic ores, in all but very flat areas, the curvature will not be employed and the gradient alone will be relied upon to supply the data required for purposes of interpretation.

Probably the greatest disadvantage of the torsion balance method lies in the slowness of its operation, and consequently the length of time required to complete any survey. In a close survey, in which a large number of stations have to be occupied within a small area, this defect will be particularly noticeable, and the expense of such a survey will be considerable. The solution of this difficulty will probably be reached by the introduction of instrumental improvements which will greatly reduce the time required for the occupation of each station.

With the standard type of instrument, the reading interval has been reduced from one hour to 40 or 45 min. by means of increased damping, Bamberg having secured the extra damping by fitting an auxiliary tube to the lower weight, and Oertling by redesigning the upper weight.

If the curvature quantity is not to be employed in general practice, it would appear uneconomical to take the instrumental observations which are required for its evaluation, and it would be desirable to observe the gradient only, if this could be done with any economy of time. Instruments on these lines have already been constructed, by which the time of observation at any station is reduced to approximately 2 hr., and there is little doubt that in the not distant future the time required to make a complete gradient determination at any station will be still further reduced to one hour or even less.

From the theoretical standpoint, also, important developments are likely to be introduced in the methods of interpretation—a science which is so far comparatively undeveloped. The methods that have hitherto been employed, although effective, are too cumbersome and laborious, and it is fairly certain that quicker and more efficient methods will be discovered by means of which the complete gravitational picture as obtained from the gradient measurements at a large number of closely placed stations, will be made to disclose the secrets of the subterranean mass distribution, and to reveal, delineate, and separate the various components of a complex arrangement, giving the position and extent of any mineral body, and thus furnishing a complete interpretation of the subterranean conditions.

A Cartographic Correction for the Eötvös Torsion Balance

By C. A. Heiland,* Golden, Colo.

(New York Meeting, February, 1928)

The Eötvös torsion balance permits the measurement of certain second derivatives of the gravity-potential, which are known as the gradients of gravity and the curvature values for an equipotential plane of gravity. These quantities are influenced on level ground by the configuration of the masses underneath. Hence, the torsion balance has gained great importance for the location of geological structures and thus indirectly for the discovery of oil and other mineral deposits. If the ground is not level, however, the measurements become more difficult, because the superficial mass-irregularities also produce more or less considerable disturbances.

It is necessary, therefore, to determine the shape of the surrounding topography by leveling as well as by consulting topographic maps and to compute its influence upon the torsion balance. The correction thus determined is called, in general, the topographic correction; it is composed of two constituents, the terrain correction and the cartographic correction. The terrain correction would be that part of the topographic correction as derived from the direct leveling of the area surrounding the station, whereas the data for the computation of the cartographic correction are obtained from a topographic map, usually beginning with a radius of 100 meters, as under most circumstances it is difficult to level along a larger concentric circle about the station.

Principle of Topographic Correction

The principle of all topographic corrections is to assume a homogeneous compositon of the masses around the station in regard to specific gravity as well as a mathematical shape of the topography. The results become the more accurate the closer this mathematical shape is to the actual configuration of the terrain, but naturally the amount of necessary computation also increases. If a standard correction with fixed distances is adopted, the middle error of this method will, therefore, increase as the irregularity of the terrain. Hence, the application of the torsion balance in mountainous areas may be successful only if the disturbances due to heavy masses underneath the surface are so large as to exceed considerably the increased middle error due to rugged topography, or,

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this not being the case, if the observer wants to undergo the burden of making very detailed computations, which will then mostly be quite out of proportion in comparison to the labor necessary for obtaining the mere instrumental records. The least possible error is about 1 to 2 Eötvös units (1 Eötvös unit equals, according to the proposal of W. Schweydar, 1.10^{-9} c. g. s.$^{-2}$).

LITERATURE

There are a number of publications dealing with the theory of topographic corrections, principal among them being those of R. v. Eötvös,[1] W. Schweydar,[2,3] B. Numerov,[4] K. Jung,[5] E. A. Ansel,[6] and H. Shaw and E. Lancaster Jones.[7] Most of these authors deal with the terrain correction and emphasize that the cartographic correction may be computed by using the same principle as proposed for their terrain correction. Eötvös, however, applied different methods for these two corrections. He used fixed distances and azimuths for the terrain correction and the contour lines of topographic maps for the cartographic correction, their azimuth and distances to be determined for the particular situation.

SEPARATION OF TERRAIN AND CARTOGRAPHIC CORRECTION

The advantages of separating terrain and cartographic correction are as follows: Close to the instrument the accuracy in the determination of the mathematical shape of the terrain has to be greater. As the topographic correction usually is figured for the effect of different sectors subdivided into ring-pieces by circles concentric about the station, close to the station a certain law has to be assumed for the variation of the elevation h, as azimuth α, and as distance ρ. These assumptions may

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[1] R. v. Eötvös: Bestimmung der Gradienten der Schwerkraft und ihrer Niveauflaechen mit Hilfe der Drehwage. Verhandlungen der XV. Allgemeinen Conferenz der Internationalen Erdmessung (1906) Seite 337–395. Berlin, 1908.

[2] W. Schweydar: Die topographische Correction bei Schweremessungen mittels einer Torsionswage. *Ztsch. f. Geophysik* (1924–25) Heft 3, Seite 81–89.

[3] W. Schweydar: Die topographische Correction bei Schweremessungen mittels einer Torsionswage, zweite Mitteilung. *Ztsch. f. Geophysik* (1927) Heft 1, Seite 17–23.

[4] B. Numerov: Graphische Methode zur Beruecksichtigung des topographischen Einflusses und des Einflusses der unterirdischen Massen auf die gravimetrischen Beobachtungen. *Ztsch. f. Geophysik* (1924–25) Heft 8, Seite 367–371.

[5] K. Jung: Diagramme zur Bestimmung der Terrainwirkung fuer Pendel und Drehwage und zur Bestimmung der Wirkung "zweidimensionaler" Massenanordnungen. *Ztsch. f. Geophysik* (1927) Heft 5, Seite 201–212.

[6] E. A. Ansel: Gravimetrische Methoden. Kapitel 53 von Gutenberg's Lehrbuch der Geophysik, Seite 501–552.

[7] H. Shaw and E. Lancaster-Jones: The Theory and Practical Employment of the Eötvös Torsion Balance. *The Mining Magazine* (April –July,1927).

be dropped as the distance becomes greater, thus simplifying the computation of the formulas.

Eötvös assumes that the average height of a topographic element bounded by two concentric circles with the radii ρ' and ρ'' and the angle $\alpha_2 - \alpha_1$ is given by a function:

$$h = c + a\alpha + b\rho\alpha + d\rho \tag{1}$$

where c, a, b and d are constants which are to be determined from the elevations h''_1, h''_2 and h'_1 and h'_2 measured in the four corners of the ring-piece as follows:

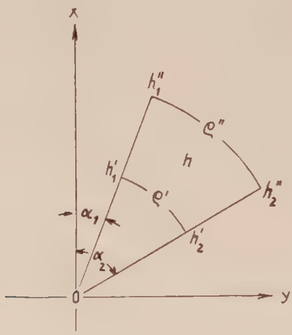

FIG. 1.

$$\left.\begin{aligned}
c &= \frac{1}{(\alpha_2 - \alpha_1)(\rho'' - \rho')}\left\{\rho''(\alpha_2 h_1' - \alpha_1 h_2') - \rho'(\alpha_2 h_1'' - \alpha_1 h_2'')\right\} \\
a &= \frac{1}{(\alpha_2 - \alpha_1)(\rho'' - \rho')}\left\{\rho''(h_2' - h_1') - \rho'(h_2'' - h_1'')\right\} \\
b &= \frac{1}{(\alpha_2 - \alpha_1)(\rho'' - \rho')}\left\{(h_2'' - h_1'') - (h_2' - h_1')\right\} \\
d &= \frac{1}{(\alpha_2 - \alpha_1)(\rho'' - \rho')}\left\{\alpha_2(h_1'' - h_1') - \alpha_1(h_2'' - h_2')\right\}
\end{aligned}\right\} \tag{2}$$

With these equations, Eötvös computes terrain formulas for use with fixed distances and azimuths. No fixed distances and azimuths, however, are used in his cartographic correction. There is also no longer any law assumed for the variation of the height in one element. One mass element is now a small piece bounded by two consecutive contour lines at a horizontal distance $d\rho$ and sides of the small angle $d\alpha$. The average height of this element is the elevation at its center, that is at the distance ρ from the instrument and the azimuth α from north. Eötvös publishes the following formulas for the cartographic correction. In these formulas, U_{xz} and U_{yz} (U = the gravity potential) are the gradients of gravity in the north and east direction, or $\frac{\partial^2 U}{\partial x \partial z}$ and $\frac{\partial^2 U}{\partial y \partial z}$,

respectively, and $2U_{xy}$ and U_Δ are the curvature values or $2\frac{\partial^2 U}{\partial x \partial y}$ and $\frac{\partial^2 U}{\partial y^2} - \frac{\partial^2 U}{\partial x^2}$, respectively.[8]

$$\left.\begin{aligned}
U_{xz} &= -3G\frac{\sigma}{2}h^2 \frac{d\rho \cos \alpha \, d\alpha}{\rho^3} \\
U_{yz} &= -3G\frac{\sigma}{2}h^2 \frac{d\rho \sin \alpha \, d\alpha}{\rho^3} \\
2U_{xy} &= +3G\sigma h \frac{d\rho \sin 2\alpha \, d\alpha}{\rho^2} \\
-U_\Delta &= +3G\sigma h \frac{d\rho \cos 2\alpha \, d\alpha}{\rho^2}
\end{aligned}\right\} \quad (3)$$

G is the gravitational constant, or 66.3 E, and σ the density.[9]

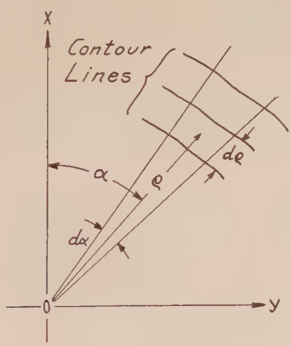

Fig. 2.

Schweydar's First Method

In Schweydar's first method, no differentiation is made between terrain and cartographic correction. The variation of height as azimuth and distance is more rigorous than in Eötvös' formula, thus taking care of more rugged topography. The variation of elevation as azimuth or the function F (α, ρ = constant) is expressed by a Fourier series of the form:

$$F_n(\alpha) = a_n + b_n \sin \alpha + c_n \cos \alpha + d_n \sin 2\alpha + e_n \cos 2\alpha + \ . \ . \ .$$

The variation of the elevation h between two consecutive radii ρ_n and ρ_{n+1} upon which the Fourier series F_n and F_{n+1} represent the variation of h

[8] There is a misprint in Eötvös' publication. The sign before $3G\sigma h$ in the third equation of group 3 is plus instead of minus as published there.

[9] These formulas result directly from the equations as given later on (see p. 551) if ζ is written o in comparison with h and ρ.

as azimuth is assumed to be linear. This variation is expressed by the proportion

$$\frac{F_{n+1} - F_n}{\rho_{n+1} - \rho_n} = \frac{h_\rho - F_n}{\rho - \rho_n},$$

which may readily be obtained from Fig. 3. Hence,

$$h_\rho = F_n - \rho_n \frac{F_{n+1} - F_n}{\rho_{n+1} - \rho_n} + \rho \frac{F_{n+1} - F_n}{\rho_{n+1} - \rho_n} \tag{4}$$

The coefficients of the Fourier series are to be determined from the levelings, namely, the coefficient c for the north gradient, b for the east

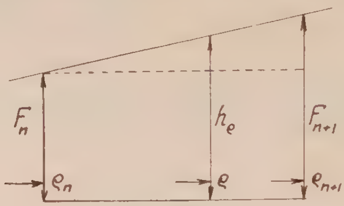

FIG. 3.

gradient, and e and d for the curvature values. Thus, except for the first circle, the north gradient for one circle n becomes, for instance,

$$\left.\begin{aligned}(U_{xz})_n = 3\sigma\pi\zeta G c_n \Bigg[& \frac{\rho_{n-1}}{3\zeta^2(\rho_n - \rho_{n-1})}\left(\frac{\rho_{n-1}^3}{\sqrt{(\rho_{n-1}^2 + \zeta^2)^3}} - \frac{\rho_n^3}{\sqrt{(\rho_n^2 + \zeta^2)^3}}\right) + \\ & + \frac{1}{\rho_n - \rho_{n-1}}\left(\frac{\rho_{n-1}^2 + \frac{2}{3}\zeta^2}{\sqrt{(\rho_{n-1}^2 + \zeta^2)^3}} - \frac{\rho_n^2 + \frac{2}{3}\zeta^2}{\sqrt{(\rho_n^2 + \zeta^2)^3}}\right) + \\ & + \frac{\rho_{n+1}}{3\zeta^2(\rho_{n+1} - \rho_n)}\left(\frac{\rho_{n+1}^3}{\sqrt{(\rho_{n+1}^2 + \zeta^2)^3}} - \frac{\rho_n^3}{\sqrt{(\rho_n^2 + \zeta^2)^3}}\right) + \\ & + \frac{1}{\rho_{n+1} - \rho_n}\left(\frac{\rho_{n+1}^2 + \frac{2}{3}\zeta^2}{\sqrt{(\rho_{n+1}^2 + \zeta^2)^3}} - \frac{\rho_n^2 + \frac{2}{3}\zeta^2}{\sqrt{(\rho_n^2 + \zeta^2)^3}}\right)\Bigg]\end{aligned}\right\} \tag{5}$$

or, the term in brackets being defined by constants:

$$(U_{xz})_n = 3\sigma\pi\zeta G c_n \cdot C_n,$$

where ζ = the distance of the center of gravity of the torsion balance from the ground.

Replacing c_n by b_n, gives the same general formula for $(U_{yz})_n$.

The curvature values may be similarly computed by using the equations given later on and Schweydar's formulas 14 and 15.[10]

Schweydar's first method does not introduce for the gradients a term containing the difference between the elevation and the height of the instrument. The gradient of gravity becomes minus instead of plus if the topographic masses are higher than the center of gravity of the

[10] W. Schweydar: *Op. cit.*

instrument. Such a case could be treated by computing a quadratic term which is to be added to the formulas of the above shape (Equation 5). The coefficients of this quadratic term, obviously, have a negative sign. The computation is very cumbersome.

Schweydar's Second Method

Schweydar introduced in his second method a term $h - \zeta$ so that this second method would be applicable for cases where the masses are underneath as well as above the instrument. The variation of $(h - \zeta)^2$ on one circle is again represented by the Fourier series, the constants B and C of which must be obtained from the levelings. The variation of $(h - \zeta)^2$ in one azimuth from circle to circle is not assumed to be linear as in the first method, but

$$(h - \zeta)^2 = M + N\rho + P\rho^2.$$

Otherwise, the computation is similar to that in the first method. The calculation of the curvature values is the same.

Graphical methods for the topographic correction as a whole have been suggested by B. Numerov[11] and computed by K. Jung.[12] I do not deal with them here but will refer to them in a later paper. So far this paper has been introductory in nature in order to familiarize the reader with the principles applied.

It was pointed out at the beginning that it is advisable to separate the terrain and the cartographic correction, as for the latter much simpler or no assumptions for the variation of the elevation may be made. This, in turn, facilitates the computation of the numerical factors of such corrections so that the observer may readily compute such a correction for any distance he may deem suitable under the particular circumstances.[13]

The Author's Method of Cartographic Correction

The cartographic correction as suggested by Eötvös, although applicable to any arbitrary distances, is sometimes very cumbersome in application because of the immense number of mass elements to be computed when using contour lines. I suggest therefore the following method, which uses instead of contour-line sections for the cartographic correction, ring-pieces such as are employed in terrain correction. The assumptions about the variation of elevations are so simple as to permit the easy computation of this correction for any suitable radii. In making these simple assumptions, emphasis was mainly placed upon the requirement that the gradients should not be influenced by any such simplifications, whereas the effect on curvature values need not be quite as accurate.[14]

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[11] B. Numerov: *Op. cit.*

[12] K. Jung: *Op. cit.*

[13] E. A. Ansel: *Op. cit.*

[14] Merely mathematically speaking, the reverse is the case $\left(U_{xz} \sim \frac{1}{\rho^3}, U \sim \frac{1}{\rho^2}\right)$.

Experience has shown that in many instances the curvature values show quantitatively less regular conformity with subterranean disturbances than the gradients do, whence in interpretation the gradients are generally given preference.

The cartographic method described here has been applied for almost two years by several oil companies in this country who use the torsion balance for their exploration work.

Effect of Surrounding Topography

The effect of the topography which surrounds the instrument may be conceived as composed of infinitesimal effects, which are due to mass elements dm. The potential U at a point $P(x, y, z)$ of such an element at the distance r from the instrument is

$$U = \frac{dmG}{r}\cdot \quad \text{Hence, the derivatives}$$

$$\left.\begin{aligned} U_{xz} &= 3\,Gdm\,\frac{xz}{r^5} \\ U_{yz} &= 3Gdm\,\frac{yz}{r^5} \\ U_{\Delta} &= 3Gdm\,\frac{y^2 - x^2}{r^5} \\ U_{xy} &= 3Gdm\,\frac{xy}{r^5} \end{aligned}\right\} \tag{6}$$

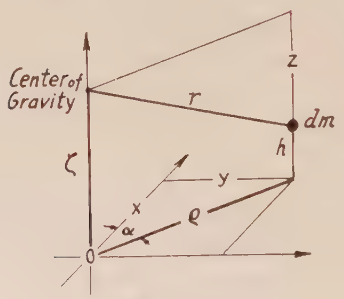

FIG. 4.

Substituting for $z : \zeta - h$, for $r : \sqrt{\rho^2 + (\zeta - h)^2}$, for $x : \rho \cos \alpha$ and $y : \rho \sin \alpha$,

$$\left.\begin{aligned} U_{xz} &= 3Gdm\,\frac{\rho \cos \alpha\,(\zeta - h)}{\sqrt{(\rho^2 + (\zeta - h)^2)^5}} \\ U_{yz} &= 3Gdm\,\frac{\rho \sin \alpha\,(\zeta - h)}{\sqrt{(\rho^2 + (\zeta - h)^2)^5}} \\ -U_{\Delta} &= 3Gdm\,\frac{\rho^2 \cos 2\alpha}{\sqrt{(\rho^2 + (\zeta - h)^2)^5}} \\ 2U_{xy} &= 3Gdm\,\frac{\rho^2 \sin 2\alpha}{\sqrt{(\rho^2 + (\zeta - h)^2)^5}} \end{aligned}\right\} \tag{7}$$

The mass elements in which the surroundings of the station are subdivided are usually zones bounded by two concentric circles with the radii; ρ_n and ρ_{n+1} and the two angles α_2 and α_1, so that

$$\int dm = \sigma \int_{\alpha_1}^{\alpha_2} \int_{\rho_n}^{\rho_{n+1}} \int_0^h \rho d\alpha d\rho dh$$

whence the total effect of the surrounding topography,

$$\left.\begin{aligned} U_{xz} &= 3G\sigma \int_0^\rho \int_0^{2\pi} \int_0^h \frac{\rho^2 \cos \alpha d\alpha d\rho dh \, (\zeta - h)}{\sqrt{(\rho^2 + (\zeta - h)^2)^5}} \\ U_{yz} &= 3G\sigma \int_0^\rho \int_0^{2\pi} \int_0^h \frac{\rho^2 \sin \alpha d\alpha d\rho dh \, (\zeta - h)}{\sqrt{(\rho^2 + (\zeta - h)^2)^5}} \\ -U_\Delta &= 3G\sigma \int_0^\rho \int_0^{2\pi} \int_0^h \frac{\rho^3 \cos 2\alpha d\alpha d\rho dh}{\sqrt{(\rho^2 + (\zeta - h)^2)^5}} \\ 2U_{xy} &= 3G\sigma \int_0^\rho \int_0^{2\pi} \int_0^h \frac{\rho^3 \sin 2\alpha d\alpha d\rho dh}{\sqrt{(\rho^2 + (\zeta - h)^2)^5}} \end{aligned}\right\} \quad (8)$$

Writing $[\rho^2 + (\zeta - h)^2]^{-5/2} = \left[\left(\rho^2 + \zeta^2\right)\left(1 + \frac{h^2 - 2\zeta h}{\rho^2 + \zeta^2}\right)\right]^{-\frac{5}{2}}$ and provided $\frac{h^2 - 2\zeta h}{\rho^2 + \zeta^2}$ is small, we may expand the above expression in series with rising powers of the latter term. Carrying out the integrations $\int_0^h dh(\zeta - h)$ and $\int_0^h dh$ we obtain for U_{xz}, for instance,

$$U_{xz} = 3G\frac{\sigma}{2} \int_0^\rho \int_0^{2\pi} \frac{\rho^2 \cos \alpha d\alpha d\rho}{\sqrt{(\rho^2 + \zeta^2)^5}} \left(2\zeta h - h^2 + \frac{5}{2}\frac{(h^2 - 2\zeta h)^2}{\rho^2 + \zeta^2} - \frac{35}{8}\frac{(h^2 - 2\zeta h)^3}{(\rho^2 + \zeta^2)^2} \ldots\right)$$

and, similarly, for the other derivatives.

Taking into consideration only the first two terms of the series at the gradients and the first one at the curvature values,

$$\left.\begin{aligned}
U_{xz} &= -3G\frac{\sigma}{2}\int_0^{\rho}\int_0^{2\pi}\frac{\rho^2\cos\alpha d\alpha d\rho}{\sqrt{(\rho^2+\zeta^2)^5}}(h^2-2\zeta h)\\
U_{yz} &= -3G\frac{\sigma}{2}\int_0^{\rho}\int_0^{2\pi}\frac{\rho^2\sin\alpha d\alpha d\rho}{\sqrt{(\rho^2+\zeta^2)^5}}(h^2-2\zeta h)\\
-U_{\Delta} &= +6G\frac{\sigma}{2}\int_0^{\rho}\int_0^{2\pi}\frac{\rho^3\cos 2\alpha d\alpha d\rho}{\sqrt{(\rho^2+\zeta^2)^5}}\cdot h\\
2U_{xy} &= +6G\frac{\sigma}{2}\int_0^{\rho}\int_0^{2\pi}\frac{\rho^3\sin 2\alpha d\alpha d\rho}{\sqrt{(\rho^2+\zeta^2)^5}}\cdot h
\end{aligned}\right\} \tag{9}$$

The integration is first now to be carried out along one concentric circle over the entire circumference. Bearing in mind, first, that,

$$\int_0^{2\pi}\zeta^2\cos\alpha d\alpha = \int_0^{2\pi}\zeta^2\sin\alpha d\alpha = \int_0^{2\pi}\zeta\cos 2\alpha d\alpha = \int_0^{2\pi}\zeta\sin 2\alpha d\alpha = 0,$$ we

may substitute in (9) instead of

$$h^2 - 2\zeta h : h^2 - 2\zeta h + \zeta^2 = H^2, \text{ and instead of}$$
$$h : h - \zeta = H.$$

We further substitute in these equations certain factors s, t, u, v, which designate the integration from o to 2π as follows, if the elevations are observed in m equidistant points on one radius ρ_n:

$$\left.\begin{aligned}
s_n &= \int_0^{2\pi}\cos\alpha d\alpha H_n^2 = \frac{2\pi}{m}\sum_m H_n^2\cos\alpha\\
t_n &= \int_0^{2\pi}\sin\alpha d\alpha H_n^2 = \frac{2\pi}{m}\sum_m H_n\sin\alpha\\
u_n &= \int_0^{2\pi}\cos 2\alpha d\alpha H_n = \frac{2\pi}{m}\sum_m H_n^2\cos 2\alpha\\
v_n &= \int_0^{2\pi}\sin 2\alpha d\alpha H_n = \frac{2\pi}{m}\sum_m H_n\sin 2\alpha
\end{aligned}\right\} \tag{10}$$

Whence, for $m = 16$,

$$
\begin{aligned}
s_n &= 0{,}393[H_1^2 - H_9^2 + 0{,}924(H_2^2 - H_8^2 - H_{10}^2 + H_{16}^2) + \\
&\qquad 0{,}707(H_3^2 - H_7^2 - H_{11}^2 + H_{15}^2) + 0{,}383(H_4^2 - H_6^2 - H_{12}^2 + H_{14}^2)] \\
t_n &= 0{,}393[H_5^2 - H_{13}^2 + 0{,}924(H_4^2 + H_6^2 - H_{12}^2 - H_{14}^2) + \\
&\qquad 0{,}707(H_3^2 + H_7^2 - H_{11}^2 - H_{15}^2) + 0{,}383(H_2^2 + H_8^2 - H_{10}^2 - H_{16}^2)] \\
u_n &= 0{,}393[H_1 - H_5 + H_9 - H_{13} + \\
&\qquad 0{,}707(H_2 - H_4 - H_6 + H_8 + H_{10} - H_{12} - H_{14} + H_{16})] \\
v_n &= 0{,}393[H_3 - H_7 + H_{11} - H_{15} + \\
&\qquad 0{,}707(H_2 + H_4 - H_6 - H_8 + H_{10} + H_{12} - H_{14} - H_{16})]
\end{aligned}
$$

which, for $m = 32$, become

$$
\begin{aligned}
s_n = 0{,}196[H_1^2 - H_{17}^2 &+ 0{,}981(H_2^2 - H_{16}^2 - H_{18}^2 + H_{32}^2) \\
&+ 0{,}924(H_3^2 - H_{15}^2 - H_{19}^2 + H_{31}^2) \\
&+ 0{,}831(H_4^2 - H_{14}^2 - H_{20}^2 + H_{30}^2) \\
&+ 0{,}707(H_5^2 - H_{13}^2 - H_{21}^2 + H_{29}^2) \\
&+ 0{,}555(H_6^2 - H_{12}^2 - H_{22}^2 + H_{28}^2) \\
&+ 0{,}383(H_7^2 - H_{11}^2 - H_{23}^2 + H_{27}^2) \\
&+ 0{,}195(H_8^2 - H_{10}^2 - H_{24}^2 + H_{26}^2)] \\
t_n = 0{,}196[H_9^2 - H_{25}^2 &+ 0{,}981(H_8^2 + H_{10}^2 - H_{25}^2 - H_{26}^2) \\
&+ 0{,}924(H_7^2 + H_{11}^2 - H_{23}^2 - H_{27}^2) \\
&+ 0{,}831(H_6^2 + H_{12}^2 - H_{22}^2 - H_{28}^2) \\
&+ 0{,}707(H_5^2 + H_{13}^2 - H_{21}^2 - H_{29}^2) \\
&+ 0{,}555(H_4^2 + H_{14}^2 - H_{20}^2 - H_{30}^2) \\
&+ 0{,}383(H_3^2 + H_{15}^2 - H_{19}^2 - H_{31}^2) \\
&+ 0{,}195(H_2^2 + H_{16}^2 - H_{18}^2 - H_{32}^2)]
\end{aligned}
$$

$$
\begin{aligned}
u_n = 0{,}196[H_1 &- H_9 + H_{17} - H_{25} \\
&+ 0{,}924(H_2 - H_8 - H_{10} + H_{16} + H_{18} - H_{24} - H_{26} + H_{32}) \\
&+ 0{,}707(H_3 - H_7 - H_{11} + H_{15} + H_{19} - H_{23} - H_{27} + H_{31}) \\
&+ 0{,}383(H_4 - H_6 - H_{12} + H_{14} + H_{20} - H_{22} - H_{28} + H_{30})] \\
v_n = 0{,}196[H_5 &- H_{13} + H_{21} - H_{29} \\
&+ 0{,}924(H_4 + H_6 - H_{12} - H_{14} + H_{20} + H_{22} - H_{28} - H_{30}) \\
&+ 0{,}707(H_3 + H_7 - H_{11} - H_{15} + H_{19} + H_{23} - H_{27} - H_{31}) \\
&+ 0{,}383(H_2 + H_8 - H_{10} - H_{16} + H_{18} + H_{24} - H_{26} - H_{32})]
\end{aligned}
$$

With these formulas, the amount of necessary computation is exactly the same as if Fourier series were used to represent the variations of h along one radius.

Simplification of Computation

The last integration over the terrain in radial direction may be considerably simplified by introducing certain assumptions on the behavior of the elevation h in the mass-elements.

1. The radial dimension of the element is such in comparison to ρ as to produce an effect upon the instrument which may be conceived as being concentrated at the center of gravity.

2. The vertical axis through the geometrical center of the element contains the center of gravity, and the elevation measured on this center is equivalent to the average height h of this element.

For this reason, the boundaries of the mass elements are so chosen as to be radially and laterally symmetrical to the points (α, ρ) where h is measured. Then the last integral may readily be approximated by a summation of the concentrical rings, so that Equation 9 becomes

$$\left.\begin{aligned}
U_{xz} &= -\frac{\sigma}{2}\left[\frac{3G\rho_n^2 d\rho_n}{\sqrt{(\rho_n^2+\zeta^2)^5}}s_n + \frac{3G\rho_{n+1}^2\, d\rho_{n+1}}{\sqrt{(\rho_{n+1}^2+\zeta^2)^5}}s_{n+1} + \cdots\right]\\
U_{yz} &= -\frac{\sigma}{2}\left[\frac{3G\rho_n^2\, d\rho_n}{\sqrt{(\rho_n^2+\zeta^2)^5}}t_n + \frac{3G\rho_{n+1}^2 d\rho_{n+1}}{\sqrt{(\rho_{n+1}^2+\zeta^2)^5}}t_{n+1} + \cdots\right]\\
U_\Delta &= -\frac{\sigma}{2}\left[\frac{6G\rho_n^3\, d\rho_n}{\sqrt{(\rho_n^2+\zeta^2)^5}}u_n + \frac{6G\rho_{n+1}^3\, d\rho_{n+1}}{\sqrt{(\rho_{n+1}^2+\zeta^2)^5}}u_{n+1} + \cdots\right]\\
2U_{xy} &= +\frac{\sigma}{2}\left[\frac{6G\rho_n^3 d\rho_n}{\sqrt{(\rho_n^2+\zeta^2)^5}}v_n + \frac{6G\rho_{n+1}^3 d\rho_{n+1}}{\sqrt{(\rho_{n+1}^2+\zeta^2)^5}}v_{n+1} + \cdots\right]
\end{aligned}\right\} \quad (11)$$

Designating

$$\frac{3G\rho_n^2 d\rho_n}{\sqrt{(\rho_n^2+\zeta^2)^5}} \text{ by } A_n, \text{ and } \frac{6G\rho_n^3 d\rho_n}{\sqrt{(\rho_n^2+\zeta^2)^5}} \text{ by } B_n.$$

where $B_n = 2\rho_n A_n$, we have

$$\left.\begin{aligned}
U_{xz} &= -\frac{\sigma}{2}(A_{n-1}s_{n-1} + A_n s_n + A_{n+1}s_{n+1} + \cdots)\\
U_{yz} &= -\frac{\sigma}{2}(A_{n-1}t_{n-1} + A_n t_n + A_{n+1}t_{n+1} + \cdots)\\
U_\Delta &= -\frac{\sigma}{2}(B_{n-1}u_{n-1} + B_n u_n + B_{n+1}u_{n+1} + \cdots)\\
2U_{xy} &= +\frac{\sigma}{2}(B_{n-1}v_{n-1} + B_n v_n + B_{n+1}v_{n+1} + \cdots)
\end{aligned}\right\} \quad (12)$$

It will be readily seen how little work is involved in the computation of the coefficients, inasmuch as set B may readily be derived from A. Hence, this method is very flexible, and for any type of topography suitable distances may be selected and the correction be figured.

There are a few advantageous arrangements of the computation, which may be applied in practice to make this method still more convenient. It is first advisable to let ρ increase in a certain progression. In the example referred to below, a geometrical series has been assumed such as ρ, ρq, ρq^2, ρq^3 . . . Now $d\rho$, obviously, should follow the same progression, at the same time obeying, for the reason of symmetry, the relation $\rho_{n+1} - \rho_n = \frac{1}{2}d\rho_{n+1} + \frac{1}{2}d\rho_n$. Furthermore, it is very advantageous to let the units in which the elevations may be expressed increase as the distance increases. In the example given below the units are meters for the first six radii, 10 meters for the second six, 100 meters for the third five radii, and 1000 meters for the remainder. This arrangement cuts down considerably on the amount of figures required and suggests automatically the moment when ζ may be dropped in comparison to h.

The following arrangement of the computation sheets proved to be very valuable in avoiding errors: There are four sheets provided, one for each U_{xz}, U_{yz}, U_Δ and $2U_{xy}$. First, the U_Δ-sheet is placed upon the $2U_{xy}$-sheet with carbon paper in between. The values of $(h - \zeta)$ are then entered in the units as indicated on the left margin, uneven azimuths on the left, even azimuths on the right side. The sheets are so arranged that those azimuths which are not needed for U_Δ are crossed out on this sheet, while there is no cross in their column on the $2U_{xy}$-sheet, and vice versa. The remainder of the sheet is self-explanatory. After this, the U_{xz}-sheet is placed upon the U_{yz}-sheet, again carbon paper in between. The values of the $(h - \zeta)$ are taken from the U_Δ-sheet, their squares are formed by means of "tables of squares of figures from 1–100," and the values of H^2 thus obtained are entered in their respective columns. The arrangement of the sheets is again so made as to cross out all values which are not needed for either U_{xz} or U_{yz}.

As an example, a cartographic correction has been computed for the following distances:

1	2	3	4	5	6
100 meters	150	225	340	500	750
7	8	9	10	11	12
1,150	1,700	2,500	3,800	5,800	8,600
13	14	15	16	17	18
13,000	20,000	30,000	45,000	67,000	100,000
19					
150,000					

Computations for Bamberg and Suess Type Balances

The computations have been made for both the Bamberg and the Suess types of the Eötvös torsion balance. The formulas are as follows:

1. For the Bamberg balance (ζ = 90 centimeters):

$$
\left.
\begin{aligned}
U_{xz} = & -\frac{\sigma}{2}E\cdot 10^{-4}[\{79{,}030\ s_1 + 35{,}510\ s_2 + 15{,}670\ s_3 + 7{,}098 \\
& \qquad s_4 + 2{,}860\ s_5 + 1{,}510\ s_6\}\text{meters} \\
& + \{62{,}740\ s_7 + 25{,}111\ s_8 + 12{,}472\ s_9 + 5{,}873\ s_{10} \\
& \qquad + 2{,}426\ s_{11} + 1{,}007\ s_{12}\}10\ \text{meters} \\
& + \{50{,}513\ s_{13} + 20{,}935\ s_{14} + 8{,}531\ s_{15} + 4{,}020\ s_{16} \\
& \qquad + 1{,}692\ s_{17}\}100\ \text{meters} \\
& + \{80{,}397\ s_{18} + 22{,}155\ s_{19}\}1000\ \text{meter-units}] \\
U_{yz} = & -\frac{\sigma}{2}E\cdot 10^{-4}[\{79{,}030\ t_1 + 35{,}510\ t_2 + \ .\ .\ .\ \text{etc.}\}\ .\ .\ .\ ] \\
U_{\Delta} = & -\frac{\sigma}{2}E\cdot 10^{-1}[\{15{,}806\ u_1 + 10{,}653\ u_2 + 7{,}052\ u_3 + 4{,}827\ u_4 \\
& \qquad + 2{,}860\ u_5 + 2{,}265\ u_6\}\text{meters} \\
& + \{14{,}430\ u_7 + 8{,}538\ u_8 + 6{,}236\ u_9 + 4{,}464\ u_{10} \\
& \qquad + 2{,}814\ u_{11} + 1{,}732\ u_{12}\}10\ \text{meters} \\
& + \{13{,}133\ u_{13} + 8{,}374\ u_{14} + 5{,}118\ u_{15} + 3{,}618\ u_{16} \\
& \qquad + 2{,}267\ u_{17}\}\ 100\ \text{meters} \\
& + \{16{,}079\ u_{18} + 6{,}647\ u_{19}\}1000\ \text{meters}] \\
2U_{xy} = & \ \frac{\sigma}{2}E\cdot 10^{-1}[\{15{,}806\ v_1 + 10{,}653\ v_2 + \ .\ .\ .\ \text{etc.}\}\ .\ .\ .\ ]
\end{aligned}
\right\} \quad 13)
$$

2. For the Suess balance: (ζ = 100 centimeters)

$$
U_{xz} = -\frac{\sigma}{2}E\cdot 10^{-4}[\{79{,}008\,s_1 + 35{,}506\,s_2 + 15{,}669\,s_3 + 7{,}098\,s_4 + 2{,}860\ s_5 + \ .\ .\ .\ \text{etc.}\}\ .\ .\ .\ ]
$$

(The other coefficients are the same as in Equation 13).

$$
\left.
\begin{aligned}
U_{yz} = & -\frac{\sigma}{2}E\cdot 10^{-4}[\{79{,}008\ t_1 + 35{,}506\ t_2 + \ .\ .\ .\ \text{etc.}\ \}\ .\ .\ .\ ] \\
U_{\Delta} = & -\frac{\sigma}{2}E\cdot 10^{-4}[\{15{,}802\ u_1 + 10{,}652\ u_2 + 7{,}051\ u_3 + 4{,}827\ u_4 \\
& \qquad + 2{,}860\ u_5 + \ .\ .\ .\ \text{etc.}\}\ .\ .\ .\ ] \\
2U_{xy} = & \ \frac{\sigma}{2}E\cdot 10^{-4}[\{15{,}802\ v_1 + 10{,}652\ v_2 + \ .\ .\ .\ \text{etc.}\}\ .\ .\ .\ ]
\end{aligned}
\right\} \quad 14)
$$

The computation of the cartographic correction for a torsion balance station is appended to demonstrate in detail the procedure. (Tables 1, 2, 3 and 4.) The results obtained by the author's method have been checked as far as is at all possible by a computation according to Schweydar's method. The results obtained by these methods are not exactly comparable, because the provided radii are different in both; consequently, the elevations on which the computations are based will also be different. The results which are best comparable are those for the 100 and 150-meter radii, because the sequence of the radii is 70,

100, 150 and 250 meters in Schweydar's method and 67, 100, 150 and 225 meters according to the assumptions of the author. For U_Δ and U_{xz}, for example, the results are as follows:

	U_Δ		U_{xz}	
	Schweydar	Heiland	Schweydar	Heiland
100 m.................	+2,66	+2,79	+0,64	+0,10
150 m.................	+6,29	+3,14	+0,17	+0,12

Taking into account the average middle error of the entire torsion-balance method, the above agreement of both corrections may be considered as perfectly satisfactory.

TABLE 1

Carthographic Correction of U_{xz}

Number	Distance	Unit	$(h-\zeta)^2$ in azimuths +3	−7	−11	+15	Sum	×0,707−I	$(h-\zeta)^2$ in azimuths +2	−8	−10	+16	Sum	×0,924−II	$(h-\zeta)^2$ in azimuths +4	−6	−12	+14	Sum	×0,383−III	$(h-\zeta)^2$ in azimuths +1	−9	+5	−13	Sum=IV	Sum I+II+III+IV	Coefficient	Result $\times 10^{-9}$
1	100	m	0,8	5,8	5,8	0,8	−10,0	−7,1	0,8	10,9	5,8	0,8	−15,1	−14,0	0,8	2,3	0,8	0	−2,3	−0,9	0,1	10,9	1,4	10,9	−10,8	−32,8	79,090	−0,25822
2	150	m	0,8	16,0	5,8	0,8	−20,2	−14,3	3,2	30,3	16,0	0,8	−45,5	−42,0	0,8	5,8	0,8	0,4	−5,4	−2,1	0,4	29,2	1,4	0,8	−28,8	−87,2	35,510	−0,31000
3	225	m	4,4	30,3	16,0	5,8	−36,1	−25,5	0	72,3	37,2	0,4	−109,1	−100,8	3,2	16,0	0,8	4,8	−8,8	−3,4	0,4	72,3	3,2	5,8	−71,9	−201,6	15,670	−0,31591
4	340	m	9,0	49,0	30,3	13,7	−56,6	−40,0	0,8	102,0	72,3	12,7	−164,0	−148,2	5,8	16,0	0,8	13,7	+2,7	+1,0	4,8	134,6	36,0	4,8	−129,8	−317,0	7,098	−0,22506
5	500	m	5,8	88,4	30,3	13,7	−99,2	−70,1	0	134,6	114,5	13,7	−235,4	−217,5	16,0	16,0	0,8	28,0	+27,0	+10,4	4,8	163,8	16,0	4,8	−159,0	−436,2	2,860	−0,12467
6	750	m	16,0	134,6	30,3	37,0	−121,9	−86,2	16,0	213,2	171,6	4,8	−364,0	−336,3	16,0	30,3	0,4	27,0	+12,3	+4,7	0,8	187,7	0,8	13,7	−186,9	−604,1	1,510	−0,09131
7	1150	10m	0	1,7	0	0,2	−1,5	−1,1	0,2	4,4	1,7	0	−5,9	−5,5	0,2	0,2	0,2	0,3	+0,1	0	0,2	4,8	0	0,2	−4,6	−11,2	63,837	−0,07150
8	1700	10m	0	7,8	0,5	0	−8,3	−5,9	0,3	7,3	4,8	0,3	−11,5	−10,6	1,2	1,7	0,2	0	−0,7	−0,3	0,5	7,8	3,6	0,2	−7,3	−24,1	25,[illegible]	−0,05175
9	2500	10m	0	7,8	0	0,8	−7,0	−5,0	1,4	18,5	0	1,7	−15,4	−14,2	2,9	0,5	0	0	+2,4	+0,9	1,7	0	1,2	0,1	+1,7	−16,6	12,472	−0,02070
10	3800	10m	2,9	13,0	9,0	0,8	−18,3	−12,9	1,7	24,0	5,8	3,6	−26,5	−24,5	0,2	4,8	2,9	0,1	−7,4	−2,8	14,4	7,8	1,7	2,6	+6,6	−33,6	5,873	−0,01973
11	5800	10m	0,2	32,5	0,6	6,3	−26,6	−18,8	39,7	18,4	0,5	0,2	+20,0	−18,5	0,2	54,8	13,7	6,3	−62,0	−2,4	6,3	7,3	19,4	13,0	−1,0	−40,7	2,426	−0,00987
12	8600	10m	0,5	42,3	0,2	29,2	−13,8	−9,8	34,8	4,0	2,3	0,6	+41,1	−38,0	17,6	84,6	9,6	39,7	−36,9	−14,1	3,2	1,7	54,8	21,2	+1,5	−60,4	1,007	−0,00608
13	13000	100m	0,5	0,4	0,1	0,8	+0,8	+0,6	0,2	0,5	0,4	0,1	+0,6	−5,5	1,0	0,6	1,0	1,0	+0,4	+0,2	0,3	0,1	1,0	4,0	+0,2	−4,5	60,373	−0,00273
																										Sum		−1,52861

$U_{xz} = -\frac{[illegible]}{2} \cdot E \cdot 0{,}383 \cdot \text{Sum} = +[illegible]\, E \cdot 0{,}6$

TABLE 2

Carthographic Correction for U_{yz}

Number	Distance	Unit	$(h-\zeta)^2$ in azimuths				Sum	$\times 0{,}707 = I$	$(h-\zeta)^2$ in azimuths				Sum	$\times 0{,}383 = II$	$(h-\zeta)^2$ in azimuths				Sum	$\times 0{,}924 = III$	$(h-\zeta)^2$ in azimuths				Sum = IV	Sum I+II+III+IV	Coefficient	Results $\times 10^{-9}$
			+ 3	+ 7	− 11	− 15			+ 2	+ 8	− 10	− 16			+ 4	+ 6	− 12	− 14			+ 1	− 9	+ 5	− 13				
1	100	m	0,8	5,8	5,8	0,8	0	0	0,8	10,9	5,8	0,8	+5,1	+1,9	0,8	5,8	0,8	0,4	+2,3	+2,1	0,1	10,9	1,4	0,8	+0,6	+4,6	79,050	0,03635
2	150	m	0,8	16,0	5,8	0,8	+10,2	+7,2	3,2	30,3	16,0	0,8	+13,5	+5,2	0,8	5,8	0,8	0,4	+5,4	+5,0	0,6	10,9	1,4	0,8	+0,6	+18,0	36,590	0,06392
3	225	m	4,4	30,3	16,0	5,8	+12,9	+9,1	0	72,3	37,2	0,4	+34,7	+13,3	3,2	16,0	0,8	4,8	+13,6	+12,6	0,6	72,3	3,2	5,8	−2,6	+32,4	15,670	0,05077
4	340	m	9,0	49,0	30,3	13,7	+14,0	+9,9	0,8	102,0	72,3	13,7	+16,8	+6,4	5,3	16,0	0,8	13,7	+7,3	+6,7	4,8	134,6	32,0	4,8	+25,4	+48,8	7,098	0,03464
5	500	m	5,3	68,4	30,3	13,7	+50,2	+35,2	0	134,6	114,8	13,7	+6,4	+2,5	16,0	16,0	0,8	28,0	+3,2	+3,0	4,8	102,8	16,0	4,8	+8,2	+52,2	2,860	0,01493
6	750	m	16,0	134,6	30,3	27,0	+93,3	+66,0	16,0	213,2	171,6	4,8	+52,8	+20,2	16,0	30,3	0,4	27,0	+18,9	+17,5	0,6	183,2	0,8	13,7	−12,9	+90,8	1,510	0,01371
7	1150	10m	0	1,7	0	0,2	+1,5	+1,1	0,2	4,4	1,7	0	+2,9	+1,1	0,2	0,2	0,2	0,3	−0,1	−0,1	0,2	4,8	0	0,3	−0,3	+1,8	63,837	0,01149
8	1700	10m	0	7,8	0,5	0	+7,3	+5,2	0,3	7,3	4,8	0,3	+2,5	+1,0	1,2	1,7	0,2	0	+2,7	+2,5	0,8	7,8	3,6	0,2	+3,4	+12,1	25,[illegible]	0,03038
9	2500	10m	0	7,8	0	0,8	+7,0	+4,9	4,4	18,5	0	1,7	+16,2	+7,0	2,9	0,5	0	0	+3,4	+3,1	1,7	0	1,2	0,1	+1,1	+16,1	12,472	0,02008
10	3800	10m	2,0	13,0	2,0	0,8	+6,1	+4,3	1,7	26,0	5,8	3,6	+18,3	+7,0	0,2	4,8	2,9	0,1	+2,0	+1,8	14,0	7,6	1,7	2,6	−0,9	+12,2	5,873	0,00716
11	5800	10m	0,2	22,5	0,6	6,3	+25,8	+18,2	39,7	19,4	0,5	0,2	+58,4	+22,4	0,2	54,8	13,7	6,3	+35,0	+32,3	6,3	7,2	19,4	13,0	+6,4	+79,3	2,426	0,01924
12	8600	10m	0,5	42,3	0,2	20,2	+12,4	+8,8	34,8	1,0	2,3	9,6	+23,9	+9,1	17,6	84,6	0,6	39,7	+52,0	+48,9	3,2	1,7	59,8	21,2	+33,6	+100,4	1,007	0,01011
13	12000	100m	0,5	0,4	0,1	0,8	0	0	0,2	0,5	0,4	0,1	+0,2	+0,1	1,0	0,6	1,0	1,0	−0,4	−0,4	0,3	0,1	1,0	4,0	−3,0	−3,3	50,513	−0,01667

Sum +0,2861

$U_{yz} = -\frac{g}{2} E \cdot 0{,}393 \cdot \text{Sum} = -\frac{g}{2} E \cdot 0{,}1$[illegible]

TABLE 3

Carthographic Correction for U_Δ

Number	Distance	Unit	$(h-\zeta)$ in azimuths								Sum = I	$(h-\zeta)$ in azimuths								Sum	$\times 0{,}707 = II$	Sum I+II	Coefficient	Results $\times 10^{-9}$
			+ 1	+ 3	− 5	− 7	+ 9	+ 11	− 13	− 15		+ 2	− 4	− 6	+ 8	+ 10	− 12	− 14	+ 16					
1	100	m	−0,3	−0,9	−1,2	−2,4	3,9	−2,4	−0,9	−0,9	−1,5	−0,9	−0,9	−1,5	−3,9	−2,4	−0,9	0	−0,9	−4,2	−3,0	−4,5	5,806	−7,01
2	150	m	+0,6	−0,9	−1,2	−4,0	−5,4	−2,4	−0,9	−0,9	−2,7	0	−0,9	−2,4	−5,5	−4,0	−0,9	+0,6	−0,9	−6,8	−4,8	−7,5	10,653	−7,99
3	225	m	+0,6	−2,1	−1,8	−5,5	−8,5	−4,0	−2,4	−2,4	−3,7	0	−1,8	−4,0	−3,5	−6,1	−0,9	+2,2	+0,6	−9,5	−6,7	−10,4	7,052	−7,33
4	340	m	+2,2	−3,0	−5,5	−7,0	−11,6	−5,5	+2,2	+3,1	−6,1	−0,9	2,4	−4,0	−0,1	−8,5	−0,9	+3,7	+3,7	−12,2	−7,2	−13,9	4,827	−6,42
5	500	m	+2,2	−2,4	−4,0	−9,4	−12,8	−5,5	+2,2	+3,7	−8,8	0	−4,0	−4,0	−11,6	−10,7	−0,9	+5,9	+3,7	−15,0	−10,6	−19,4	2,860	−7,84
6	750	m	−0,9	−4,0	−0,9	−11,6	−13,7	−5,5	+3,7	+5,2	−17,4	−4,0	−4,0	−5,5	−14,6	13,1	+0,6	+5,2	+2,2	−25,8	−18,2	−35,6	2,265	−8,06
7	1150	10m	−0,4	+0,2	+0,2	−1,3	−2,2	−0,2	+0,5	−0,9	−3,3	0,4	+0,4	−0,4	−2,1	−1,3	+0,4	+0,5	+0,2	−4,5	−3,2	−6,5	14,810	−9,63
8	1700	10m	−0,7	−0,1	+1,9	−2,8	−2,8	−0,7	+0,4	−0,1	−5,8	−0,5	+1,1	−1,3	−2,7	−2,2	+0,4	+0,2	−0,5	6,3	−4,5	−10,3	8,538	−8,79
9	2500	10m	−1,3	−0,1	+1,1	−2,8	+0,2	+0,2	+0,3	−0,9	−2,5	−1,2	+1,7	−0,7	−4,3	−0,1	+0,2	−0,1	−1,3	−8,0	−5,7	−8,2	6,236	−5,11
10	3800	10m	−3,8	+1,7	−1,3	−3,6	−2,8	+3,0	+1,6	−0,9	−6,9	−1,3	−0,4	−2,2	−5,1	+2,4	+1,7	+0,3	−1,9	−5,3	−3,7	−10,6	4,446	−4,71
11	5800	10m	−2,5	−0,4	−4,4	−5,7	−2,7	+0,8	+3,6	+2,5	−4,4	−1,3	−0,4	−7,4	−4,4	−0,7	+3,7	+2,5	−0,4	−10,2	−7,2	−11,6	2,880	−3,33
12	8600	10m	−1,8	−0,7	−7,4	−6,5	−1,3	+1,1	+4,6	+5,4	−0,3	−5,9	−4,2	−9,2	−1,0	−1,5	+3,1	+6,3	+3,1	−1,3	−0,9	−1,2	1,732	−0,21
13	13000	100m	0,5	−0,7	1,0	−0,6	−0,3	−0,3	+2,0	+0,9	−1,8	−0,4	−1,0	−0,8	−0,7	+0,6	+1,0	+1,0	−0,3	−1,0	−0,7	−2,5	13,133	−3,28

Sum −77,53

$U_\Delta = -\frac{g}{2} E \cdot 0{,}383 \cdot \text{Sum}$ | $\frac{g}{2} E\ 30{,}5$

TABLE 4

Carthographic Correction for $2U_{xy}$																									
Number	Distance	Unit	(h-5) in azimuths								Sum-I	(h-5) in azimuths								Sum	× 0,707 = II	Sum I+II	Coefficient	Results × 10⁻⁴
			+1	+3	−5	−7	+9	+11	−13	−15		+2	+4	−6	−8	−10	+12	−14	−16					
1	100	m	−0,9	−0,9	−1,2	−2,0	−3,8	−2,4	−0,9	−4,9	0	−0,9	−0,9	−1,5	−3,3	−2,4	0,9	0	−0,9	+0,6	+0,4	+0,4	15,806	+0,06
2	150	m	+4,2	−0,9	−1,2	−4,0	−5,6	−2,4	−0,9	−0,0	+1,6	0	−0,9	−2,4	−5,5	−4,0	−0,9	+0,6	−0,9	+2,4	+1,7	+3,3	10,653	+3,51
3	225	m	+0,6	−2,1	−1,8	−5,6	−8,5	−8,0	−2,4	−2,4	+1,8	0	−1,8	−4,0	−8,5	−6,1	−0,9	+2,2	+0,6	+0,9	+0,6	+2,4	7,052	+1,69
4	340	m	+2,9	−3,0	−3,5	−7,0	−1,6	−5,5	+2,7	+3,7	−5,2	−0,9	−2,4	−9,0	−10,1	−8,5	−0,9	+3,1	+3,7	−6,0	−4,2	−9,4	4,827	−4,54
5	500	m	+2,2	−2,4	−4,8	−9,4	−12,8	−5,5	+2,9	+3,7	−2,2	0	−4,0	−8,0	−11,6	−12,7	−0,9	+5,3	+3,7	−9,0	−6,4	−8,6	2,860	−2,46
6	750	m	−0,9	−4,0	−0,9	−8,6	−13,5	−5,5	+3,7	+5,2	−3,1	−4,0	−4,0	−5,5	−14,6	−13,1	+0,6	+5,2	+2,2	−7,8	−5,5	−8,6	2,265	−1,95
7	1150	10m	−0,4	+0,2	+0,3	−1,3	−2,2	−0,2	+0,5	+0,4	+0,9	−0,4	+0,4	−0,4	−2,1	−1,3	+0,4	+0,5	+0,2	+0,1	0	+0,9	14,810	−1,33
8	1700	10m	−0,7	−0,1	+4,9	−2,8	−2,8	−0,7	+0,4	−0,1	+2,1	−0,5	+1,1	−1,3	−2,7	−2,2	+0,4	+0,2	−0,5	+3,1	+2,2	+4,3	8,385	+3,67
9	2800	10m	−1,3	−0,1	+1,1	−2,8	+0,2	+0,2	+0,3	0,9	+3,8	−1,2	+1,7	−0,7	−4,3	−0,1	+0,2	−0,1	−1,9	+7,0	+4,9	+8,7	6,236	+5,43
10	3800	10m	+3,8	+1,7	−1,3	−3,6	+2,8	+8,0	+1,6	−0,9	+9,2	−1,3	−0,6	−2,2	−5,1	−2,4	+1,7	+0,3	−1,9	+8,3	+8,0	+17,2	4,446	+8,71
11	5800	10m	−2,5	−0,4	+4,4	−5,7	+2,7	+0,8	+3,6	+2,5	+3,6	−1,3	−0,6	−7,4	−4,4	−0,7	+3,7	+2,5	−0,4	+8,9	+7,8	+11,4	2,880	+3,28
12	8600	10m	−1,8	−0,7	−1,4	−1,5	−1,3	+1,1	+2,6	+5,4	+1,5	−5,9	−4,2	−9,2	−1,0	−1,5	+3,1	+6,3	+3,1	−7,7	−5,4	−3,9	1,730	−0,67
13	13000	100m	−0,5	−0,7	−1,0	−0,6	−0,3	−0,3	−2,0	+0,9	−1,3	−0,4	−1,0	−0,8	−0,7	+0,6	+1,0	+1,0	−0,3	+1,0	−0,7	−2,0	13,133	−2,62
																							Sum		+7,8
																				$2U_{xy} = \frac{\sigma}{2} E \cdot 0,393$ Sum				$\frac{\sigma}{2} E \cdot 5,0$

DISCUSSION

[This discussion refers also to the paper by E. Lancaster-Jones, which begins on page 505.]

F. W. LEE, Washington, D. C.—The use of the torsion balance for interpreting the geological structure is invaluable, yet the methods proposed by these two papers for interpreting the indications of the balance are of an academic nature, and their practicability for field use may be questioned.

E. LANCASTER-JONES (written discussion).—Mr. Lee's doubts as to the practicability of the employment of refined methods of computation in field problems of gravity surveying probably arise from a healthy distrust of over elaborate mathematics, and a suspicion that scientists are endeavoring to surround the Eötvös balance with a cloud of noxious symbols. Doubtless, also, he is influenced by the fact that the majority of gravity surveys have been made over large geologic structures such as anticlines, salt domes and the like, which are revealed in great prominence in the total subterranean values obtained from the gravity survey.

In such cases, a mere inspection of the gravity map may reveal all the characteristics of the structure which are required in practice. On the other hand, my own field experience—which has extended now for several years—has been confined to the direct location of mineral deposits, in which the feature sought is gravitationally quite subordinate to the total complex of subterranean irregularities. In these cases, only refined methods of analysis can differentiate with the requisite precision between the more prominent but valueless constituents and the minor but valuable residuals. A reference to H. Shaw's paper[15] will amplify this point.

[15] Gravity Surveying in Great Britain. See page 530.

If, therefore, gravity methods are to be applied to any but the simplest geological problems, analytical methods must be used to dissect the constituent subterranean features. Frequently a simple approximate analysis will suffice, but there is surely every advantage in having the more precise and refined methods available if required. There is usually no extreme urgency in prospecting for minerals, or determining geologic structures, and a gravity survey is not completed in the field, but in the office.

C. A. HEILAND (written discussion).—I regret to say that Mr. Lee has obviously completely overlooked the fact that my paper does not deal with the interpretation of the indications of the torsion balance at all but only with a correction to be applied for the elimination of terrain effects. Even assuming that Mr. Lee has been only incorrect in his expressions and means this correction instead of interpretation, he has overlooked the statement contained in the paper that the method proposed has been used for two years in practice by several oil companies in this country. This will prove sufficiently that Mr. Lee's view, that the practicability in field use may be questioned, cannot be correct.

I take the liberty of answering also Mr. Lee's criticism of Mr. Lancaster-Jones' paper. This is because of the fact that the analytical methods used for the computation of terrain and subterranean effects are very much allied; if we prove that Mr. Lee's criticism of methods used in interpretation is unfounded it means that it does not hold for similar analytical methods either.

Mr. Lee must have overlooked the following sentence in Mr. Lancaster-Jones' paper: "The present paper is intended to act as a survey of the gravitational computation." Therefore, Mr. Lancaster-Jones did not intend to propose any new methods. He reviewed mostly principles that were published in part more than 20 years ago by other authors, which have been proved sufficiently in practice (Eötvös, Nikiforov, Meisser, Numerov, Jung, Mekel and many others.) After more than five years field practice with the torsion balance, I can assert that these methods are not of an academic nature but have proved their practicability in the field because the computation of depths and size of the disturbing formations based on all of these methods which I have made so far for American companies have been proved correct by later drilling. Not much has been published of such successes; of the published material, the articles of Nikiforov, Mader, Schumann, Matuyama, Koenigsberger and Barton must be consulted. A sentence in Dr. Barton's most recent paper[16] should answer Mr. Lee's criticism; *i. e.*, "For compilation of more usable formulas see the publication of E. Lancaster-Jones."

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[16] See page 481.

Experiments with Eötvös Torsion Balance in the Tri-State Zinc and Lead District

By P. W. George,* Baxter Springs, Kansas

(New York Meeting, February, 1928)

The rapid increase in cost of discovering new orebodies by churn drilling in the Tri-State district has led to some attempts to lessen the expense by using geophysical methods.. Electrical prospecting was tried three or four years ago, but no successful results were recorded.

In 1926, the Federal Mining and Smelting Co. engaged the North American Exploration Co., of Houston, Texas, to make some tests with the torsion balance. At first, it was thought that the most favorable conditions for discovery of ore deposits with the torsion balance would be in locations where the ore was known to occur at rather shallow depths,

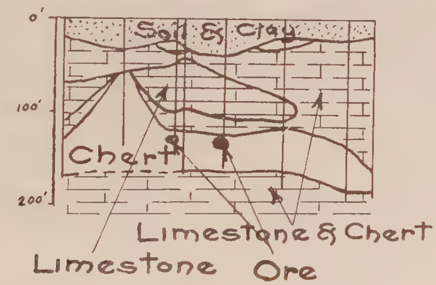

Fig. 1.—Cross-section of Duenweg lease.

therefore the first trial was made at Duenweg, east of Joplin, where some zinc ore had been developed at a depth of 130 ft. The gravity gradients at 61 stations on a tract of about seventeen acres indicated plainly one large zone of maximum gravity and a smaller narrow zone of minimum gravity. Fourteen churn-drill holes, half of which struck ore, show that all the ore found was located outside the zones of maximum and minimum gravity. In the zone of maximum gravity the limestone beds were closest to the surface, and the first chert beds encountered were at the greatest depth. The results (shown in Table 1) indicated that the gravity of the limestone was greater than the gravity of the fractured chert together with its orebodies, and probably also greater than the bouldery soil and clay on top of the limestone. A cross-section of the Duenweg lease is shown in Fig. 1.

* Superintendent, Federal Mining & Smelting Co.

TABLE 1.—*Record of Torsion Balance and Churn-drill Prospecting at Duenweg*

Gravity Zone	No. of Holes	Depth of Soil, Clay and Boulders in Top of the Holes or on Top of the Limestone, Feet	Depth from Surface to the Top of the First Well-silicified Beds, Feet	Total Thickness of Silicified Beds as Far as Drilled, Feet
Maximum	2	20	135	37
Intermediate	10	27	125	32
Minimum	2	34	110	38

The second experiment was made in a well-drilled corner of the Federal Brewster lease, containing ore runs of better than average size and grade at a depth of about 215 ft. below the surface. The resulting gravity gradients indicated clearly a wide zone of minimum gravity which covered an area known to be silicified, fractured and brecciated to a large extent and to contain an ore run about 100 ft. wide and 15 to 20 ft. high. The ore-bearing brecciated chert was known to contain a number of small caves, openings and crevices which evidently more than offset any

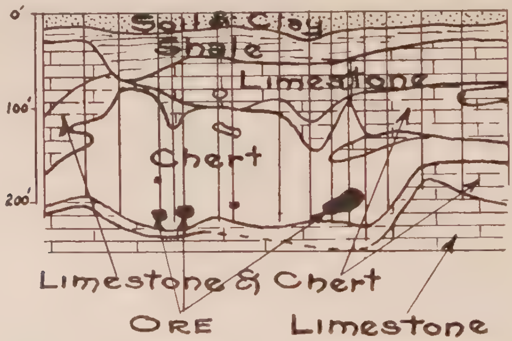

FIG. 2.—CROSS-SECTION OF BREWSTER LEASE.

increase in gravity caused by the included zinc ore. The specific gravity of the limestone is about 2.67; the gravity of the solid chert about 2.54. It would take only 1.1 per cent. of openings in the chert to balance the added weight of the orebody. The soil and the shale with pyrite inclusions was of about the same specific gravity as the limestone. The silicified beds were thicker in the zone of minimum gravity, while the cover of soil and shale was of about the same thickness over the whole area surveyed (see Fig. 2). On the whole, the test on the Brewster lease confirmed our belief that the torsion balance would be useful only in differentiating between zones containing silicified and fractured beds, with or without ore, and zones of unaltered limestone which as a rule do not contain any orebodies.

The third trial was made on a wildcat lease west of Picher, with the hope that determination of the gravity gradients would cut down the

number of churn-drill holes necessary for exploration of the tract. The lease consisted of 120 acres. The total number of torsion balance stations

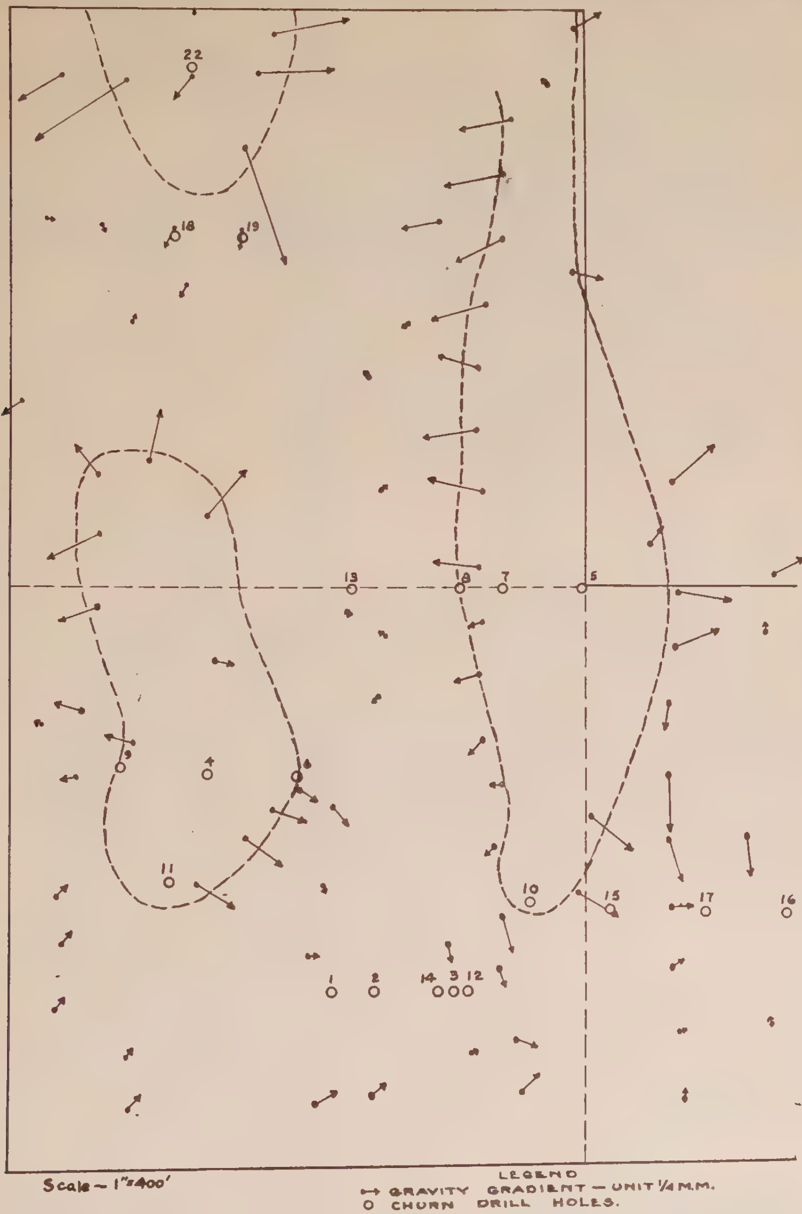

FIG. 3.—WEST PART OF WILDCAT LEASE NEAR PICHER SHOWING GRAVITY GRADIENTS AND CHURN-DRILL HOLES.

was 106. The gravity gradients indicated three zones of minimum gravity in the west part of the tract; 23 holes were drilled to check the torsion

balance results and to hunt for orebodies in or near the areas of minimum gravity. The zones of gravity and drill holes in the west part of the lease are shown in Fig. 3.

The drilling checked the torsion balance observations as shown in Table 2, although the drilling in the silicified and fractured beds did not result in discovery of any orebodies.

TABLE 2.—*Wildcat Lease West of Picher*

Drill Hole Number	Depth of Soil, Clay, Shale and Sandstone in Top of the Holes, Feet	Depth from Surface to the Top of the First Well-silicified Beds, Feet	Total Thickness of Silicified Beds above Second Limestone, Feet
Holes in Zones of Minimum Gravity			
5	120	120	20
8	120	160	80
6	124	200	95
11	125	205	70
9	128	150	115
7	130	155	70
10	135	95	120
4	140	140	130
22	200	200	70
Average.........	136	158	86
Holes in Remainder of Lease			
18	98	295	10
19	99	275	30
13	103		0
20	125	275	40
21	125	265	30
23	125	190	65
1	130		0
2	130		0
3	130	275	30
12	130	275	25
14	130		0
15	130	290	25
16	130	275	30
17	130	235	25
Average.........	123	265	22

Hole 5, in the zone of minimum gravity, struck light porous sandstone from 85 to 90 ft. and hole 7 drilled through sandstone from 90 ft. to 130 ft. No sandstone was struck in any of the other holes. Generally sandstone is not common enough in the Tri-State district to interfere with the use of the torsion balance for determination of silicified areas.

Hole 23, outside the zone of minimum gravity, shows considerable silicification. Such exceptions are to be expected because the torsion balance stations were not located close enough for discovery of anything but the largest areas of minimum gravity.

The average thickness of the shale cover in the Tri-State district is usually slightly greater in the silicified areas than in the areas underlaid by limestone. This is due to slumpage of the silicified and brecciated beds. Where the silicified beds are thicker they are also most frequently closer to the surface. If the drill holes on the wildcat lease that show approximately the same thickness of shale cover are compared, it seems evident that the variation in the silicified beds is the main cause of the change in gravity gradients shown by the torsion balance.

Holes with Thickness of Shale, Soil and Sandstone, 124 to 130 Ft., Inclusive

	Thickness of Soil, Shale and Sandstone in Top of Drill Holes, Feet	Depth to Upper Chert Beds, Feet	Thickness of Chert Beds, Feet
Minimum gravity area—average....	127	177	87
Remainder of lease—average.......	129	260	25

The fourth and last experiment with the torsion balance was made in the west part of the Federal Jarrett lease. The main ore horizon on this lease was known to be at about the 300-ft. level—deep enough to make it

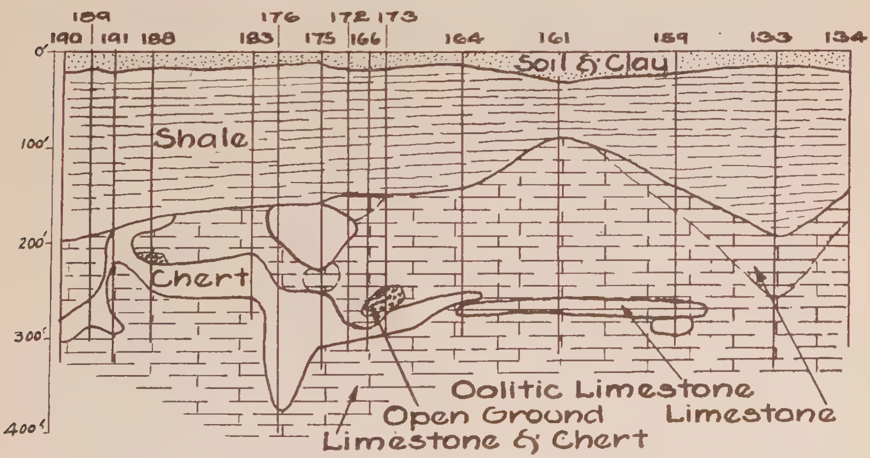

Fig. 4.—Cross-section of Jarrett lease.

profitable to use the torsion balance for elimination of unnecessary drilling of limestone areas. Part of the tract covered by torsion balance observations in this area is not considered in the following, partly on account of the disturbing influence of a hill and partly due to the lack of drilling.

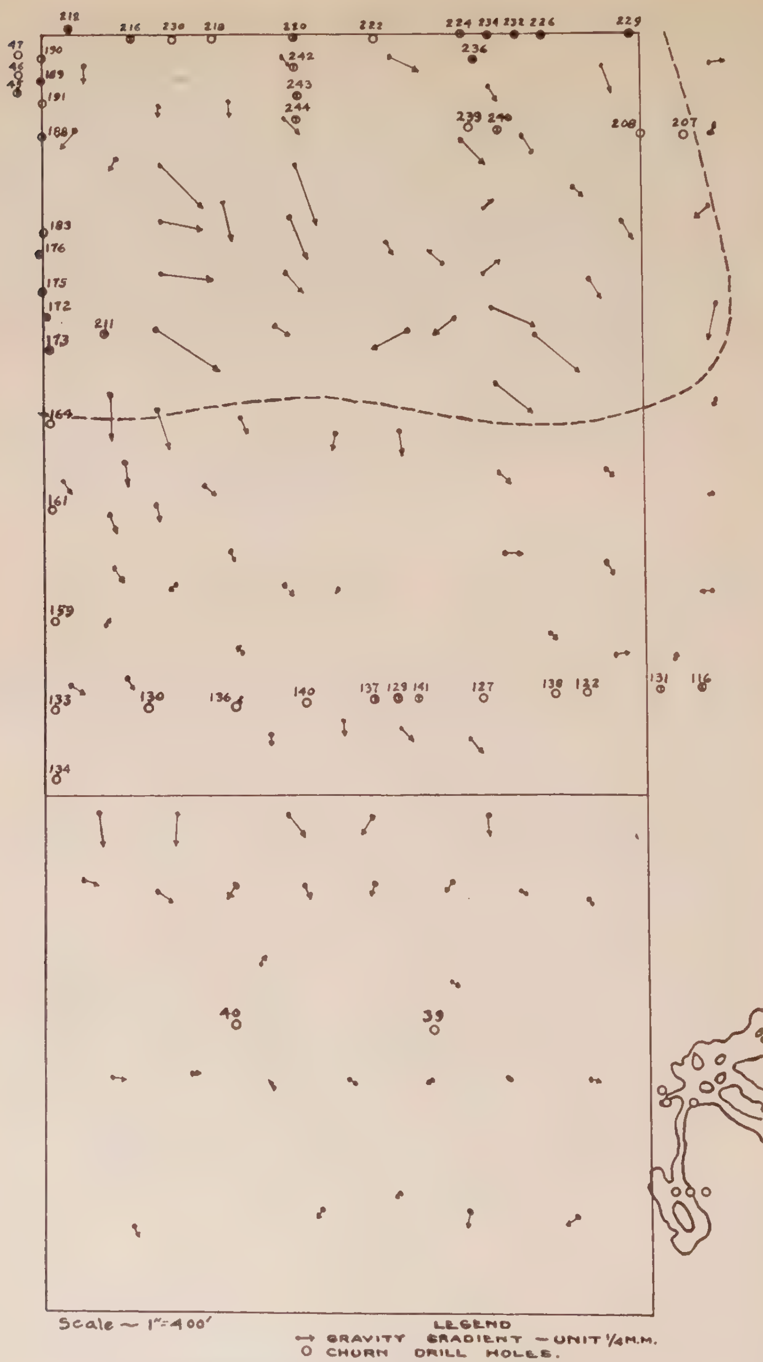

FIG. 5.—GRAVITY GRADIENTS AND CHURN-DRILL HOLES ON PART OF JARRETT LEASE.

TABLE 3.—*Jarrett Lease, West of Picher*

Drill Hole Number	Depth of Soil, Clay and Shale in Top of the Holes, Feet	Depth from Surface to the Top of the First Well-silicified Beds, Feet	Total Thickness of Silicified Beds above the Second Limestone, Feet
Holes in 30-acre Area of Low Gravity			
229	150	180	110
172	150	180	23
173	150	270	More than 61
224	155	185	125
236	155	215	77
234	155	225	More than 61
232	155	185	90
226	155	225	95
207	155	285	40
240	156	205	102
239	157	235	60
208	157	290	75
176	160	160	170
175	160	160	125
183	165	210	45
188	175	175	50
243	175	175	150
222	180	240	95
216	185	225	90
191	187	200	85
242	190	205	130
244	190	190	130
189	195	260	30
190	197	275	25
47	198	195	65
46	198	155	102
211	200	205	90
215	200	260	45
212	205	205	130
230	205	260	65
220	210	280	70
Average............	175	217	More than 84
Holes in Remainder of Lease			
116	110	280	10
122	112	285	10
138	117	295	23
131	119	265	49
127	120	160	5
141	129	273	38
129	130	280	50
39	130	260	10
137	133	270	67
136	136		0
130	139		0
40	140		0
140	140	265	15
161	140		0
134	145		0
159	145	280	20
164	145	255	5
139	145	240	55
142	145	275	10
145	145		0
148	150		0
41	150		0
42	155		0
151	165		0
154	165		0
133	210		0
Average............	141	263	14

Note.—The most silicified zones were drilled more thoroughly than the areas of unaltered limestone. The difference between silicified zones and unaltered limestone would therefore be greater than shown in Table 3 if the drill holes had been spaced regularly. To correct this condition to some extent, holes located nearer than 50 ft. to the previous holes were not considered.

The remainder of the tract consists of 130 acres with 118 torsion balance stations. The observations show an area of about 30 acres in the north part of the tract with gravity gradients indicating lower gravity than normal in unsilicified areas. The gravity gradients in the 100 acres at the south and southwest show uniform high gravity as in the previously observed areas of unaltered limestone. Good orebodies have been struck in drill holes located in the 30-acre low-gravity zone. No ore has been found so far in the remainder of the tract. A cross-section of the lease is shown in Fig. 4 and the arrangement of drill holes and torsion balance tests are given in Fig. 5.

Comparing the records of the drill holes showing the same thickness of shale in the two groups of Table 3, the silicified beds appear again as the main cause of the gravitational minima.

Holes with Thickness of Shale and Soil from 150 to 165 Feet Inclusive

	Thickness of Soil and Shale in Top of Drill Holes, Feet	Depth to Upper Chert Beds, Feet	Thickness of Chert Beds, Feet
Minimum gravity area—average....	156	214	More than 84
Remainder of lease—average.......	157	No chert	No chert

The variations in the depth of the chert beds and in their thickness are too great, and the observations made are too few in number to make any very definite determination of the effect on the torsion balance of variations in the thickness of the shale. However, the relatively high specific gravity of the shale, with its inclusions of pyrite, and the many openings and crevices in the lighter chert, indicate plainly that the effect of variations in shale cover is of minor importance and that the depth and the thickness of the chert beds are the chief determining factors of the gravitational minima. For practical purposes there is no need to differentiate between the effect of the thickness of the chert beds and their depth, because both these factors tend to produce the same gravitational effect at the same places. The general slumping of the thickest chert beds has been followed by a thickening of the overlying shale. The tendency is consequently for thicker shale cover over the most silicified areas. Even assuming that the shale would be slightly lighter in specific gravity than the limestone, the torsion balance would therefore probably still show gravitational minima only above the silicified areas where the shale cover is thicker and the upper limestone surface more depressed.

The North American Exploration Co. used two torsion balances of photographic recording type, making a total of about 30 observations per week. A Ford truck was used to move the balances and the observation

houses. The crew consisted of five to six men, three of whom were used to level the places for the stations and to move the instruments.

Summary.—In general the results described indicate that the torsion balance would be useful in the Tri-State district only under favorable conditions and only for outlining the silicified and fractured areas previous to drilling. The Brewster and Kansouri sections given in Figs. 2 and 6 illustrate typical relations between the orebodies and the silicified and fractured zones above the Grand Falls chert horizon. Favorable conditions for use of the torsion balance preliminary to drilling in the Tri-State district are as follows:

1. Comparatively level surface free from boulders, dumps or caves.
2. No mine workings.

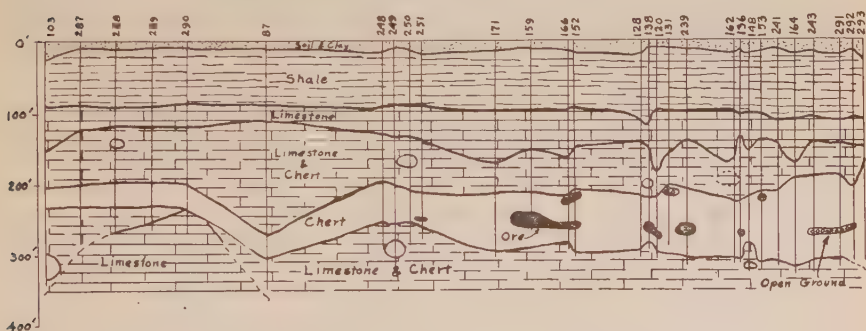

FIG. 6.—CROSS-SECTION OF KANSOURI LEASE.

3. No sandstone of great enough thickness to cause a minimum gravity zone above an area of unaltered limestone.
4. High cost of churn drilling, such as exists in the west part of the Picher field where the orebodies are relatively deep.
5. Conditions in which a comparatively small number of torsion balance observations may differentiate between silicified zones and areas of unaltered limestone.
6. Low cost of torsion balance observations.

It is well known that the richest orebodies in the district are most frequently found near the edges of the silicified zones. Ore deposits located near the center of very extensively silicified zones are generally of lower tenor.

DISCUSSION

D. H. McLaughlin, Cambridge, Mass.—This paper is of particular importance as an example of an effective combination of geophysical prospecting with sound geology. Obviously, metallic deposits rarely cause gravity anomalies great enough to be of practical use as indications of ore. Even the largest orebodies are relatively small compared with salt domes or other structures with which the oil geologists work. But if a characteristic type of rock alteration accompanying the ore can be recognized and

is capable of detection by some geophysical method, the target is considerably enlarged and the chances of success greatly increased. In the Tri-State district, the torsion balance was of use not in direct location of orebodies but in the detection of the silicified limestone in which the ore occurred. In general, it is certain that geophysical methods will be of greatest service when combined intelligently with geologic observation, as in the work described in this paper.

C. Carmean, Baxter Springs, Kans. (written discussion*).—The Federal Company has tried two methods for finding deposits of lead and zinc ores other than by the usual method of churn drilling.

The first method tried was what is known as the Lundberg, or Swedish method. Briefly, this depends on the electrical conductivity of the orebodies, using an outside generated electrical field.

In practice we had a 10-kw. 220-volt alternating generator mounted on a Ford chassis and driven by the Ford engine. This arrangement gave us a portable outfit which was easily moved from place to place. The energy from the generator was led to two 3/8-in. galvanized stranded wires that were buried about a foot under ground. These wires were each about 1000 ft. long; they were practically parallel to each other and 800 to 1400 ft. apart.

Within this energized area we took readings in ohms at each corner of lots 100 ft. square. These readings were taken by means of a very sensitive potentiometer. The two binding posts leading from the potentiometer were connected by insulated wires to two copper finding rods. These rods were placed at the corners of the square areas in succession and the readings on the potentiometer were logged.

When the readings were all platted up it looked very pretty and we were sure that we would be able to find a relationship between our curves and the orebodies, which, in some cases, we knew were there, but after repeated trials we gave it up. Our experiments led us to believe that the pyrite in the shale had an effect, where this method was used on orebodies that were under shales. On orebodies where the shale had been eroded, the only way we could account for inconsistent readings was that the surface water might have a disturbing effect.

The other method we tried was the Eötvös torsion balance. This method, perfected in Germany, depends for its success upon a marked difference in specific gravity of the underlying rocks. The North American Exploration Co., of Houston, Texas, supplied both instruments and men to do our testing.

We thought that the ore-bearing rocks would have a greater specific gravity than the surrounding flint, and that the torsion balance would show these areas, at least in a general way, but we found that it could not do this. The greatest obstacle was the difference in specific gravity of limestone and flint, solid limestone having a specific gravity of 2.67 and solid flint a specific gravity of 2.54. Our ores in this district are mostly deposited in a flint bed, in fractures and open areas in the flint. This open flint ground, of course, made the difference in specific gravity still more marked. In places where we had a known ore deposit, the weight of the ore in this lighter, open, crevicey flint failed to offset the higher specific gravity of the adjacent more solid limestone. If an orebody was extremely large and in solid flint ground, of course a difference could be noticed, but we are not favored in this district with such deposits, except occasionally.

We believe, from our experience, that the torsion balance could be used to advantage in new areas where it would be advantageous to outline the silicified areas in a general way. In this district, such work would prevent much useless drilling, for we would not drill the barren limestone areas.

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* Discussion presented before Joplin-Miami Zinc Section, A. I. M. E.

G. B. Corless, Amarillo, Tex. (written discussion).—Although these experiments did not furnish a clue to the problem of locating individual orebodies, they have added another method of locating zones of silicification in the Tri-State district. These zones are the loci of all profitable orebodies but so far no one has been able to establish suitable criteria for ore hunting even by underground exploration.

Mr. George calls attention to the fact that slumpage of the silicified beds often causes a thickening of the surface shale formation. This relationship has been the basis of recent "shale-drilling" campaigns that have been very successful in locating silicified areas. It would be valuable to have a comparison of the costs of securing the ame information by shale drilling and by the torsion balance.

P. W. George.—Comparing the relations between the silicified zones and the depressions in the old surface on which the shale was laid down in the Oklahoma-Kansas field, it is evident that there are a great many important silicified ore-bearing zones which are not accompanied by slumpage of the beds and therefore not discoverable by "shale drilling." Some deep shale zones may also indicate erosion of the underlying beds instead of slumpage. For these reasons I would consider the torsion balance more dependable than shale drilling for roughly outlining the silicified areas, provided conditions otherwise are favorable for its use.

The cost of shale drilling is about 50 c. per foot above 100 ft. depth and 75 c. per foot below this depth. A 150-ft. hole would cost $87.50. For outlining the sub-shale topography by drilling, about 50 holes would be required to a 40-acre tract, or a total expense of $4375. I am not very familiar with the cost of torsion-balance work but I would estimate that in the above case 60 observations with the torsion balance should give more valuable information at about half the cost of the shale drilling. With an average shale depth of 70 to 80 ft. it is probable that the cost of the two methods would be about the same.

The Seismic Method of Mapping Geologic Structure

By Donald C. Barton,* Houston, Texas

(Boston Meeting, August, 1928)

The elastic earth waves produced naturally by earthquakes have been used for a long time as evidence from which to draw conclusions in regard to the constitution of the interior and crust of the earth. The elastic earth waves produced artificially by occasional explosions and recorded by seismological observatories were used to draw conclusions regarding the geology of the intervening area. Intentionally produced elastic earth waves in the past five years have come to be used extensively in the investigation of very local geologic structure. The seismic method of working geologic structure, which makes use of artificially controlled explosions and the resulting earth waves, has developed almost clairvoyant power in handling certain geologic situations; and, in the discovery of salt domes in the Gulf Coastal Plain region of Texas and Louisiana, it has scored the most brilliant success.

The first proposal for the use of artificial earthquakes in the study of velocity of elastic earth waves in the surface formations of the earth's crust were made before 1888, by the English seismologists Mallet and Abbot. Partly on the basis of their work, A. Schmidt in 1888 proposed the use of time-distance graphs of artificial earthquakes to study the variation of velocity with depth. Belar in 1902 proposed the practical application of such investigation in connection with boring tunnels. Galitzen repeatedly (1912, 1913) proposed the use of explosions to study the velocity of the longitudinal and transverse waves in the uppermost formations and pointed out that the velocity depended in a high degree on the physical character of the beds and that from changes in the velocity, conclusions could be drawn in regard to the composition of the beds. Somewhat the same thought was proposed by von dem Borne (1908), by Benndorf, Udden and others.[1]

First Use of Artificially Excited Earth Waves to Determine Local Structure

The first application of the use of artificially excited elastic earth waves to the determination of local geologic structure was worked out

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[1] Historical summary largely after W. Schweydar and H. Reich. See bibliography at end of paper.

by Fessenden who in 1913 in a series of field experiments near Framingham, Mass., developed the instruments and technique to a point of practical applicability and patented his method. He used an adaptation of his method of sonic sounding for depth in water; a sonic sounder, immersed in water in a bore hole, was used to set up a controlled series of compression waves in the water, which in turn set up elastic earth waves in the surrounding ground; sonic receivers were immersed in water in other bore holes and connected with photographically recording galvanometers; from the reflection, refraction, and absorption of the waves, conclusions were drawn in regard to the character of the intervening ground.

L. Mintrop and O. Hecker started experimenting early during the Great War, Mintrop with a mechanical seismograph and Hecker with a microphone and recording galvanometer. Working as a junior colleague of Wiechert of Göttingen, Mintrop perfected his instruments and technique to the point of practicability and in 1919 received a basic patent, since revoked, on the application of the seismic method to the working of local geologic structure. By 1921, he had demonstrated the potentiality of the method but apparently had not done much actual field surveying of geologic structure.

In this country, Eckhart, Hasemen, Karcher, and McCollom experimented with a seismic method in Oklahoma in 1921. Their results then seemed rather negative but the apparent failure at that time was due largely to lack of encouragement, to limited financial resources, and to the attempted application of the method in an area of slightly too complicated geology.

In the early summer of 1923, Mintrop's method was introduced in Mexico by the Royal Dutch Shell. In the late summer or autumn of the same year, his method was introduced by the Marland Oil Co. in Oklahoma and in the fault line district north of Powell, Texas, and in the spring of the following year in the Gulf Coast salt dome district of Texas. The discovery of several salt domes late in 1924 by a troop of Mintrop's "Seismos" company, working for the Gulf Production Co., gave great impetus to the use of the method.

By the spring of 1926, Anderson, Eckhart, Karcher, McCollom, Ricker, Rieber and Trueman in this country had perfected seismographs of varying degrees of fieldworthiness and of varying types, some of them radically different from Mintrop's seismograph, and some of the instrumental technique of the method being radically improved. The most important improvements were the application of wireless to the timing of the shots and to communication between shot point and receivers and the use of the air wave from the explosion for the determination of the distance rather than the time of the shot. Other seismographs have been designed since by other physicists.

Up to 1926, the "refraction" method was the only one in practical use, although there had been some experimentation in the attempt to perfect a method of using waves reflected directly back at a high angle; the "technique" of a reflection method was perfected during 1926 by the Geophysical Research Corpn. to the point of practical applicability. The seismic method has proved its ability in the reconnaissance for new salt domes and its ability roughly to delimit salt domes. It has shown ability in places to handle the nonsalt-dome types of petroliferous structures. The technique of taking the observations and of interpreting the results is being studied and considerable improvement has been made in the ability of the method to handle the more difficult problems of detailing structure such as detailing the flanks of salt domes and as working all but certain seismically simple geologic structures. There is potentiality and probability of considerable continuing improvement in the power of the method. In 1927, the method was applied by the Geophysical Research Corpn. to the mapping of oil wells to determine their crookedness.

Elastic Earth Waves (Seismic Waves)

The fundamental physical property on which seismic geophysical prospecting is based is the variation in speed of transmission of the elastic earth waves in different geologic formations. If a formation with a higher speed of transmission underlies one with a lower speed of transmission, elastic earth waves from the surface are reflected back to the surface from the top of the bed with the higher speed of transmission or are refracted along the upper surface of that bed and then re-refracted back to the surface. From the travel time of the earth waves, the presence of and certain facts about the lower bed can be determined.

The speed of transmission of the elastic earth waves is dependent upon the elastic properties of the transmitting medium. The mathematical formulas for the speed of transmission in terms of the elastic properties of the medium are:

$$V_c = \sqrt{\frac{\lambda + 2\mu}{D}} = \sqrt{\frac{E}{D}\frac{1 - \sigma}{(1 + \sigma)(1 - 2\sigma)}}$$

$$V_T = \sqrt{\frac{\mu}{D}} = \sqrt{\frac{E}{D}\frac{1}{2(1 + \sigma)}}$$

where:

λ and μ are Lamé coefficients, μ is the coefficient of rigidity,

E is Young's modulus which is dependent upon the stretching of a rod of unit cross-section under given tension,

σ is Poisson's constant which is dependent on the contraction of the cross-section of the rod under the tension,

V_c is the velocity of the longitudinal (compression) waves in km. per second,

V_T is the velocity of the transverse waves in km. per second,
D is the specific gravity.

The speed of transmission of the waves geologically may be said to be proportional to the denseness and compactness of the formation. Unconsolidated sands, shales, and marls transmit the waves with a low velocity; weak sandstones and limestones with slightly higher speeds, and massive crystalline rocks (limestones, rock salt, schists, gneisses and plutonic igneous rocks) with very much higher speeds. The approximate velocities for various kinds of rock are given by Sieberg[2] on the

TABLE 1.—*Velocity of Transmission of Longitudinal Earth Waves in Various Rocks (after Sieberg)*

Rocks	Minimum Velocity, Km. per Sec.	Mean Velocity, Km. per Sec.	Maximum Velocity, Km. per Sec.
Crystalline schists	5.5	6.3	7.0
Plutonic rocks	2.1	5.3	8.0
Eruptive rocks	1.4	2.8	4.2
Sediments in general	1.2	3.6	5.9
Limestone	3.8	5.0	5.7
Sandstone	1.4	1.8	2.2
Archaean rocks	4.1	5.6	7.0
Paleozoic sediments	2.2	4.5	5.7
Mesozoic sediments	1.8	3.4	5.9
Tertiary sediments	1.2	2.1	3.0
Pleistocene sediments	1.4	2.2	3.6

basis of laboratory determinations of Young's modulus of elasticity by Nagaoka, Kusakabe and Oddone, as shown in Table 1. The velocities of some formations as determined by field surveys with the seismic method are given in Table 2.

Several types of waves are recognized in seismology, but the applied seismic method as yet makes use only of the longitudinal wave, which is the compression-rarefaction wave in the direction of movement. The transverse or shearing waves are also recorded by the field seismographs. In the transverse wave, the particles oscillate at right angles to the direction of propagation of the wave. The velocity of the transverse wave is about 0.57 that of the corresponding longitudinal wave. It comes in behind the corresponding longitudinal wave and is picked up with less amplitude by the vertical component seismographs which are used in the field seismic method. The Rayleigh and Love waves are waves that travel at the surface of a formation with a velocity slightly less than that of the transverse waves. They are of no importance in the applied seismic method.

[2] A. Sieberg: Erdbebenkunde, 171–172. 1923.

TABLE 2.—*Velocities of the Longitudinal Waves in Some Geologic Formations*

Formation	Location	Velocity, Km. per Sec.
Plio-Pleistocene sediments	Mississippi-Texas Gulf Coast[1]	2.0±
Upper Miocene in part[1]	Louisiana-Texas Gulf Coast	2.4 to 2.7
? Oligocene	Mississippi[1]	3.8 to 4.3
Middle Eocene	Texas-Louisiana[1]	4.2 ±
Rock salt of salt domes (less toward the surface, greater at greater depth)[1]	Texas-Louisiana	4.7 to 5.2
Pecan Gap Chalk (in the subsurface)[1]	Texas	3.0 to 3.6
Austin Chalk (in the subsurface)[1]	Texas	3.6 to 4.2
Wet Sand (Hecker)[2]		1.4
Sand (Barré, Schnell)[2]		2.0 ± 0.6
Muschelkalk (Hubert)[2]		1.7
Cretaceous limestone (Maurin, Elbé)[2]	France	2.1
Granite (Barré, Schnell)[2]		8½ ± ¾
Basement[2]	Europe	5.6
Water[3]		1.4 to 1.5
Ice(4)*		3.4
Diluvium underlaid by Miocene sands	Kümmersdorf, Germany(5)	1.0
Diluvium	Sperenberg, Germany	0.87 to 0.91(5)
	Lüneburg Heath, Germany(6)	1.7 ± 0.1(6)
Muschelkalk limestone	Rüdersdorf, Germany(7)	4.3
Zechstein gypsum	Sperenberg, Germany(7)	3.5

* Numbers in parentheses refer to bibliography at end of paper.

[1] Current experience in the Gulf Coast. These values give only the general magnitude of the velocities. The actual velocity of a formation may vary considerably horizontally and vertically.

[2] B. Gutenberg: Lehrbuch der Geophysik., 608. 1928.

[3] B. Gutenberg: *Loc. cit.*, 583.

MATHEMATICAL THEORY

The use of the seismic method in mapping geologic structure depends on the fact that elastic earth waves traveling through rock of relatively low speed of transmission of the waves and impinging on the upper surface of an underlying formation with a higher speed of transmission will in part be reflected back to the surface of the ground and in part refracted along the upper surface of that formation and then refracted back to the surface of the ground. In the diagrammatic vertical section of Fig. 1, the upper formation has a speed of transmission of the longitudinal earth waves of V_1 km. per sec., and the lower formation of V_2 km. per sec. V_2 is larger than V_1. The speed is assumed to be constant in all directions. The waves produced by an explosion at A will travel outward in all directions with a spherical wave front. That part of the wave

which starts off parallel to the surface will continue indefinitely on that course, as for example along the path I of Fig. 1. Each part of the wave which starts off downward will in part be reflected by the upper surface of the lower formation upward, as for example along the paths II and III and in part be refracted along path IV along the upper surface of the lower formation and the re-refracted back upward along paths V and VI.

By the use of the times of arrival of the reflected waves, it is possible to determine the depth to the top of the underlying high-speed bed. In the diagrammatic vertical section of Fig. 2, the upper formation has a velocity of transmission of the longitudinal earth waves of V_1 and the lower formation of V_2, $V_1 < V_2$, Z is the depth to the top of the lower formation, X is the distance between the shot point O and the seismograph station A, and T is the travel time.

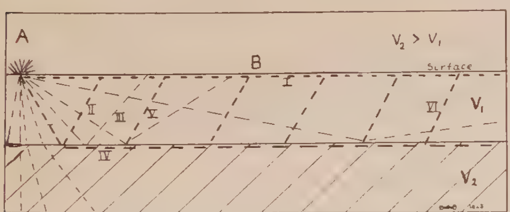

FIG. 1.—PATHS OF EARTH WAVES PRODUCED BY AN EXPLOSION WHERE THERE IS A HORIZONTAL UNDERLYING HIGH-SPEED BED.

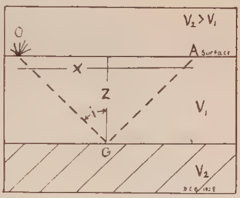

FIG. 2.—EARTH WAVE REFLECTED FROM AN UNDERLYING HIGH-SPEED BED.

The equation of the time-distance curve is:

$$T_2 = \frac{OC}{V_1} + \frac{CA}{V_1} = \frac{2}{V_1}\sqrt{\frac{x^2}{4} + Z^2} = \frac{\sqrt{x^2 + 4Z^2}}{V_1} \qquad (1)$$

If the equation is squared and is treated as a function of $T_2{}^2$ and x^2 and if Z remains constant, the curve becomes a straight line with a slope of $1/V_1{}^2$.

$$(T_2{}^2) = \frac{1}{V_1{}^2}(x^2) + \frac{4Z^2}{V_1{}^2} \qquad (2)$$

The velocity V_1 can be determined also from the time of arrival of the longitudinal waves which travel near the surface.

$$T_1 = \frac{x}{V_1} \text{ and } V_1 = \frac{x}{T_1} \qquad (3)$$

But the value of V_1 determined by (2) gives a mean value for the whole upper formation, if there is any vertical variation of V_1, and gives a more accurate value for the same absolute error of observation of T_1. If the surface of the underlying high-speed stratum is sloping, the value of Z can be maintained constant by moving the points O and A equally in opposite directions.

The depth to the top of the bed B can be calculated for each shot by formula (3) derived from (2) if V_1 has been determined.

$$Z = \frac{1}{2}\sqrt{V_1^2T^2 - x^2} \tag{3a}$$

The maximum amount of energy should be reflected to any point at the surface approximately when the angle of incidence, i, is the critical angle; *i. e.*, when $\sin i = \frac{V_1}{V_2}$. The distance x between the shot point and the seismograph location to obtain this maximum reflection is:

$$x_m = 2Z \tan i = 2Z\frac{V_1}{V_2}\frac{1}{\frac{\sqrt{V_2^2 - V_1^2}}{V_2}} = \frac{2ZV_1}{\sqrt{V_2^2 - V_1^2}} \tag{4}$$

The value of V_2 as well as of V_1 in unknown territory may be obtained by preliminary profiles by the refraction method. But in many areas the probable values of V_1 and V_2 and the approximate value of Z are known. The value of X_m can be estimated approximately and the value of V_1 can be checked by a profile of reflection shots.

The formulas (1) to (4) are only approximately valid in most actual situations in practice. The value of V normally increases downward and also the formation A of Fig. 3 may be composed of subordinate zones, each of which may have a slightly different value of V. A superficial zone with an abnormally small value of V is not uncommonly present.

The reflection method has the advantages that the depth, Z, can be calculated from each "shot," whereas a profile is necessary by the refraction method and that the velocity, V_2, in the reflecting bed does not enter the formula for the depth. The method has the disadvantage that the identification of the reflecting bed is impossible by reflection shooting alone and that beds may be jumped without any warning of the fact. As the value, V_2, of the velocity in the refracting bed is determined by the refraction method and is approximately constant for each formation and slightly to very different for different formations, different beds commonly can be told apart by the value of V and are not likely to be jumped unless two beds happen to have closely similar values of V.

The refraction (mirage) method takes advantage of the fact that the path $abcd$ is quicker for a wave than the direct path ad, provided the velocity, V_1, of stratum A is less than the velocity, V_2, of stratum B, and that the ratio, $\frac{Z}{ad} > \frac{1}{2}\sqrt{\frac{V_2 - V_1}{V_2 + V_1}}$ (Fig. 3). If the ratio, $\frac{Z}{ad} < \frac{1}{2}\sqrt{\frac{V_2 - V_1}{V_2 + V_1}}$ the path ad is quickest and if $\frac{Z}{ad} = \frac{1}{2}\sqrt{\frac{V_2 - V_1}{V_2 + V_1}}$, the travel times along the two paths are the same. Waves normally travel over both paths. If the seismograph records only the wave traveling over the path ad, the

inference is warranted that no higher speed bed is present within the depth $Z = \frac{x}{2}\sqrt{\frac{V_2 - V_1}{V_2 + V_1}}$. If the refracted wave traveling over some path, *abcd*, is recorded by the seismograph and if a profile is "shot," V_1, V_2 and the critical distance x_0, at which $Z = \frac{x}{2}\sqrt{\frac{V_2 - V_1}{V_2 + V_1}}$, can be read from the time-distance graph for the profile and from the values V_1, V_2 and x_0, the depth Z, to the top of the high-speed bed B.

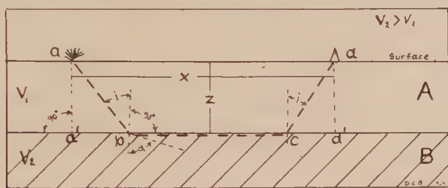

FIG 3.—PATH OF THE EARTH WAVE IN THE REFRACTION METHOD.

The equation for the travel time T_1 for the wave moving over the direct path *ad* is:

$$T_1 = \frac{x}{V_1} \tag{5}$$

The equation for the travel time T_2 for the wave traveling over the path *abcd* is:

$$T_2 = \frac{ab}{V_1} + \frac{bc}{V_2} + \frac{cd}{V_1} \tag{6}$$

According to the law of incidence and refraction, $\frac{\sin i}{\sin \alpha} = \frac{V_1}{V_2}$. If the wave is to travel horizontally along the path *bc* in the horizontal upper surface of bed B, the angle α must equal 90° and,

$$\frac{\sin i}{1} = \frac{V_1}{V_2}\sin 90^\circ = \frac{V_1}{V_2}$$

The same condition holds for the angle of refraction, i', back upward along *cd*, *i. e.*, $\sin i' = \frac{V_1}{V_2}$.

As the surface and *bc* are assumed as horizontal, $ab = cd = \frac{Z}{\cos i}$.

The distance $bc = x - a^1b - cd' = x - 2a'b = x - 2z \tan i$.

Therefore,

$$T_2 = \frac{2Z}{V_1 \cos i} - \frac{2Z \tan i}{V_2} + \frac{x}{V_2} = \frac{2Z}{\cos i}\left(\frac{1}{V_1} - \frac{\sin i}{V_2}\right) + \frac{x}{V_2}$$

$$= \frac{2Z}{V_1 \cos i}(1 - \sin^2 i) + \frac{x}{V_2}$$

And,

$$T_2 = \frac{2Z \cos i}{V_1} + \frac{x}{V_2} \tag{7}$$

If the point a is not at the same elevation as d but is at a distance $+E$ algebraically above d, a correction has to be applied to (7) and the travel-time equation becomes:

$$T_2 = \frac{2Z\cos i}{V_1} + \frac{E}{V_1 \cos i} + \frac{x}{V_2} - \frac{E \tan i}{V_2}$$

$$= \frac{\cos i}{V_1}(2Z + E) + \frac{x}{V_2} \qquad (8)$$

where Z is the depth below the level of d and E is relatively small.

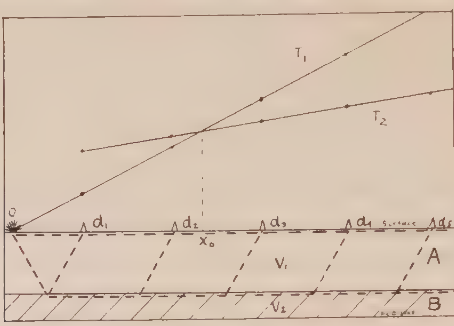

FIG. 4.—TIME-DISTANCE CURVES IN THE REFRACTION METHOD, WHERE A LEVEL FLAT-TOPPED HOMOGENEOUS ISOTROPIC HIGH-SPEED BED UNDERLIES AN ISOTROPIC, HOMOGENEOUS LOWER SPEED STRATUM.

The values of V_1 and V_2 can be read from the time-distance graph. The lower part of Fig. 4 is a repetition of the diagrammatic section of Fig. 3, with the addition of the paths for the waves at the series of stations, d. The upper part of the figure gives the time-distance curves for equations (5) and (7).

$$\frac{dT_1}{dx} = \frac{1}{V_1} \text{ and } \frac{dT_2}{dx} = 0 + \frac{1}{V_2} \qquad (9, 9a)$$

That is, the velocity in the upper stratum is the reciprocal of the slope of the equation for T_1 and the velocity in the lower stratum is the reciprocal of the slope of the equation for T_2. The values for V_1 and V_2 therefore can be read from the graph. The critical distance x_0 at which the travel time is the same for the direct and the refracted wave is the abscissa of the point of intersection of the two lines, T_1 and T_2.

A convenient formula for the depth, Z, to the top of high-speed bed, B, can be obtained in terms of V_1, V_2 and x_0, where x_0 is the critical distance at which the travel time is the same for the direct and the refracted waves.

$$\frac{x_0}{V_1} = T_0 = \frac{2Z\cos i}{V_1} + \frac{x_0}{V_2} \qquad \text{(from (5) and (7))}$$

$$2Z = \frac{x_0 V_1}{\cos i}\left(\frac{1}{V_1} - \frac{1}{V_2}\right) = \frac{x_0 V_1}{\frac{\sqrt{V_2^2 - V_1^2}}{V_2}}\left(\frac{V_2 - V_1}{V_1 V_2}\right)$$

And,

$$Z = \frac{x_0}{2}\sqrt{\frac{V_2 - V_1}{V_2 + V_1}} \tag{10}$$

For a local area and the same formations, A and B, $\frac{1}{2}\sqrt{\frac{V_2 - V_1}{V_2 + V_1}}$ is a constant and (10) becomes

$$Z = Kx_0 \tag{11}$$

As $\frac{x_0}{V_2} = T_0$, (11) can be written in terms of T_0 instead of x_0,

$$Z = KV_2T_0 = kT_0 \tag{12}$$

For the case where point, a, is at an elevation of $+E$ algebraically above point, b, (11) and (12) become,

$$Z = Kx_0 + \frac{E}{2} \text{ and } Z = kT_0 + \frac{E}{2} \tag{13}$$

The error in the determination of the depth, Z, varies directly with the error of the determination of x_0 and with any uncorrected values for difference of elevation between the shot point and seismograph stations. As K commonly is about 0.3, the value of x_0 must be determined with an error of less than three times the desired limit of error in the determination of the depth, Z. If the topographic relief exceeds twice the desired limit of error in the determination of Z, care must be taken to place the shot point and stations at approximately the same elevation or in the plotting of the time-distance curves corrections must be applied to reduce the observed time and distance for each shot to some standard elevation.

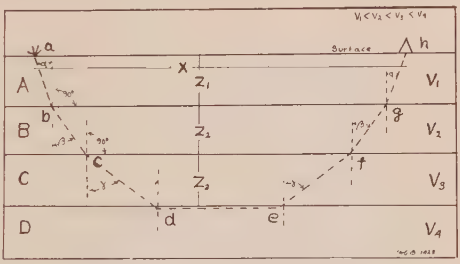

FIG. 5.—PATH OF WAVE IN REFRACTION METHOD IN A SERIES OF HORIZONTAL STRATA OF SUCCESSIVELY HIGHER SPEED DOWNWARD.

If several horizontal beds, A, B, C, D, are present, Fig. 5, and if their respective velocities, V_1, V_2, V_3, V_4, have the relation $V_1 < V_2 < V_3 < V_4$:

$$\left.\begin{aligned} \sin \gamma_1 &= \frac{V_3}{V_4} \\ \frac{\sin \beta_2}{\sin \gamma_1} = \frac{V_2}{V_3} \text{ and } \sin \beta_2 &= \frac{V_2}{V_4} \\ \frac{\sin \alpha_3}{\sin \beta_2} = \frac{V_1}{V_2} \text{ and } \sin \alpha_3 &= \frac{V_1}{V_4} \end{aligned}\right\} \tag{14}$$

for the path a-b-c-d-e-f-g-h; but for the path a-b'-c'-f'-g'-h:

$$\frac{\sin \beta_1}{1} = \frac{V_2}{V_3} \text{ and } \sin \alpha_2 = \frac{V_1}{V_3} \tag{15}$$

The equation of the travel time T_4 of the wave traveling path *a-b-c-d-e-f-g-h* may be derived in a similar manner to (7) and is,

$$T_4 = \frac{2Z_1 \cos \alpha_3}{V_1} + \frac{2Z_2 \cos \beta_2}{V_2} + \frac{2Z_3 \cos \gamma_1}{V_3} + \frac{x}{V_4} \tag{16}$$

and for the travel time, T_3, for the wave traveling path, $a\text{-}b'\text{-}c'\text{-}f'\text{-}g'\text{-}h$,

$$T_3 = \frac{2Z_1 \cos \alpha_2}{V_1} + \frac{2Z_2 \cos \beta_1}{V_2} + \frac{x}{V_3} \tag{17}$$

The angles α and β of equation (17) are not exactly the same respectively as the angles α and β of equation (16).

The depths Z_C and Z_D, respectively, to the tops of beds C and D can be written simply in terms of V_1, V_2, V_3 and V_4, the abscissas, x_1, x_2, x_3 of the respective points of intersection of the time-distance curves (7) and (17), and (17) and (16), for any area where V_1, V_2, and V_3 remain constant.

From (17), (16), (7) and (11),

$$Z_3 = \frac{x_3}{2}\sqrt{\frac{V_4 - V_3}{V_4 + V_3}} + \frac{V_3 Z_1}{V_1 \cos \gamma_1}(\cos \alpha'' - \cos \alpha''') + \frac{V_3 Z_2}{V_2 \cos \gamma_1}(\cos \beta' - \cos \beta'') \tag{18}$$

$$Z_2 = \frac{x_2}{2}\sqrt{\frac{V_3 - V_2}{V_3 + V_2}} + \frac{V_2 Z_1}{V_1 \cos \beta^1}(\cos \alpha' - \cos \alpha'') \tag{19}$$

$$Z_1 = \frac{x_1}{2}\sqrt{\frac{V_2 - V_1}{V_2 + V_1}} = K_1 x_1 \tag{20}$$

The following expressions are constants and may be written:

$$\frac{\cos \alpha'' - \cos \alpha'''}{\sin \alpha'' \cos \gamma'} = A_2, \quad \frac{\cos \alpha' - \cos \alpha''}{\sin \alpha' \cos \beta'} = A_1$$

$$\frac{\cos \beta' - \cos \beta''}{\sin \beta' \cos \gamma'} = B_1, \quad \frac{1}{2}\sqrt{\frac{V_2 - V_1}{V_2 + V_1}} = K_1$$

$$\frac{1}{2}\sqrt{\frac{V_3 - V_2}{V_3 + V_2}} = K_2, \quad \frac{1}{2}\sqrt{\frac{V_4 - V_3}{V_4 + V_3}} = K_3$$

Adding (18), (19) and (20) and substituting the values of Z_1 and Z_2 from (19) and (20),

$$Z_D = (1 + A_1 + A_2 + A_1B_1)\, K_1x_1 + (1 + B_1)\, K_2x_2 + K_3x_3 \tag{21}$$
$$Z_C = (1 + A_1)\, K_1x_1 + K_2x_2 \tag{22}$$

or as the coefficients of x_1, x_2, x_3 are constants,

$$Z_D = M_2x_1 + N_2x_2 + K_3x_3 \tag{21a}$$
$$Z_C = M_1x_1 + K_2x_2 \tag{22a}$$

If the lower high-speed bed, V_2, is discontinuous as in Fig. 6, the travel-time equation of the refracted wave is given by equation (7) up

to the point x_n, but for the next point, $x_n + \Delta x$, and points to the right, the refracted wave has no high-speed bed through which to travel for the added distance Δx, but must be cut diagonally through the low-speed bed with the following equation for the travel time:

$$T_2^1 = \frac{Z \cos \alpha}{V_1} + \frac{s}{V_2} + \frac{\sqrt{(x - s)^2 + Z^2}}{V_1} \tag{23}$$

The point, x_n, is the abscissa of the point of intersection of (7) and (23) and can be read from the time-distance graph. Then:

$$s = x_n - Z \tan \alpha \tag{24}$$

If the face of bed B is gently inclined or if the corner is rounded, equation (23) holds only approximately; there may be difficulty in determining the point x_n and s can be determined only approximately.

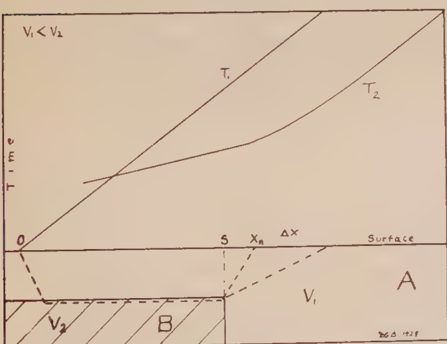

FIG. 6.—TIME-DISTANCE CURVES AND PATH OF REFRACTED WAVE WHERE THE HORIZONTAL BURIED HIGH-SPEED BED IS CUT OFF BY A VERTICAL SCARP.

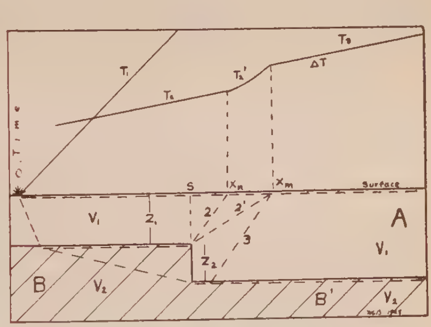

FIG. 7.—TIME-DISTANCE CURVES AND PATHS OF REFRACTED WAVES, WHERE THE BURIED HORIZONTAL HIGH-SPEED BED IS CUT BY A VERTICAL FAULT.

If bed B has been faulted down on the right (Fig. 7), with a displacement of Z_2, there is a point x_m beyond which wave 3 traveling through the upthrown mass and refracted back to the surface from the top of the downthrown mass arrives earlier than wave 2′ with travel time given by equation (23). The travel-time equation of wave 3 is:

$$T_3 = \frac{2Z_1 \cos i}{V_1} + \frac{\sqrt{s^2 + Z_2^2}}{V_2} + \frac{x - s}{V_2} + \frac{Z_2 \cos i}{V_1} \tag{25}$$

The throw, Z_2, commonly is small compared to the distance, s, and $\sqrt{s^2 + Z_2^2}$ may be replaced by s.

The amount of the throw may be expressed in terms of V_1, V_2 and ΔT, where ΔT is the mean difference in time between the line T_3' representing equation (25) and the prolongation of T_2, representing equation (7), and also in terms of Z_1, s, i and x_m, where $\sin i = \frac{V_1}{V_2}$ and x_m is the

abscissa of the point of intersection of T_2' and T_3 (equations 23 and 25). Subtracting equation (7) from equation (25):

$$\Delta T = T_3 - T_2 = \frac{Z_2 \cos i}{V_1}$$

and

$$Z_2 = \frac{V_1 V_2 \cdot \Delta T}{\sqrt{V_2^2 - V_1^2}} \text{ or } V_2\, \Delta T \tan i \tag{26}$$

Or from equations (23) and (25):

$$\frac{Z_1 \cos i}{V_1} + \frac{x - s}{V_2} - \frac{\sqrt{(x - s)^2 + Z_1^2}}{V_1} + \frac{Z_2 \cos i}{V_1} = 0$$

and

$$Z_2 = \frac{\sqrt{(x - s)^2 + Z_1^2}}{\cos i} - (x - s) \tan i - Z_1 \tag{27}$$

The preceding formulas are based on the assumption that the formations are horizontal or parallel to the surface. If the formations have an

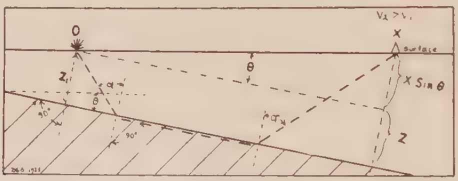

FIG. 8.—PATH OF WAVE IN REFRACTION METHOD, WHERE THE SURFACE OF THE BURIED HIGH-SPEED BED IS DIPPING.

appreciable dip, those formulas do not hold. The travel-time equation for the refracted wave represented in Fig. 8, can be formed in the same manner as (7):

$$\begin{aligned} T_2 &= \frac{Z \cos \alpha}{V_1} + \frac{(Z + x \sin \theta) \cos \alpha}{V_1} + \frac{x \cos \theta}{V_2} \\ &= \frac{2Z \cos \alpha}{V_1} + \frac{x}{V_1} (\sin \theta \cos \alpha + \cos \theta \sin \alpha) \\ &= \frac{2Z \cos \alpha}{V_1} + \frac{x}{V_1} \sin (\theta + \alpha) \end{aligned} \tag{28}$$

Formula 28 is based on the assumption that point O in Fig. 8 remains fixed and point x moves down the dip, *i. e.*, that the profile is shot down the dip. If point x moves updip from an O', that is, if the profile is shot up the dip, then

$$T_2' = \frac{Z' \cos \alpha}{V_1} + \frac{(Z' - x \sin \theta) \cos \alpha}{V_1} + \frac{x \cos \theta}{V_2}$$

$$T_2' = \frac{2Z' \cos \alpha}{V_1} + \frac{x}{V_1} (\sin \alpha \cos \theta - \cos \alpha \sin \theta)$$

$$= \frac{2Z' \cos \alpha}{V_1} + \frac{x}{V_1} \sin (\alpha - \theta) \tag{28a}$$

The slopes of (28) and (28a) are respectively,

$$\frac{dT_2}{dx} = \frac{\sin(\alpha + \theta)}{V_1}; \quad \frac{dT_2'}{dx} = \frac{\sin(\alpha - \theta)}{V_1} \tag{29, 29a}$$

V_1 can be read from (5), α and θ can be calculated from (29) and (29a), and V_2 from α and V_1,

$$\sin(\alpha + \theta) = V_1\frac{dT_2}{dx} = V_1S \text{ and } \alpha + \theta = \text{arc sin } V_1S$$

$$\sin(\alpha - \theta) = V_1\frac{dT_2'}{dx} = V_1S' \text{ and } \alpha - \theta = \text{arc sin } V_1S'$$

where $S = \frac{dT_2}{dx}$ = the slope of (28) and $S' = \frac{dT_2'}{dx}$ = the slope of (28a). Hence,

$$\alpha = \frac{\text{arc sin } V_1S + \text{arc sin } V_1S'}{2} \tag{30a}$$

$$\theta = \frac{\text{arc sin } V_1S - \text{arc sin } V_1S'}{2} \tag{30b}$$

And,

$$V_2 = \frac{V_1}{\sin\left(\frac{\text{arc sin } V_1S + \text{arc sin } V_1S'}{2}\right)} \tag{30c}$$

By comparison of equations (28) and (28a) with (7), and by inspection of Fig. 8, it is evident that the slope of (28) is steeper and of (28a) gentler than the slope which (7) would have if bed B were horizontal. If a profile is shot down a dip, the apparent velocity according to (9) is too low and if the profile is shot up a dip, the apparent velocity is too large. If V_2 is not known, the presence of a dip will not be suspected if a profile is shot in one direction only. If V_2 is known, an abnormally high or low value for V_2 as determined from the time-distance graph of a profile indicates the presence of a dip and the necessity of reshooting the profile in the opposite direction.

The depth D at the constant point O, O of the profile can be determined in terms of $\cos i$ where $\sin i = \frac{V_1}{V_2}$, of V_1, of the slope of (28) or (28a), and of x_P, the abscissa of the point of intersection of (5) and of (28) or (28a):

$$\frac{x_P}{V_1} = \frac{2Z\cos\alpha}{V_1} + \left(\frac{\sin\theta\cos\alpha}{V_1} + \frac{\cos\theta}{V_2}\right)x_P$$

$$Z = \frac{x_P(1 - \sin(\alpha + \theta))}{2\cos\alpha} \tag{31}$$

and,

$$D = \frac{x_P(1 - \sin(\alpha + \theta))}{2\cos\alpha\cos\theta} \tag{32}$$

The movement of the elastic earth waves according to the normal law of refraction and reflection which has been assumed in the derivation of the preceding formulas has been challenged on the basis of Schweydar and Reich's determination that the angle of emergence of the waves is nearly vertical. The suggestion has therefore been made by Schweydar and Reich[3] that the wave from the explosion travels vertically downward, sets up oscillations in the buried relatively high-speed bed; that the oscillations are propagated horizontally along its upper surface and excites vertical oscillations in the overlying formation. Such an assumption has not been found necessary in the quantitative work in Texas, Louisiana and Oklahoma and a very high angle of emergence seems probable theoretically, with refraction of the waves under the normal law. The velocity of the relatively thin surficial zone apparently is of the order of 0.85 to 1.3 km. per sec. and of deep high-speed beds 4 to 5½ km. per sec.; the angle of emergence therefore, should have a deviation from the vertical ranging from 10° to 17°, provided that the refracting surfaces are horizontal.

If one or more of the refracting surfaces dips toward the firing point and if its dip is not compensated by a dip in the opposite direction by one or more refracting surfaces across which the wave must pass the emergence of the wave may be vertical under the normal law of refraction. In the case of a shot across a salt dome, as for example on the extreme shot of Fig. 19, the wave cannot have traveled vertically downward to the salt but must have traveled diagonally downward, in many cases at a low angle; the lengths of shots across domes range up to 8 miles and the salt dome may have a diameter of less than 2 miles and project for thousands of feet into homogeneous beds of much less velocity than the salt. The so-called "absorption" manifested on some shots across the center of domes and interpreted by the writer as divergent refraction seems to indicate normal refraction of the wave.

Crooked Drill Holes

Crooked drill holes can be mapped by the electric type of seismograph. If the geophone is dropped down a drill hole to some position, C' (Fig. 9), if shots are set off at A, B, E and D, and if the velocity is uniform in all directions, the horizontal position of the well at that point is given by the relation,

$$\frac{AC'}{C'B} = \frac{T_{AC}}{T_{CB}} \tag{33}$$

[3] W. Schweydar and H. Reich: *Loc. cit.* For a criticism of the suggestion see O. von Schmidt: Angewandte Seismik. (See bibliography at end of this paper.)

and the depth, CC', by the formula,

$$CC' = \frac{\sqrt{T_{AC}^2V^2 - (AC')^2} + \sqrt{T_{CB}^2V^2 - (C'B)^2}}{2} \tag{34}$$

where T_{AC} is the travel time from A to C and T_{BC} is the travel time from B to C.

If several horizontal formations of different velocities make up the section between the surface and the position of the geophone at C, the relation (33) still holds, but the depth formulas are more complicated than (34), and the calculation of the depth is somewhat uncertain within the accuracy desirable. A first approximation to the depth may be obtained by using the apparent depth as determined by an accurate steel line measurement and a series of position determinations by the seismic method from the surface down to the bottom of the hole. If the positions of points A, B, C, D, etc., are determined by the seismic method as (x_1, y_1), (x_2, y_2), . . . $(x_n\ y_n)$, if s_1, s_2, s_3, . . . s_n are the respective horizontal distances from O to A, A to B, B to C, C to D, . . . $(n-1)$ to n, and if $(D_{SL})_{A,B,C\ etc.}$, are the depths of A, B, C, etc., according to the steel line measurement, and D_A the actual depth, then

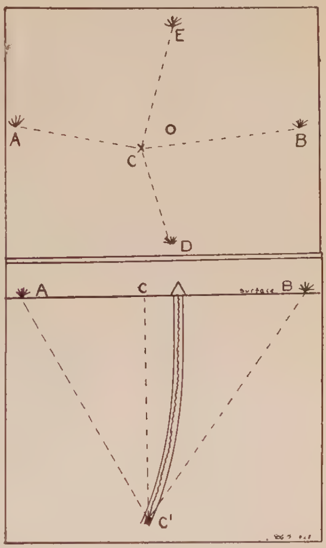

FIG. 9.—MAPPING A CROOKED DRILL HOLE BY THE ELECTRIC TYPE OF SEISMOGRAPH.

$$D_A = \sqrt{(D_{SL})_A{}^2 - S_1{}^2} + \sqrt{(D_{SL})_{A\ to\ B} - S_2{}^2} + \sqrt{(D_{SL})^2{}_{B\ to\ C} - S_3{}^2} + \cdot\cdot\cdot\cdot \tag{35}$$

For more accurate determination of the depth, profiles must be run out from the well in order to give the velocity relations in depth.

If the formations have considerable dip, the seismic method of mapping the depth and position of a bore hole becomes so complicated as to be impracticable.

A great advantage of the seismic method is that if an accurate position determination of the bottom or any other point in the well is desired, the determination can be made for that point without mapping the entire hole down to that point.

Application of Formulas and Curves under Actual Conditions

The formulas and time-distance curves of the preceding pages are in general only approximately valid in actual application in geologic map-

ping. They are based on the assumption that each formation is homogeneous and isotropic and therefore on the assumption that within each formation the wave path is a straight line. The actual situation practically is not so simple. In a region such as the Texas-Louisiana Gulf Coast where there is an enormously thick section of clays and sands, the successively deeper and therefore older beds should tend to show an increasing degree of compaction and therefore an increasing speed of transmission of the seismic earth waves. A wave therefore should undergo continuous refraction and its path should be an arc and not a straight line. The rate of increase downward of the speed of transmission of the seismic waves in some areas may be rather high and the resulting continuous refraction may be sufficient seriously to affect the applicability of the formulas and the time-distance curves of this paper. But in many areas, these formulas and curves may be regarded as fair first approximations, and locally, empirical compensatory constants in some cases can be determined for use with the formulas.

No great thickness of formations furthermore is homogeneous or uniformly changing in character from top to bottom, but there will be greater or lesser variations of inherent character due to the variation of the source of the materials of the sediments, the conditions of their deposition, and alteration of the sediments subsequent to their deposition. The speed of transmission of the seismic waves should therefore vary in a similarly irregular manner. Sufficient horizontal variation in character to affect the speed of transmission of the seismic waves is common in formations. Complex and irregular refraction consequently should arise. Irregularities in the boundary surfaces between formations should cause additional irregularities in refraction. Such irregularities in the speed of transmission of the seismic waves and of their refraction, in the main, can not be allowed for and will tend, as it were, to throw the results of the seismic method out of clear-cut focus.

The anisotropism of sedimentary formations may also produce an appreciable effect. Sedimentary formations are stratified and even although homogeneous commonly show a tendency to the orientation of the constituent particles in reference to the stratification. It is therefore distinctly possible that the elastic earth waves may be propagated differently along, from perpendicularly to, the stratification. Incipient metamorphism produced through the dynamic effect of the upthrust of the salt, or of folding or faulting might also produce a tendency to a differently oriented anisotropism.

The formulas and time-distance curves of this paper are based on the assumption of the simple conditions of homogeneity and isotropism within each formation and sharp plane or smoothly curving contacts between formations or bodies. These simple conditions do not exist in nature. The actual time-distance curves obtained in practical work will

therefore deviate from the simple time-distance curves of this paper to the approximate degree that the actual conditions vary from the simple conditions assumed.

Geological Application

The seismic method of mapping geologic structure is applicable where the velocity of transmission of the elastic earth waves increases downward. A massive limestone or any other high-speed bed screens everything below it, if the underlying formation has a lower speed of transmission of the elastic earth waves. A thin limestone or other high-speed bed tends to reduce but may not effectually eliminate the refraction or reflection of the waves back to the surface. The method in the present state of its technique, therefore, is impracticable in areas such as the caliche-covered area of southern and western Texas or the many parts of southwestern Texas where the Cretaceous limestones are at or near the surface. In the west Texas Permian basin, the top of the salt series should be mapable, but it should be difficult if not impossible effectively to get through the salt and anhydrite to the top of the "Big Lime," the key horizon used by the geologists of the area. But rather fortunately, the velocity tends rather commonly to increase with depth in the areas in which the oil geologist is interested; the increase of velocity with depth is presumably a function of the increasing degree of compaction with increase of the age and depth of burial of the formation.

The greater the contrast between the velocity of the high-speed key bed and the overlying lower speed formation or formations, the more clean-cut are the results with the seismic method. The brilliance of the success of the method in the coastal salt dome area of Texas and Louisiana is due to the sharp exaggerated structural and physical contrast of the pluglike salt masses with a velocity of 5 km. per sec. intruded into weak Tertiary sediments with a velocity of 2 to 4 km. per sec. In the search for oil deposits, those limitations of the necessary velocity-depth relations preclude the use of the seismic method in some areas and limit its use in others, but there would seem to be many areas in which it will be successfully applied to structures other than salt domes. In the search for metalliferous deposits, its applicability would seem to be very limited as few mineral deposits present the necessary conditions of a relatively high-speed orebody of rather definite geometric shape in and overlaid by rocks of definitely lower velocity. A very large number of orebodies are not of large enough size or definite enough geometric shape to produce a usable seismic effect, even though the velocity relations were favorable. But the majority of metalliferous mineral deposits are in country rock of very high velocity or of very complicated velocity relations.

Reflection Method

If the assumption is made that a relatively high-speed bed underlies relatively lower speed overlying rocks, theoretically, by the reflection method, a depth determination is made by each shot and to map the dip, folding, or faulting of the upper surface of the high-speed bed, it is necessary only to scatter "shots" over the area to be mapped and draw structure-contours or profiles from the results. Practically the application of the method is somewhat uncertain and far from as simple as it seems theoretically. Reflections, unexpectedly, unexplainably and not uncommonly, are not obtainable without regard to the size of the charge of the explosion. The wave seemingly must be diffused by an irregular surface, refracted so that it does not get back to the surface or diffracted. The accuracy of a depth determination is not always satisfactory and a sufficient number of shots must be taken approximately at a given point to obtain a satisfactory mean value.

The impossibility of recognizing the reflecting bed is a serious disadvantage of the reflection method in comparison to the refraction method. In the latter method, the velocity, V_2, of the lower bed is determinable and in many places is a sufficient criterion for the identification of the lower, refracting bed. The velocity, V_2, of the reflecting bed is not determinable by the reflection method and it is therefore impossible to identify the reflecting bed from the results of the reflection shooting and beds may be jumped without suspicion of the fact.

Refraction Method

By the refraction method, the presence or absence of any considerable mass of relatively high-speed rock within a certain distance of the surface

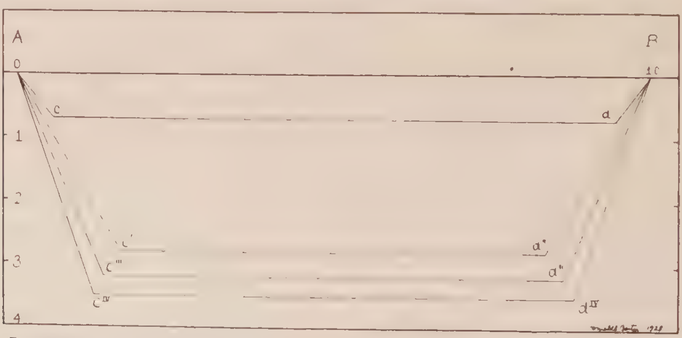

Fig. 10.—Limiting conditions for detection of buried high-speed mass in connection to position and depth in reference to the "shot" length.

may be determined, the position, limits, depth to the top of the mass, and form of the upper surface of the mass may be determined with fair, and in many places, good accuracy.

If no relatively high-speed mass is present within the limiting conditions discussed on page 578, the only waves arriving at the seismograph will have traveled at or near the surface. Conversely, if the seismogram of a "shot" shows the arrival of only a single series of waves, no relatively high-speed mass is present within those limiting conditions of the depth, with, however, the following two important limitations which are illustrated by Fig. 10. In that figure, the paths represent the path of the refracted wave which has the same travel time as the direct wave traveling near the surface, respectively for the velocities of $V_2 = 1.5$, 2.0, 2.5, and 3.5 km. per sec. For the refracted wave to arrive first and be usable, first the high-speed, lower formation must have a length longer than *cd* above the depth *cd* or having a shorter length than *cd* must be enough nearer the surface so that the ratio of its length to depth is greater than the ratio of *cd* to its depth, and second the effective length of the higher speed mass must lie between Ac^{IV} and Bd^{IV}. If the higher speed mass lies below the line, $c^{IV}d^{IV}$, to the left of the line, Ac^{IV}, or to the right of the line Bd^{IV}, it is not detectible by this shot. A blind spot therefore exists immediately under the points of the explosion and of the seismograph.

The diagram of Fig. 10 has been referred to tacitly as a vertical section; it is equally valid as a horizontal plan; strictly it is valid for the plane passing through AB and being perpendicular to the plane of which *cd* is the trace; that is, if a relatively high-speed mass lies to one side of the line AB, the relations of Fig. 10 hold the same as if the high-speed mass were below the line AB; in the former case, the plane of the figure is inclined and in the latter vertical. The space covered by each "shot" is, therefore, a half cylinder whose diameter is dependent on the length of the shot and the velocity of the two formations. If a series of shots is to cover a given area to a given depth, the firing and receiving points must be so placed that the adjacent effective cylinders of the shots overlap or must be placed close enough together, so that the hoped for salt dome or other high-speed mass will have a chance to fall between the cylinders of effect of adjacent shots.

If a higher speed bed underlies the surface bed in such a way as to be detectible on a given shot, the seismogram will show the arrival of a wave in advance of the direct or "surface" wave, and conversely, the presence of such an advanced wave on the seismogram indicates the presence of some relatively high-speed mass within the cylinder of effect of the shot. (See seismogram B of Fig. 27.)

If the higher speed mass is a horizontal stratum of great horizontal extent and if a profile is shot across it, the time-distance graph will consist of two intersecting straight lines, T_1 and T_2, and the slopes of T_1 and T_2 will be the same for all azimuths. (Fig. 12.)

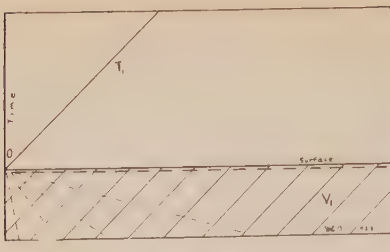

FIG. 11.

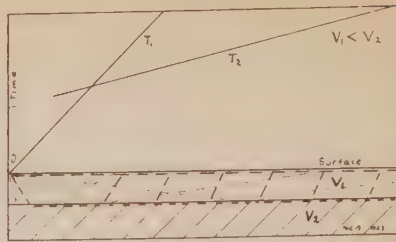

FIG. 12.

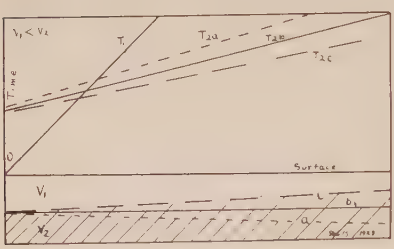

FIG. 13.

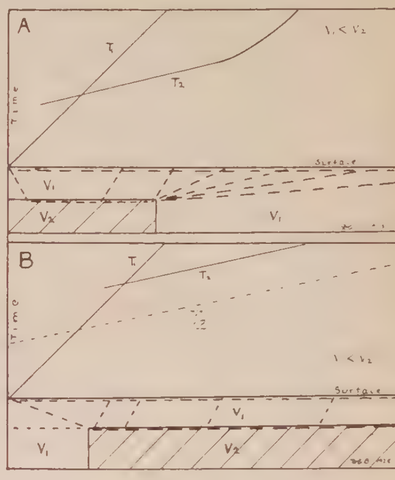

FIG. 14.

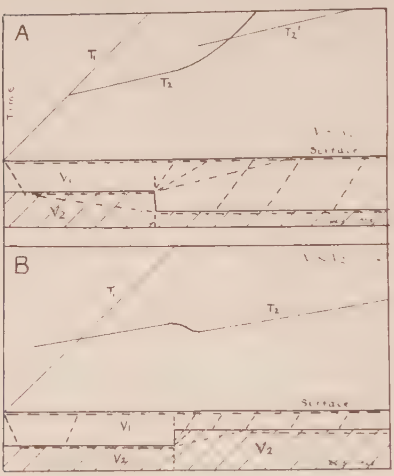

FIG. 15.

FIG. 16.

FIGS. 11–21.—TIME-DISTANCE CURVES AND PATHS OF WAVES FOR A SERIES OF TYPES OF SIMPLE GEOLOGIC STRUCTURE. EACH FORMATION IS ASSUMED TO BE HOMOGENEOUS AND ISOTROPIC.

FIG. 11.—NO BURIED HIGH-SPEED FORMATION PRESENT.

FIG. 12.—AN INFINITELY EXTENSIVE LEVEL FLAT-TOPPED BURIED HIGH-SPEED FORMATION PRESENT.

FIG. 13.—A SLOPING PLANE SURFACE TO THE INFINITELY EXTENSIVE BURIED HIGH-SPEED FORMATION.

If the stratum is dipping at a uniform rate, the time-distance graph will consist of the same two straight lines but the slope of T_2 will vary with the azimuth of the profile and will be at a maximum if the profile is shot down the dip and at a minimum if it is shot up the dip. (Fig. 13.) The angle of dip and the velocity of the lower formation can be calculated by formulas, from the profiles in which T_2 has the maximum and minimum slope respectively, and the direction of the dip is the azimuth of the profile in which T_2 has the minimum slope.

If the stratum is terminated by a scarp (a) on a profile shot from a position over the relatively high-speed mass across the scarp, T_2 is straight up to a critical point beyond the scarp and then bends upward to become asymptotic to a line parallel to T_1 (see A, Fig. 14); (b) on a profile shot in the reverse direction, T_2 is a straight line parallel to but above the position which T_2 would have if the V_2 formation extended to the left under the firing point, and the graph is the same as if the V_2 bed were deeper and extended under the firing point. In case (a), the position can be calculated by formula (24).

If the relatively high-speed stratum is faulted, and is present on both the upthrow and downthrow sides of the fault, T_2 is offset at a critical point in front of the fault, (a) upward if the profile is shot from the upthrown to the downthrown side and (b) downward if the profile is shot in the reverse direction. The position of the fault in the high-speed stratum can be calculated by formula (24) and the throw of the fault by formula (26) (see Fig. 15).

If the face of the scarp of the stratum is gently inclined, T_2 is essentially two straight lines intersecting at a critical point, x_m. The left portion of T_2 is the straight line corresponding to the horizontal surface, ab, and for gentle slopes, the right portion of T_2 is a straight line. If the dip of bc exceeds a certain critical angle, the right portion of T_2 lies above and is therefore obscured by the right portion of T_{2d} of the situation illustrated by Fig. 16. If the profile is shot up the slope of the scarp, as in Fig. 17, T_2 is a broken straight line; the right portion has the slope of the T_2 which would be produced by the surface of the V_2 bed and the left portion has a slope depending on the slope of the scarp. If the face of the scarp is considered to consist of a number of plane faces of successively greater dip with greater depth, and if the number of faces is very great,

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Fig. 14.—The level flat-topped buried high-speed formation cut off by a vertical scarp. A, the profile buried is "shot" from over the high-speed formation across the scarp. B, the profile is "shot" from off the high-speed formation across the scarp.

Fig. 15.—The level flat-topped buried high-speed formation is cut by a vertical fault A. The profile is "shot" from the upthrown side, and B from the downthrown side, across the fault.

Fig. 16.—A level flat-topped buried high-speed formation is terminated by a sloping plane surface on one side and profile is "shot" in the direction of down that slope.

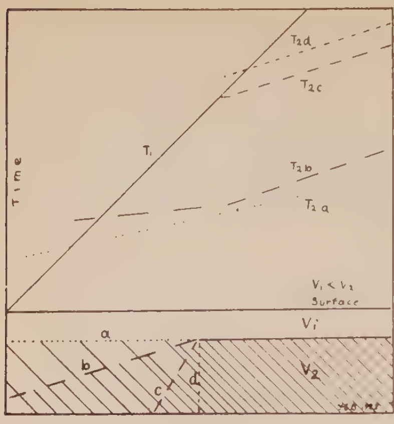

FIG. 17.

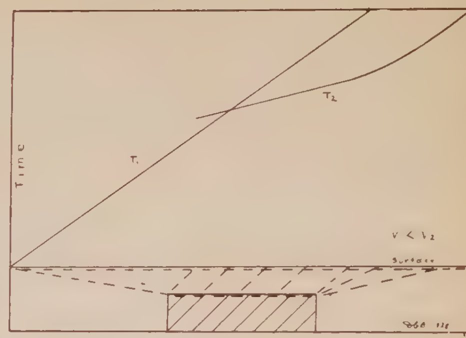

FIG. 20.

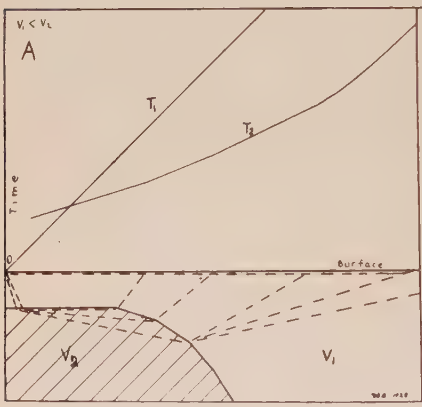

FIG. 18.

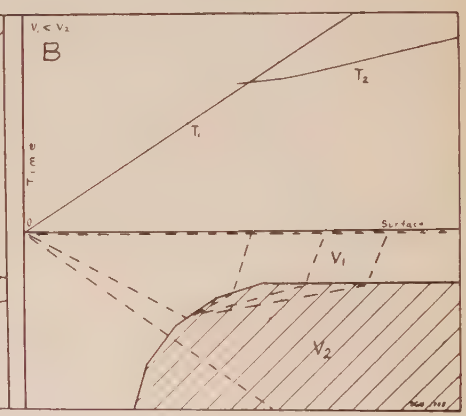

FIG. 19.

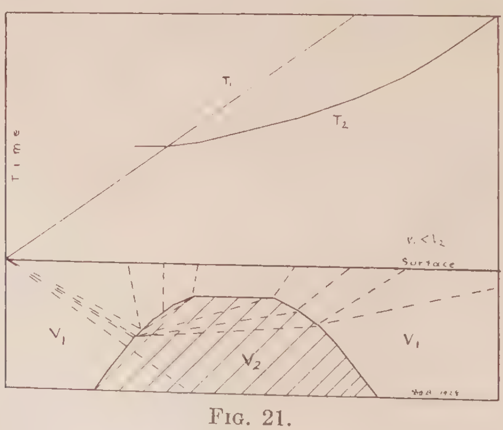

FIG. 21.

FIGS. 17 TO 21.—(Captions on opposite page.)

T_2 for the dotted curved face is the limit of T_2 for an infinite number of such faces and consists of a straight line which goes over into a curved line which finally goes over into a curved line of the type of right portion of T_2 in Fig. 14*A*. If the lower relatively high-speed stratum is a prism at right angles to the profile, T_2 is a combination of T_2 of Figs. 14*B* and and 14*A*, as is shown in Fig. 20.

If a vertical relatively high-speed mass has a cross-section of the general type shown in Fig. 21, T_2 is a curved line gradually becoming asymptotic to a line parallel to T_1 and is composed of the T_2 of both Figs. 18 and 19.

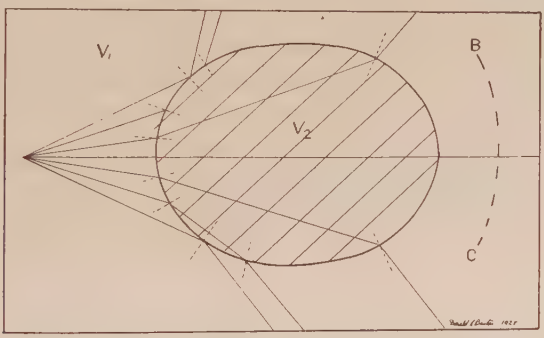

FIG. 22.—MAP SHOWING DIVERGENT REFRACTION OF WAVE PATHS BY A SALT DOME.

If the horizontal cross-section of the high-speed mass is circular or elliptical, the mass acts as a diverging lens and at any point in the sector *BC*, Fig. 22, directly behind the high-speed mass from the point of explosion, *A*, the wave energy getting through to the seismograph is very greatly reduced and on the seismogram the amplitude of the wave is very greatly reduced.[4] A blind spot therefore may exist directly behind the center of the dome from the point of the explosion.

If the top of a vertical high-speed mass is overturned as in Fig. 23, the overturned portion may act as a diverging lens and the amplitude of the waves refracted back to the surface may be very greatly reduced. A blind spot therefore may exist. If the overturning does not extend

[4] In the seismic prospecting on the Texas-Louisiana Gulf Coast, this effect is spoken of as "absorption of the wave."

FIG. 17.—A LEVEL FLAT-TOPPED BURIED HIGH-SPEED FORMATION IS TERMINATED BY A SLOPING PLANE SURFACE ON ONE SIDE AND PROFILE IS "SHOT" IN THE DIRECTION OF UP THAT SLOPE.

FIG. 18.—A LEVEL FLAT-TOPPED BURIED HIGH-SPEED FORMATION IS TERMINATED BY A CONVEXLY CURVING SLOPE ON ONE SIDE AND PROFILE IS "SHOT" IN THE DIRECTION OF DOWN THAT SLOPE.

FIG. 19.—A LEVEL FLAT-TOPPED BURIED HIGH-SPEED FORMATION IS TERMINATED BY A CONVEXLY CURVING SLOPE ON ONE SIDE AND PROFILE IS "SHOT" IN THE DIRECTION OF UP THAT SLOPE.

FIG. 20.—THE BURIED HIGH-SPEED MASS IS A HORIZONTAL RECTILINEAR PRISM.

FIG. 21.—THE BURIED HIGH-SPEED MASS IS THE TOP OF A SALT DOME OR SIMILAR BURIED HIGH-SPEED MASS.

to a great depth and if the points of explosion and of the seismograph are moved farther from the dome, the wave may come through with sufficient energy to be detected. But that will not be the case if the overturning extends to great depth.

A dome-shaped relatively high-speed mass with an overturned crest will have the double probability of an area of shadow behind the dome, an area to which no waves can be gotten through even with very greatly increased charges of explosives.

The so-called "absorption," which is really this divergent refraction, is a rather common phenomenon on shots across the center of a salt dome in the Texas-Louisiana Gulf Coast salt dome area and in a few cases the waveless area of shadow has been picked up. The T_1 wave which does not go through salt commonly is affected as well as the T_2

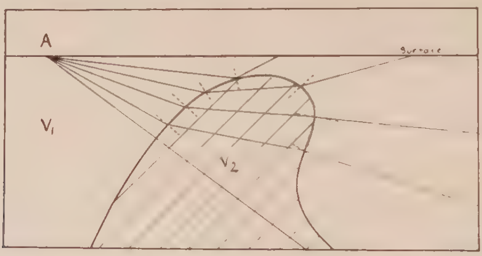

FIG. 23.—VERTICAL DIVERGENT REFRACTION OF WAVE PATHS BY A POSSIBLE SALT DOME.

wave which goes through the salt. T_1 must penetrate some distance into the ground and be affected by doming and (or) compacting of the supersalt sediments. Absorption is sometimes evident on shots across the flank of a dome. Faulting is common on the flanks of salt domes and should lead to irregularities of refraction and diversion of the direction of the wave path which would produce apparent absorption.

Objectives of Seismic Prospecting for Salt Domes

In prospecting salt domes, the seismic method has two different tasks, the one to discover new salt domes, and the other to delimit newly discovered domes or the unknown portions of older domes.

In the exploration for undiscovered domes, much of the work consists of broadside covering of extensive areas by a series of overlapping "fans." In shooting a fan, the receiving units circle along convenient roads at a radial distance of 4 to 8 miles from a central firing point and thus give a fan of shots radiating in all directions from the central firing point. The seismograph stations are so spaced that the cylinders of effect of adjacent shots overlap. The firing point is then moved over 8 to 16 miles and another fan is shot and so on until the desired area has been completely covered. If the adjacent shots of contiguous fans do not

overlap but run to a common boundary road between the fans, one or more tangential shots are made down the boundary line to cover the blind spots under the receiving stations.

If a definite prospect or small area has to be shot, a firing point is chosen slightly off the prospect and a fan of shots is thrown across the prospect or area. A second firing point may be chosen and a second fan of shots may be thrown across at an angle to the shots of the first fan.

If the exploration extends into an area in which the normal seismic reaction of the subsurface is not unknown, one or two profiles are shot in order to determine whether any high-speed or semihigh-speed beds are present, their respective depths, velocities and dip. Over much of the area along the Gulf Coast, a series of so-called "first semihigh-speed beds" rather commonly come in at a depth of 1000 to 2500 ft. with a velocity of 2.6 to 3.2 km. per sec. and a so-called "second semihigh-speed bed" at a depth of 4500 to 8000 ft. and with a velocity of 4.2 to 4.5 km. per sec. These "first and second semihigh-speed beds" are rather indefinite, are not present everywhere, and may not be correlatable from area to area.

If a salt dome lies within a fan, one or two shots probably will be directly across the dome and one or two shots will sidewipe the dome. The shots directly across the dome will show a higher speed wave than normal and may show the so-called absorption or divergent refraction. The sidewipe shots will show a slightly higher speed wave than normal. Theoretically, it is advisable to repeat the shots and to throw a fan across the suspected dome from another angle. But unless the rival scouts can be sidetracked or unless the suspected dome is wholly under land controlled by the company, any apparent halting to reshoot most commonly will bring one or more rival troops on the ground within 48 hr. The attempt therefore ordinarily is made to sneak two or three additional shots across the dome without seeming to be doing any reshooting. The company then blocks the prospecting as quietly as possible and later sends a troop back in to determine whether the relay key stuck on sending out the time of the explosion, whether a stray limestone bed may have been picked up or whether a salt dome is present, and if so, to outline it and determine its depth.

To delimit a newly discovered dome and determine the depth to the top of the cap, to delimit the unknown edge of a known dome, or definitely to establish or disprove a prospect, profiles are shot by the refraction method across the dome or prospect at different angles. From the profiles, the high-speed bed can be identified through its velocity, and if it is the salt, the depth to the top of the cap, in most cases the edge of the dome, in practically all cases, the approximate edge of the dome, and in some cases the slope of the flanks can be determined from the profiles. In detailing a dome, the reflection method is sometimes used.

Mapping structure on the top of the first semihigh-speed bed by means of refraction profiling was practiced considerably in the earlier days of seismic work in the Gulf Coast but fell into disrepute partly on account of certain uncertainties in the results, but very largely on account of the more profitable application of the method to the discovery of new domes. It is coming into use again and much work will probably be done in mapping on the upper surfaces of the beds which are rather loosely designated as the first and second semihigh-speed beds. Considerable mapping of this type was done in East Texas.

In mapping structure not of the salt dome type, the method of rapid reconnaissance by fan shooting ordinarily will not be possible. The fan shooting with long shots is possible only because the high-speed salt core of a salt dome rises thousands of feet above the uppermost beds of approximately the same velocity. Most structures which the oil geologist wishes to map have a structural relief of less than some hundreds of feet. To map such structures, it will be necessary in most cases to run profiles either by the refraction or reflection methods and map the surface of a relatively high-speed bed or beds.

The Instruments

The seismic method of geophysical exploration of geologic structure depends chiefly on the use of the velocity of transmission of the artificially produced earth waves. To determine the velocities of the waves it is necessary to observe accurately (1) the time of arrival of the waves, (2) the time of the explosion causing the waves, and (3) the distance from the explosion to the point of observation.

Seismograph

The types of seismograph with which the bulk of the work of recording the arrival of the artificial earth waves is being done, are two: photographically recording mechanical seismographs, and photographically recording inductive electric seismographs (geophones). A seismograph consists fundamentally of a suspended heavy mass which tends not to move when its support moves under earth shocks, a device to magnify the differential movement of the heavy mass and its support, and a recording device to record those magnified movements.

The Mintrop mechanical seismograph, Fig. 24, has a circa 10-kg. lead sphere as the heavy mass. The sphere is supported by a horizontal leaf spring and carries a conical vertical aluminum arm, the upper end of which turns a thin cylindrical spindle by friction contact. The spindle carries a small mirror. The oscillation of the pendulum is damped magnetically. From the recording apparatus, about a meter away from the seismograph, a spot of light plays on the mirror and is reflected back

and its oscillation is recorded on an unreeling strip of sensitized paper. A magnification of over 1,000,000 times is obtained. The seismograph and recording unit rise about 1 m. off the ground and each is easily carried by one man. The seismograph is set up on the ground and has to be protected from the wind.

The Schweydar seismograph, designed by Professor Schweydar of Berlin and made by the Askania Werke, is a mechanical seismograph

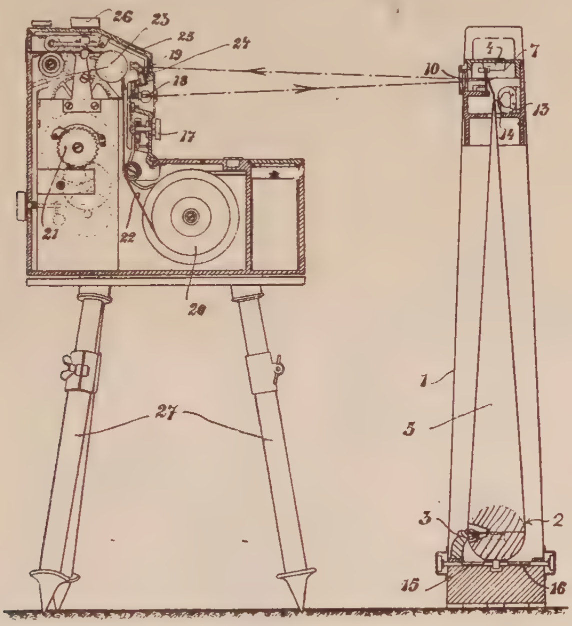

Fig. 24.—Diagram of the Mintrop field seismograph (after Mintrop).

1, case; 2, lead heavy mass; 3, horizontal leaf spring and carrier; 4, level bubble; 5, a light aluminum arm; 7, a friction block bearing against a thin spindle carrying a mirror; 10, a converging lens; 13 and 14, magnetic damping device consisting of an iron arm, 14, playing in the field of the magnet, 13; 18, low amperage D. C. lamp; 19, lens focusing beam of light on a strip of photographic paper, 22, which is drawn from the reel, 20, by the clockwork mechanism of 21; 23, a short pendulum with a short arm which vibrates as a shutter across a second beam of light from 18 to the photographic paper.

registering both the vertical component and a horizontal component. It is said to be rather similar in general design to Mintrop's mechanical type of seismograph. It is the only one of the field seismographs in practical use in which a horizontal component is recorded. It is being used in this country by the Roxana Petroleum Corpn.

Both the Mintrop and Schweydar mechanical seismographs have the disadvantages that they are much more greatly affected by surface tremors and by air vibration than is a geophone at the bottom of a small 3-in. hole and that they are impracticable for use on water or very soft marsh.

Field seismographs of the mechanical type have been designed in this country by Trueman, Ricker, Bazzoni, Taylor and others. The details of their designs are not known, but they are supposed to be similar to the Mintrop seismograph in a general way. The Trueman seismograph, employed exclusively by the Humble Oil & Refining Co., and the Schweydar seismograph are the only mechanical field seismographs other than the Mintrop seismograph, which have had extensive use in geological exploration. The Geophysical Exploration Co. is using a mechanical seismograph.

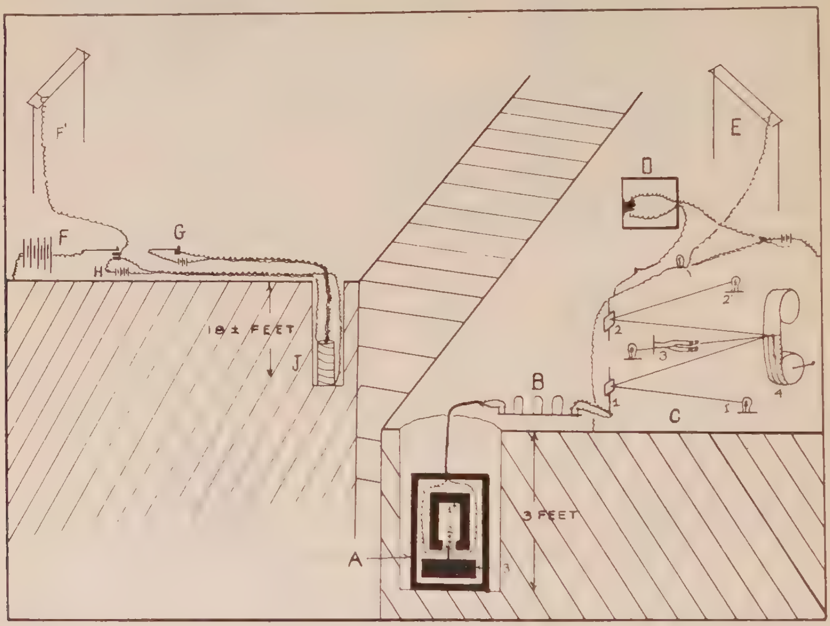

FIG. 25.—DIAGRAMMATIC DRAWING OF AN INDUCTION-TYPE ELECTRIC SEISMOGRAPH. *A*, geophone; 1, permanent magnet; 2, coil of wire playing in the field of the magnet and borne by the heavy mass, 3; *B*, three-stage radio amplifier; *C*, oscillograph, one element handling oscillation of the current from the geophone and two from the radio-receiving set, *E*, and the blastophone, *D*; 3, tuning fork timing device; 4, moving strip of cardiograph photographic paper; *F*, wireless sending and receiving set; *G*, firing key and circuit; *H*, circuit when closed holding wireless sending key down; *J*, dynamite.

The electric seismograph consists fundamentally of a heavy mass whose oscillations differentially compared with its support produce a fluctuating electric current, of an amplifying device, and of a photographically recording galvanometer (oscillograph). Several types of electric seismographs are possible, but those of practical importance at the present time are all of the inductive type except Rieber's.

The inductive type of electric seismographs (Fig. 25) have a heavy mass carrying an attached coil of wire within the field between the poles of a fixed permanent magnet or an attached permanent magnet with a fixed coil of wire within the field of its poles. The oscillations of the coil

of wire across the lines of force of the magnetic field cause an induction of an electric current in the coil. The current can be led off and the fluctuation of its intensity amplified and recorded. The fluctuation of the intensity of the current corresponds to the oscillations of the heavy mass. The electric seismographs are not uncommonly called geophones or seismicrophones.

Amplification of the fluctuations of the very faint induced current is obtained by the use of a one, two, three or four-stage amplifier similar in general principle to the familiar amplifier of radio sets. The amplification is under the control of the observer and ranges from a few hundred times to several million times. An upper limit is given to the practicable magnification by the ground tremors; the amplification is kept low enough

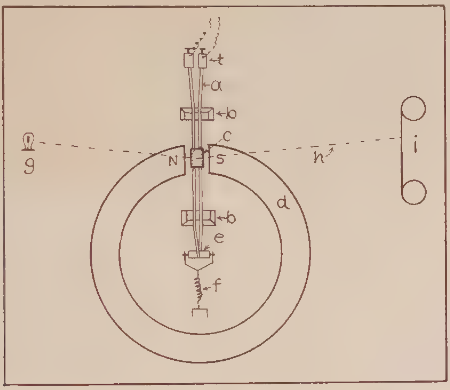

Fig. 26.—Diagrammatic sketch of an oscillograph (partly after Westinghouse Electric Co.).

a, very thin narrow silver-alloy ribbon; *b*, ivory bridge; *c*, mirror; *d*, magnet; *e*, equalizing pulley; *f*, tension spring; *t*, terminals; *h*, beam of light from lamp, *g*, to moving strip of photographic paper, *i*.

more or less to smooth out the oscillations due to the ground tremors, and a sufficient charge of explosive is used to produce an earth shock which will be appreciable with that amplification.

The recording device is a two, three, four, or five-element oscillograph (Fig. 26) which is essentially a photographically recording galvanometer. The fluctuations in the intensity of the electric current run through an element of the galvanometer causes a corresponding oscillatory rotation of a small mirror which reflects a spot of light to an unreeling sensitized strip of photographic paper. One or more geophones may be connected to an oscillograph and record simultaneously on the same strip. Each geophone is connected to a separate element of the oscillograph. One element is connected to a wireless receiving set and records the time of the explosion. A blastophone, which picks up the air wave of the explosion, may be connected to another element or cut in by the observer on the line from the wireless receiving set after the time of the explosion has

been received. The Geophysical Research Corpn., McCollom Exploration Co., the Humble Oil & Refining Co., The Texas Co., and Mintrop are all using electric seismographs which are understood to be of the inductive type.

Several additional types of seismographs have been tried out in the work of mapping geologic structure, but none of them are in considerable use at the present time in that work in this country.

The Wood-Anderson torsion seismograph has a small heavy mass attached to one side of the middle of a metal torsion ribbon. The heavy mass is very easily set to swinging by any movement in the plane of the ribbon and perpendicular to its longer axis. The oscillations can very easily be watched or recorded by means of a beam of light thrown to and reflected back from a mirror on the face of the heavy mass.[5]

Several types[6] of electric seismographs are possible in which the differential movement of the heavy mass and its support modulates an electric current. In the carbon granule type, the heavy mass presses against carbon granules through which a current is flowing; modulation of the current is produced by variation of the pressure. The current is led off directly to an oscillograph or to an amplifier and then to the oscillograph. Very great amplification is not possible with the carbon granule seismographs on account of the "frying" associated with the passage of the current from granule to granule. The earlier models of of McCollom seismograph according to report were of the carbon granule type. The heavy mass in another type of seismograph produces changes of capacity in an adjacent circuit.

Electric Type Field Seismograph

All the field seismographs of the electric type in commercial use are designed to record only the vertical component of the earth, although units have been designed and used experimentally to record a horizontal component.

A field seismograph unit of the electric type is extremely portable. The amplifying and recording apparatus, the accessory sending and receiving wireless set, and the accompanying batteries of a Geophysical Research Corpn. receiving unit are mounted on a ½-ton Chevrolet truck or on a boat. The geophone is cylindrical, about 12 in. long and 3 in. dia. and is placed at the end of a cable. To set up a station a 3-in. hole is dug 2 or 3 ft. deep, the geophone cable is reeled out, the geophone is dropped down the hole, one or two bamboo poles carrying the wireless antenna are stuck up, and the unit is ready to receive a shot. The station

[5] J. A. Anderson and H. O. Wood: A Torsion Seismometer. *Jnl.* Optical Soc. of America (1924) **8**, No. 6, 817.

[6] R. Ambronn: Methoden der Angewandten Geophysik, 170. (See bibliography at end of this paper.)

can be set up ready for operation within 20 min. and can be packed and ready to move in 5 min. Over 18 stations, all at different places, have been occupied by one receiving unit in one day. On water, the geophone is dropped overboard and set upright on the bottom. In mapping the crookedness of oil wells, the geophone is lowered several thousand feet down the hole. The number of stations that can be occupied in a day practically is limited more by the time necessary to move and other extraneous factors than by the time necessary to set up a station, take a shot, and pack up to move.

Fessenden Geophone

The Fessenden geophone strictly is not a seismograph and is affected only by the compression-rarefaction wave in water (or other liquid). A diaphragm closes the only open end of a small gas-filled cylindrical case and is pushed respectively in and out by the compression and rarefaction phases of the wave. The oscillation of the diaphragm is made to induce or modulate an electric current in the same ways as with the electric seismographs. The oscillations of the current are amplified and recorded similarly as with the electric seismograph. The Fessenden geophone has to be placed in a hole full of water, or a pail of water set into the ground, or a well or pond, etc., in order to detect the elastic earth waves. The longitudinal elastic waves (compression-rarefaction) in the ground produce similar waves in the water. Transverse waves are impossible in the water, but the transverse waves in the ground produce compression-rarefaction waves in the water. Both types of waves therefore are picked up by the Fessenden geophone.

Radio

Light portable wireless sending and receiving sets of several slightly different types are used for communication between the firing unit and the receiving units, for the instantaneous transmission of the time of the explosion to the receiving units, and, by one consulting seismic company, for firing the explosion from the receiving unit. Wireless telephone is used by some companies and wireless telegraph is used by others.

Blastophone

Air wave receivers, sometimes called blastophones, are used by some companies, to pick up the arrival of the air wave from the explosion. Mechanical seismographs usually register the arrival of the air wave, as, being above the surface of the ground, they are well exposed to the air wave. The geophones, being placed commonly 2 or 3 ft. under water, do not always pick up the air wave with sufficient distinctness. A specially designed air wave receiver is, therefore, placed above ground

in association with the geophone. It consists essentially of a small box with its only open side closed by a diaphragm, whose oscillations modulate or induce an electric current similarly as in the electric seismographs. The current is run through an amplifier or directly to the oscillograph.

Lesser accessory apparatus used by some companies comprises anemometers to measure the wind velocity, wind vanes, thermometers, and in some cases other apparatus for determining the wind velocity correction.

Field Practice in the Texas-Louisiana Gulf Coast

The seismic exploration in the Texas-Louisiana salt dome district is done by parties, rather commonly called troops, whose organization varies with the company and with the character of the work. For work with the mirage method, a troop rather commonly consists of one firing unit, two, three, or four receiving units, a squad of hole diggers, a chief of party, a "landman," a calculator, and in some cases a crew of surveyors, and in some a hole-filling crew. The personnel of the party consists of a chief of party, two, three, or four observers, and a calculator, all of whom are technical men, a firing master, a landman, a straw boss, in charge of the hole-digging squad, assistants, who are young technically trained men, in some cases a surveyor and his assistant, and laborers, and unofficially, often a scout or two from a rival company.

Constitution of Seismic Troop

The firing unit under the charge of the firing master has the duty of placing and firing the charge. It is equipped with the necessary apparatus to fire the charge and with many companies, it is also equipped with meteorological apparatus and with a sending and receiving wireless set, which is used to communicate with the receiving units and to send out the instant of the explosion. The wireless set, the batteries for firing the charge, and the firing key may be permanently mounted on a Ford or Chevrolet or on board boat.

The receiving unit under the charge of an observer and an assistant is equipped with a seismograph, and with many companies, a wireless sending and receiving set, and meteorological apparatus. The amplifying and recording apparatus of the Geophysical Research Corpn. has the wireless set and the necessary batteries permanently mounted on a ½ or ¾-ton Ford or Chevrolet truck or on a boat. The geophone is on a cable which can be reeled out. To set up a station, a 3-in. hole is dug to a depth of 3 ft.; the geophone cable is reeled out and the geophone is dropped down the hole; a pair of bamboo poles are stuck up, the antenna wire is reeled out and stretched between the poles, and the unit is ready to receive a shot. The seismographs and wireless sets of most

of the other companies are not permanently mounted on automobiles but are set up in tents at each station.

The chief of party usually has the degree of Ph. D. in geology, physics, or electrical engineering or is a man of high training and experience. He runs the troop and interprets the results of their shooting. He reports to a division or chief geologist.

The calculator is a man with mathematical training who acts as assistant to the chief of party in working up the seismograms.

The landman obtains the shooting permits from the land owners for the tracts on which the explosions are set off, bargains with the landowner for a release from damages from the shooting and (or) the necessity of filling the explosion craters, makes arrangements for filling the holes, settles the lesser claims for damages from the explosions, and in some cases acts as general utility assistant to the chief of party.

A scout from a rival company not uncommonly is set to watch the troop and to report their activity to his company and especially to report anything to indicate that possibly they may have picked up a salt dome. He often gets to be on good terms with the troop, but at critical moments they go to all sorts of strategy to outwit him.

Program of Work

A characteristic program for a day's work is as follows. The day's work has been planned with great care. Firing permits, which are temporary permits from the landowner allowing the setting off of the explosions on his land, have been obtained for certain designated locations. The amount of dynamite necessary has been estimated and has been delivered or is being delivered at the firing location. If the practice of the company is to fire the charge down a hole, the hole-digging squad has dug the holes at the first firing location. The position of passable roads has been determined. The movement of the receiving units has been completely planned and the successive stations for each have been designated.

In the morning, the firing unit and the receiving units move to their respective stations. The firing unit plants the charge, connects up the firing apparatus, sets up its wireless antenna, and signals over the wireless, which is ready. Each receiving unit takes its station, usually at the side of a road, plants its geophone, sets up its wireless antenna, and signals that tentatively it is ready to receive. When all have signaled their readiness, each receiving unit signals again when the road is clear of traffic. When all the receiving units have signaled their O. K. for firing, the firing master sets his wireless sending out a continuous wave note, signals that he is preparing to fire, waits a standard short interval and then fires the charge. The wireless key is held down by a circuit which

goes around the dynamite. The explosion instantaneously breaks the circuit, causes the release of the wireless key and the instantaneous cutting off of the wireless note. The receiving unit has been picking up and recording the sustained wireless note which is abruptly cut off at the instant of the explosion. When the record has been completed, the receiving units then pack up and move on to their next station. The firing unit plants the charge in another hole and prepares to fire again, or may move to another firing location. If it moves, the hole-digging squad has dug the requisite number of holes and moved out before the arrival of the firing unit.

The practice just described is characteristic of one of the consulting companies. Another consulting company fires at a prearranged time and either stations its receiving units off the road or attempts to flag and hold up the traffic at the time of firing. Another company is trying out a method of firing the charge by wireless. One company has an automatic developing device and therefore the observer knows immediately in a general way the results of the shot. Other companies take the records into the local headquarters to be developed. The explosive used is 60 per cent. dynamite.

The size of the charge varies with the type of seismograph, local conditions, and the type of the work. The average charges used by the Geophysical Research Corpn. on shots of 2 to 7 miles range from 40 to 250 lb. For the same shots, Seismos Gesellschaft would use two to three times as large a charge. The size of the charge varies somewhat as the length of the shot but is much more affected by the character of the soil. Dry loose sand absorbs the energy of the explosion to a high degree; in some places where the Lissie or Wilcox sands are at the surface, the maximum charges have to be used and the length of the shots shortened in order to get detectible waves through to the seismographs. Clayey and wet soils transmit the energy of the explosion well and in them a minimum charge can be used. In an exceptionally favorable terrane in the area of the Beaumont clay, better records have been obtained by an electrical seismograph from charges of 0.8 lb. dynamite than have been obtained by the same instruments with charges of 200 lb. in certain areas of the Lissie. If a strong wind is blowing, especially if the wind direction is from the seismograph station to the firing point, an extra large charge may have to be used at the surface to get the air wave through.

In the reflection type of shooting, the charge is very much smaller than in mirage shooting, as the distances are short in reflection shooting compared with those in refraction shooting. If the seismograph is on the surface of the ground, the charge has to be larger than if the seismograph is below the surface; a seismograph on the surface of the ground picks up more ground and air tremors than a geophone placed slightly below the surface and therefore a larger charge must be used to get the

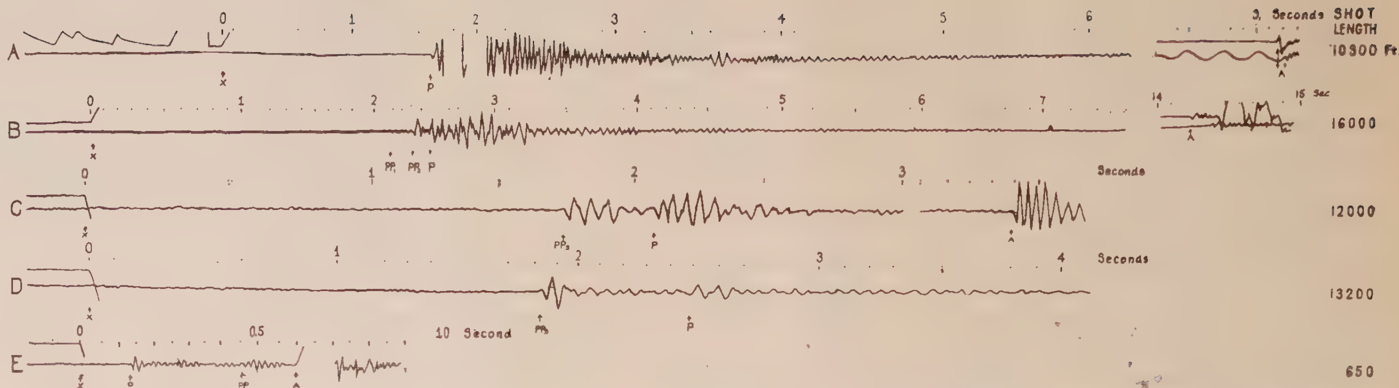

Fig. 27.—Characteristic seismograms: A, no relatively high-speed bed present within the critical depth; B, two semihigh-speed beds present, one at about 2000 ft. and one at 5000 ft. in depth; C, shot across the edge of a salt dome; D, shot across the center of the same dome; E, a seismograph of a reflection shot.

x, instant of the explosion; p, direct wave; pp_1, refracted wave via first semihigh-speed bed; pp_2, refracted wave via second high-speed bed; pp_s, refracted wave via the salt; A, air wave; pp, reflected wave.

wave through with sufficient amplitude to show distinctly above the tremors. The minimum charge that might be used in a given situation ordinarily is not used but rather a charge of sufficient power to put the wave through with sufficient factor of safety for the distinctness of its impact in the seismogram.

The practice in placing the charge varies from company to company. One consulting company places its main charge 17 to 25 ft. down a 6-in. hole and the auxiliary charge at the surface. The latter is used to produce an air wave. The holes are dug with hand augers. If a very large charge is to be fired, a cavity may be produced by firing one or two sticks of dynamite at the bottom of the hole. Other companies

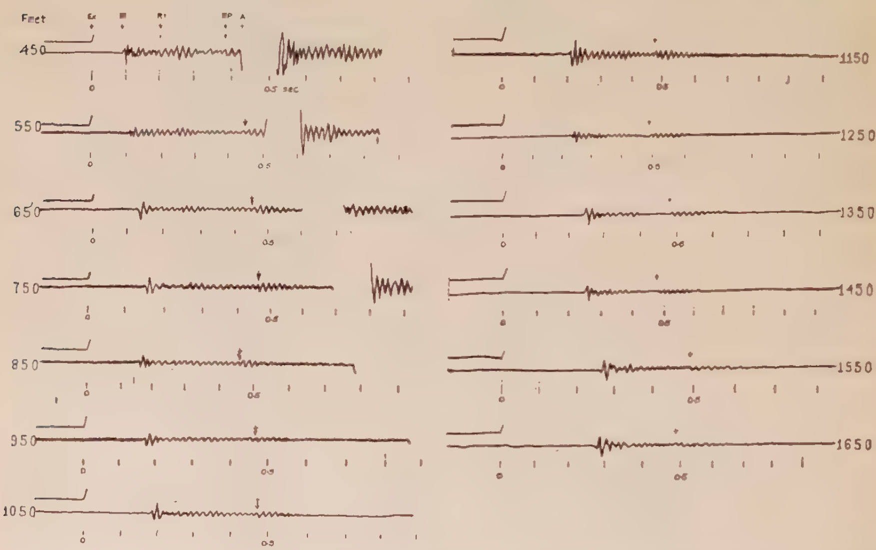

FIG. 28.—TRACINGS OF THE SEISMOGRAPHS OF A PROFILE BY THE REFLECTION METHOD.

place their charge on the surface or in the craters of previous explosions.

The shot length, that is the distance between the firing point and the seismograph, varies with the type of work and the local situation. In the reflection method, the shot length is a function of the depth and the critical angle for reflection and ranges from 1.2 to 1.8 times the depth of the formation which it is desired to map or to avoid and the respective velocities. In the refraction method, it always is more than 3 to 5 times the depth of the formation which it is desired to map and is less than 3 to 5 times the depth of any lower high-speed stratum which is to be avoided. On account of certain theoretical reasons and of certain practical limitations to the technique of the seismos method, the shot lengths were restricted to less than 3½ miles in the earlier work on the Texas-Louisiana Gulf Coast areas. The shot length now commonly used

in reconnaissance for salt domes ranges from 5 to 7 miles along the coast of Texas and Louisiana, slightly less along the Mississippi coast, and about 3½ to 5 miles in east Texas. Where a formation having a

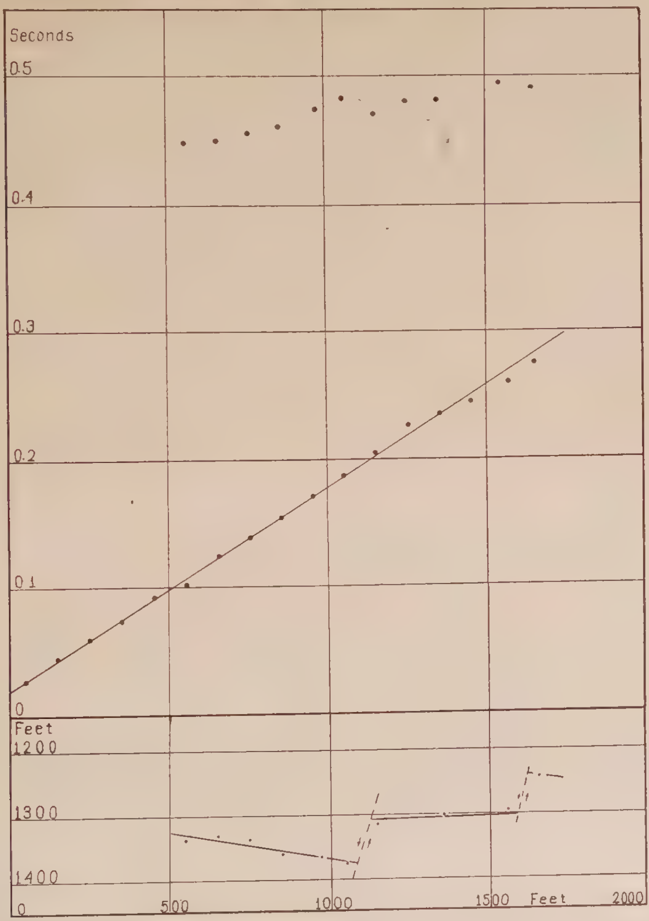

FIG. 29.—TIME-DISTANCE GRAPH AND THE STRUCTURAL SECTION CALCULATED FROM IT FOR THE REFLECTION PROFILE OF FIG. 28.

velocity almost that of rock salt is present, the shot length has to be reduced to less than 3 to 5 times the depth of that formation in order to eliminate its effects.

SEISMIC WORK IN THE GULF COASTAL PLAIN

The work of the seismic method in the Gulf Coastal Plain area[7] is illustrated by Figs. 27 to 37. A set of characteristic seismograms is

[7] Examples of seismic work in the Gulf Coastal Plain are presented in this paper through the courtesy of colleagues, who request that no acknowledgment be made and that the locality of the shots be designated no more specifically than "in the Gulf Coastal Plain."

given in Fig. 27. In each seismogram P represents the impact of the direct longitudinal wave which travels essentially at the surface and

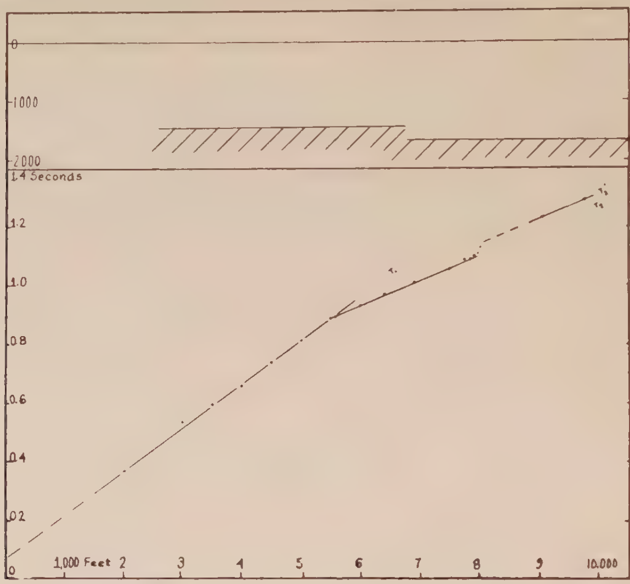

FIG. 30.—PROFILE BY REFRACTION METHOD ACROSS A KNOWN FAULT.

which commonly is spoken of as the "surface wave," PP represents the impact of the longitudinal wave refracted back to the surface by the first semihigh-speed bed and PP_2 by the second semihigh-speed bed, while PP_S is the longitudinal wave refracted back to the surface by the salt core of a salt dome. The seismograph used records only the vertical component; the upper line at the left of each seismogram is the time line and x is the instant of the explosion received via wireless; the zigzags of the time line of A represent—.—— of a code letter from the firing master.

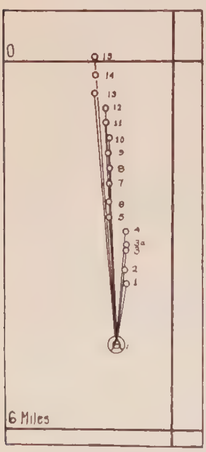

FIG. 31.—MAP OF SHOTS ON A PRELIMINARY PROFILE.

The case where no high or semihigh-speed bed is present within the critical depth for the shot length is represented by A. The case where two semihigh-speed beds are present within the critical depth is represented by B; the first semihigh-speed bed is at a depth of 2100 ft. (680 m.) and has a velocity of 3.1 km. per sec. An edge shot across a salt dome is given by C and a shot across the center of the salt dome by D; the salt is at a depth of 1600 ft. (510 m.); a slight amount of the effects of divergent refraction, the so-called absorption of energy

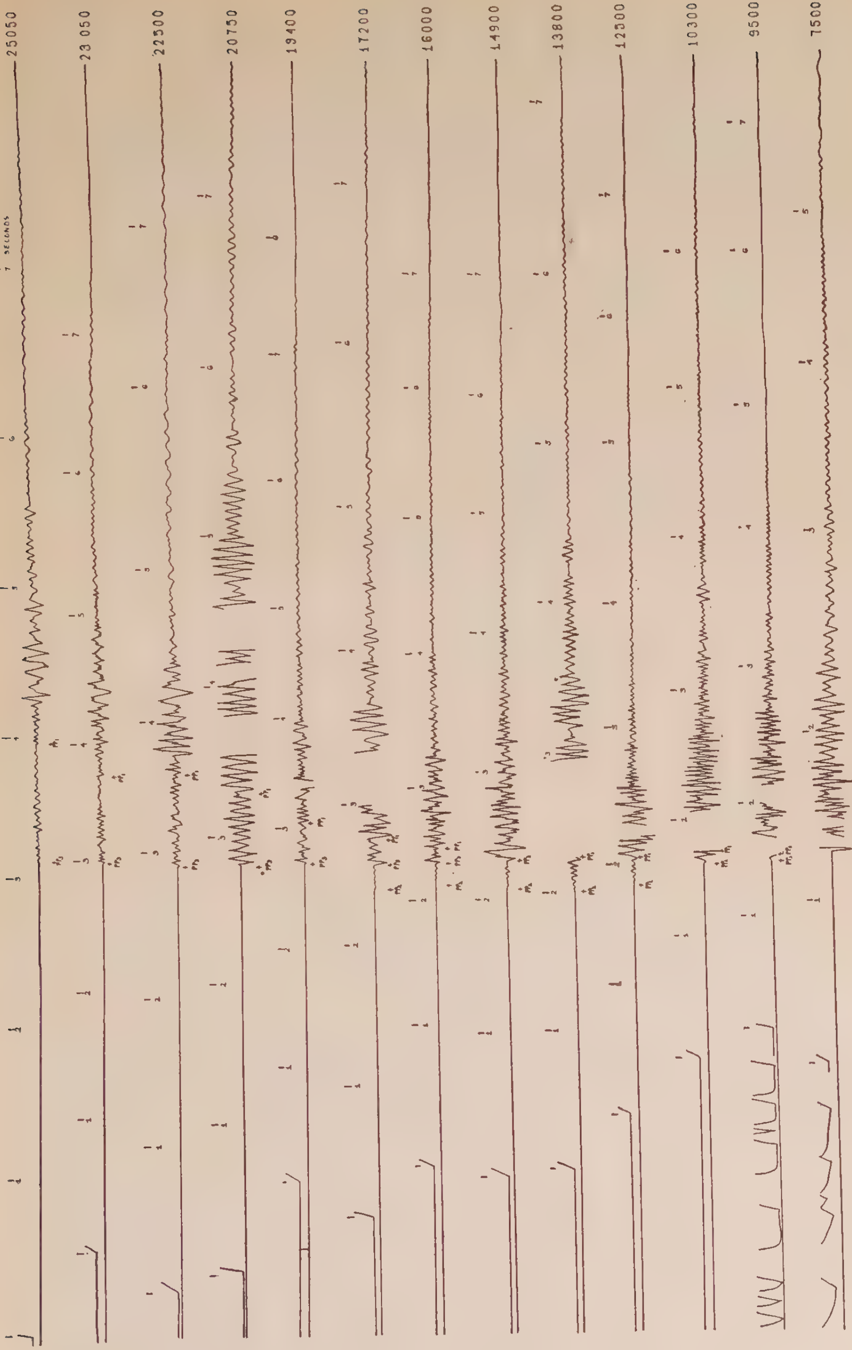

FIG. 32.—TRACINGS OF THE SEISMOGRAMS OF THE PROFILE SHOWN IN FIG. 31.

by the dome, is shown by D. A reflection seismogram is shown by E. The time spacing of the original seismograms from which A to D were traced was $\frac{1}{50}$ sec., and of E $\frac{1}{100}$ second.

A reconnaissance reflection profile of a fault is illustrated by Figs. 28 and 29. The tracings of the seismograms of the profile are given in Fig. 28. The time-distance graph is given in Fig. 29. By the use of formula (3a) the depth to the top of the high-speed stratum was obtained for each shot. As the straight, lower line, the time-distance curve of the "surface" wave, does not go directly through the origin, a thin superficial zone of low velocity apparently must be present and must

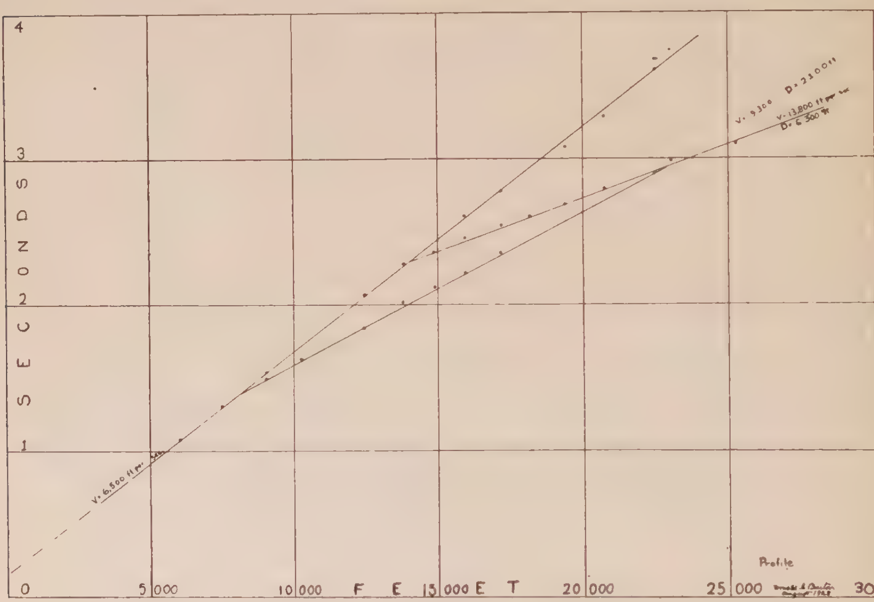

FIG. 33.—TIME-DISTANCE GRAPH FOR PRELIMINARY PROFILE OF FIG. 31.

be allowed for in calculating the depth. The structure profile obtained by the calculations is plotted at the bottom of Fig. 29 and indicates a gentle dip to the right and one and possibly two faults with upthrow on the down dip side. The data on the topography and the geology of this profile are not known to the writer. In the calculations, the assumption was made that the observation points were at the same elevation as the firing point. The general area in which the profile lay is one characterized by a regional dip of 75 to 100 ft. to the mile with many faults, more than half of which are upthrown on the down dip side.

A profile by the refraction method across a known fault is shown in Fig. 30. From the data of the time-distance graph and formulas (10), (29) and (26), the depths of the high surface of the high-speed bed, and

the throw and position of the fault are calculable as 1455 ft. to the surface of the high-speed bed on the upthrow side, throw 232 ft., position of the fault 6700 ± ft. to the right of the firing point. As the observations do not give accurately the break in the T_2 line, the position of the fault cannot be given accurately. The position of the known fault in this case was given within the accuracy corresponding to the spacing of the stations.

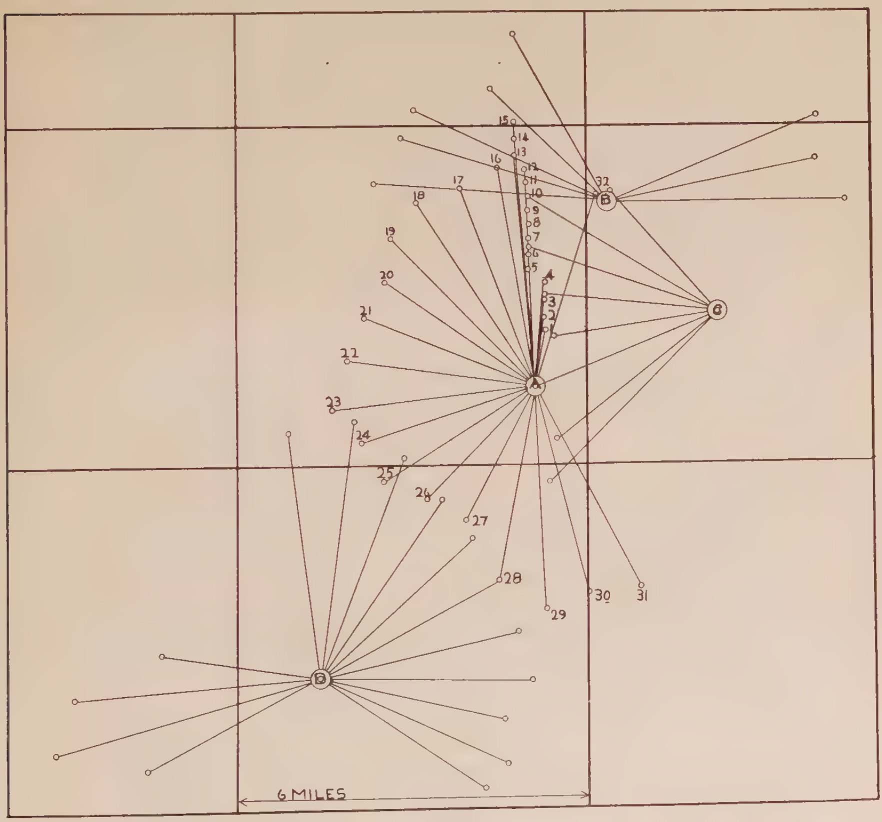

FIG. 34.—MAP OF SHOTS IN A PRELIMINARY PROFILE AND IN SUBSEQUENT RECONNAISSANCE FANS.

A preliminary profile is illustrated by Figs. 31, 32 and 33. A jump had been made into unexplored territory. This profile was run in order to determine the normal seismic make-up of the subsurface in this area. Fig. 31 shows the location of the shots; the observation points presumably were occupied three at a time. Tracings of the seismograms recorded at stations 2 to 9, and 10 to 15, inclusive, are shown in Fig. 32. The time-distance graph is shown in Fig. 33. The time-distance graph

shows the presence of two semihigh-speed beds and a low-speed surface zone and velocities of 2.9 km. per sec. (9300 ft. per sec.) for the first and second semihigh-speed beds, respectively, and 2.0 km. per sec. (6500 ft. per sec.) for the upper part of the section. The velocity of the surficial low-speed zone is not determinable from the graph of Fig. 33, as no sta-

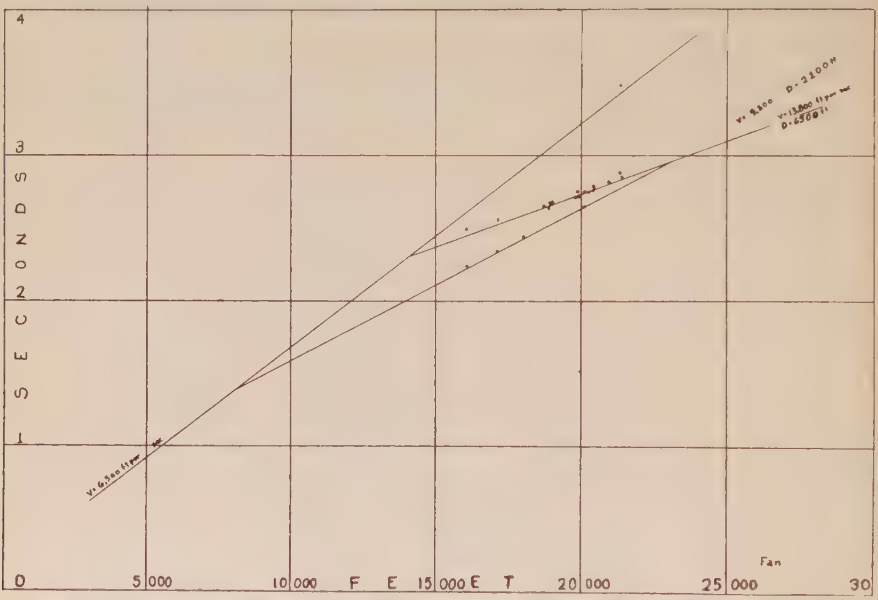

FIG. 35.—TIME-DISTANCE GRAPH FOR FAN FROM FIRING POINT *A* OF FIG. 34.

tion was near enough to the firing point. By the use of equations (7), (10) and (16) and the assumption that the velocity of the upper low-speed zone is two-thirds of the velocity in the formation below, the depths and thickness of the formations are calculable as follows:

Zone	Depth to Top of Zone	Thickness
1.3 km. per sec.	0 ft.	360 ft.
2.0 km. per sec.	360 ft.	1740 ft.
2.9 km. per sec.	2100 ft.	4200 ft.
4.1 km. per sec.	6300 ft.	

The normal reconnaissance by fan shooting is shown by Figs. 34 and 35. The preliminary profile of Fig. 31 was shot from firing point *A* of the map in Fig. 34. After the determination of the seismic make-up of the subsurface by that profile, the fan of positions 16 to 31 was shot from the same firing point, *A*. The time-distance graph for that fan is given by Fig. 35. The first waves to arrive at each station fall on the V = 9300 ft. per sec. and V = 13,800 ft. per sec. lines and therefore within the critical depths no salt dome is present within the area covered

by the fan. Firing points B, C and D were then occupied in succession and the fans shot as shown in Fig. 34. The irregularity in the fans was caused by the presence of marsh and water. Most of the blank area of the map of Fig. 34 was covered subsequently by other fans which are not shown.

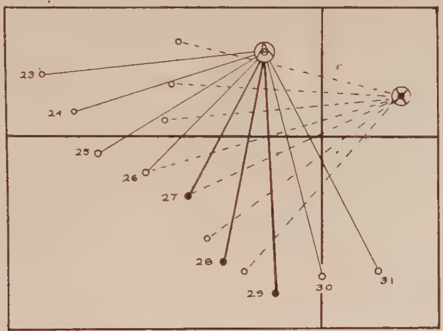

FIG. 36.—SUBSEQUENT FAN TO CHECK THE AREA COVERED BY SHOTS 27, 28 AND 29 OF MAP IN FIG. 34.

If, as was not the case, shots 27, 28 and 29 had a wave distinctly ahead of either or both of the two normal semihigh-speed waves, it would have been desirable then to shoot a corroboratory fan such as that from firing point x in Fig. 36.

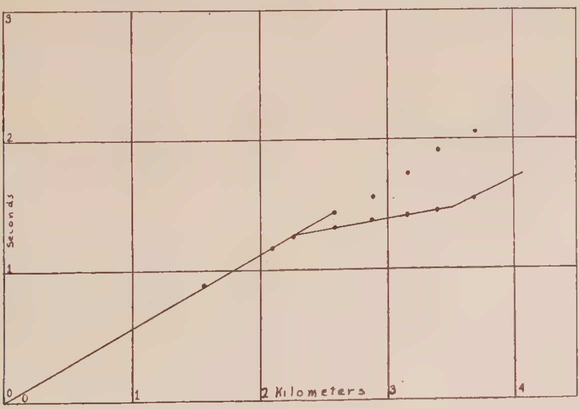

FIG. 37.—TIME-DISTANCE GRAPH ACROSS A SALT DOME.

The time-distance graph for a profile by the refraction method across a salt dome is shown in Fig. 37. Seismograms C and D, Fig. 27, were across the same dome. The apparent depth to the top of the salt and the position of the edge of the salt are calculable by formulas (10) and (29) as respectively 2825 ft. below the surface and 3.25 km. to the right of the shot point.

Results of Seismic Prospecting

The results of the use of the seismic method of prospecting in the salt dome areas of Texas and Louisiana have been, and are continuing to be, most brilliant and in the discovery of new domes, the method is almost clairvoyant in the brilliance of its success. The list of the domes and a few structures discovered by the seismograph are given in Table 3. A tabular comparative historical view of the discovery of salt domes in the area of the coastal domes of southern Louisiana and southeastern Texas and in the area of the interior domes in Texas is given in Table 4. Of the 42 coastal domes (and Goose Creek, Orange and Welsh) of the coastal group already known in 1924, about three-fourths were discovered in the

Table 3.—*Historical Survey of the Discovery of the Salt Domes in Southern Louisiana and Southeast Texas and in Northeast Texas*

Southern Louisiana and Southeast Texas

How Discovered	Proved by Drilling	Not Yet Proved	Discoveries, Information Hazy
By geology and accident:			
Prior to July 1, 1913 (inclusive of Goose Creek and Welsh)	34		
July 1, 1913, to July, 1, 1922 (inclusive of Orange)	6		
July 1, 1922, to July 1, 1924	2		
July 1, 1924, to July 1, 1928		1	
Total	42	1	
By seismic exploration:			
Autumn, 1924	2		
1925 (also 1 Oligocene High)	3		
1926	6	1	
1927	10	5	
1928		10	5
Total	21*	22	5

* Primary credit for the discovery of three of the domes belongs to the torsion balance.

Northeast Texas

How Discovered	Proved by Drilling	Not Yet Proved	Discoveries, Information Hazy
By geology and accident:	4		
Prior to 1910	2		
1910 to 1919	1		
1924			
Total	7		
By seismic exploration in 1927	5	6	

(Three of the untested domes commonly are carried as structural highs.)

five years immediately subsequent to the completion of the Lucas gusher on Spindletop.

Gulf Coast

Immediately preceding the introduction of the seismic method in the Gulf Coast, two domes had been found on the basis of H_2S in water wells, but in general, the Gulf Coast geologists were very pessimistic in regard to the value of surface work in search for salt domes, and the trend of exploration was to shallow drilling with portable rigs to a depth of 1000 ± ft., a slow and costly method of no value for deeply buried domes. The seismic method has to its credit during its four years of application in the Gulf Coast 21 salt domes and one Oligocene high which have been proved by subsequent drilling in addition to 27 + domes yet to be tested. One company during the past year discovered by the seismic method more domes than all companies discovered in the decade before 1924. In the first six months of 1928, more domes have been discovered by the seismic method than were discovered in the decade before 1924.

Interior Texas

In the area of the interior domes of Texas, the results were as brilliant as in the area of the coastal domes. Prior to the introduction of the seismic method, six salt domes were known and of these two had been discovered during the decade before 1927. During the spring and summer of 1927, there were discovered by the seismic method 11 salt domes and structural highs which are probably deeply buried salt domes. Five of the 11 have already been proved to be salt domes and the remainder are untested or being tested.

Efficacy of Seismic Method in Gulf Coast and Interior Texas

Most of the domes discovered by the seismic method in the Gulf Coast area and some of those discovered in the interior would never have been discovered except by accident or by some geophysical method. In the Gulf Coast area, some of the domes were discovered by shooting faintly suspicious prospects, localities where there was something slightly out of the ordinary, but yet not definitely enough indicative of the presence of salt domes to have led to drilling. With increasing keenness of the search for salt domes, and in the absence of more favorable prospects, many of those faintly suspicious localities would have been drilled ultimately and the salt dome discovered. But at many of the domes discovered by the seismic method, there is absolutely nothing at the surface to indicate the presence of a salt dome; the Calcasieu Lake, Cypremort Point, Lake Pelto, Caillou Island, and Lake Barre salt domes, for example, are out in large lakes or in bays and apparently are not marked by escapes of gas. Many of the domes are in the swamps of the back bottoms and delta of the Mississippi, where the surface indications of the dome would be obscured by stream activity, and where faint indications might not be discovered for decades.

TABLE 4.—*Salt Domes Discovered by the Seismic Method of Exploration*

Structure		County; Parish if in Louisiana	State	Date	Oil Company Making Discovery	Seismic Troop Making Discovery	Discovery Apparently Proved	Oil	Remarks
Orchard	Salt dome	Fort Bend	Texas	1924	Gulf Production Co.	Seismos	Yes	Small oil field	First indication of dome given by torsion balance.
Long Point*	Salt dome	Fort Bend	Texas	1924	Gulf Production Co.	Seismos	Yes	Small oil well	Apparently a fair commercial sulfur dome.
Fannett	Salt dome	Jefferson	Texas	1925	Gulf Production Co.	Seismos	Yes	Small shows	Many tests drilled.
Hawkinsville	Salt dome	Matagorda	Texas	1925	Gulf Production Co.	Seismos	Yes	Dry	Many tests drilled.
Starks	Salt dome	Calcasieu	Louisiana	1925	Gulf Production Co.	Seismos	Yes	Yes	Several good wells.
Matagorda	Oligocene High	Matagorda	Texas	1925	Marland Oil Co.	Seismos	Yes	Dry	
White Castle	Salt dome	Iberville	Louisiana	1926	Roxana Petroleum Corpn.†	Seismos	Yes	Small shows	
Moss Bluff	Salt dome	Liberty	Texas	1926	Gulf Production Co.	Geophysical Research Corpn.	Yes	Dry	Many tests drilled.
Port Barre	Salt dome	St. Landry	Louisiana	1926	Gulf Production Co.	Geophysical Research Corpn.			Untested.
Fausse Pointe	Salt dome	Iberia	Louisiana	1926	Gulf Refining Co.	Seismos	Yes	Yes	Several fair wells.
Napoleonville	Salt dome	Assumption	Louisiana	1926	Gulf Refining Co.	Seismos	Yes		
Sorrento	Salt dome	Asconsion	Louisiana	1926	Gulf Refining Co.	Seismos	Yes	Yes	Apparently first-class oil field.
Bayou Bleu	Salt dome	Iberville	Louisiana	1926	Calcasieu Oil Co. Union Sulphur Co., and Standard Oil of La.	Calcasieu Oil Co.	Yes		
Haynesville	Salt dome	Wood	Texas	1927	Gulf Production Co.	Seismos			Untested.
Mt. Sylvan	Salt dome	Smith	Texas	1927	Humble Oil & Refining Co.	Humble Oil and Refining Co.			Untested.
East Tyler	Salt dome	Smith	Texas	1927	Humble Oil & Refining Co.	Humble Oil & Refining Co.	Yes		
White House	Salt dome	Smith	Texas	1927	Humble Oil & Refining Co.	Humble Oil & Refining Co.	Yes		
Bullard	Salt dome	Smith	Texas	1927	Gulf Production Co.	Geophysical Research Corpn.	Yes		
White Castle	Salt dome	Iberville	Louisiana	1927	Roxana Petroleum Corpn.	Seismos	Yes		
Troup	Salt dome?	Smith	Texas	1927	Amerada Petroleum Corpn.	Geophysical Research Corpn.			Untested.
La Rue	Salt dome	Henderson	Texas	1927	Roxana Petroleum Corpn.	Seismos			Untested.
Bethel	Salt dome	Anderson	Texas	1927	Pure Oil Co.	Geophysical Research Corpn.	Yes		
Oakwood	Salt dome	Freestone	Texas	1927	Roxana Petroleum Corpn.	Seismos	Yes		
Cronin	High	Anderson	Texas	1927	Roxana Petroleum Corpn.	Seismos			Untested.
Van	High	Van Zandt	Texas	1927	Pure Oil Co.	Geophysical Research Corpn.			Untested.

Bayou Choctaw.......	Salt dome	Iberville	Louisiana	1927	Gulf Refining Co.	Geophysical Research Corpn.			Untested.
Chacahoula (Lutcher).	Salt dome	La Fourche	Louisiana	1927	Gulf Refining Co.	Seismos	Yes		
Darrow..............	Salt dome	Ascension	Louisiana	1927	Gulf Refining Co.	Seismos	Yes	Small shows	
Lost Lake............	Salt dome	Chambers	Texas	1927	Pure Oil Co.	Geophysical Research Corpn.	Yes	Small shows	
San Felipe*..........	Salt dome	Waller and Austin	Texas	1927	Gulf Production Co.		Yes		
Dewalt*.............	Salt dome	Harris	Texas	1927	Humble Oil and Refining Co.	Humble Oil and Refining Co.	Yes	Good production	
Bayou des Glaise......	Salt dome	Iberville	Louisiana	1927	Calcasieu Oil Co. with Union Sulphur Co. and Standard Oil Co. of La.	Calcasieu Oil Co.	Yes	Small shows	
East Hackberry.......	Salt dome	Cameron	Louisiana	1927	Calcasieu Oil Co.	Seismos	Yes	Yes	Several good wells.
Calcasieu Lake........	Salt dome	Cameron	Louisiana	1927	Louisiana Land & Exploration Co.	Geophysical Research Corpn.	Yes		
Cypremort Point or Vermillion Bay.....	Salt dome	Iberia	Louisiana	1927	Louisiana Land & Exploration Co.	Geophysical Research Corpn.	Yes		
Black Bayou..........	Salt dome	Cameron	Louisiana	1927	Roxana Petroleum Corpn.	Seismos	Yes		
Arriola...............	Salt dome	Hardin	Texas	1927	Gulf Production Co.	Seismos			Untested.
East Bay Junop......	Salt dome	Terrebonne	Louisiana	1927	Louisiana Land & Exploration Co.	Geophysical Research Corpn.			Untested.
Dog Lake............	Salt dome	Terrebonne	Louisiana	1927	Louisiana Land & Exploration Co.	Geophysical Research Corpn.			Untested.
Lake Pelto...........	Salt dome	Terrebonne	Louisiana	1928	LouisianaLand & Exploration Co.	Geophysical Reseach Corpn.			Untested.
Lake Barre...........	Salt dome	Terrebonne	Louisiana	1928	Louisiana Land & Exploration Co.	Geophysical Research Corpn.			Untested.
Bayou Four Isle......	Salt dome	Terrebonne	Louisiana	1928	Louisiana Land & Exploration Co.	Geophysical Research Corpn.			Untested.
Bayou St. Elaine.....	Salt dome	Terrebonne	Louisiana	1928	Louisiana Land & Exploration Co.	Geophysical Research Corpn.			Untested.
Caillou Island........	Salt dome	Terrebonne	Louisiana	1928	Louisiana Land & Exploration Co.	Geophysical Research Corpn.			Untested.
Spanish Pass.........	Salt dome	Plaquemine	Louisiana	1928	Gulf Refining Co.	Geophysical Research Corpn.			Untested.
Lake Hermitage.......	Salt dome	Plaquemine	Louisiana	1928	Gulf Refining Co.?	Seismos			Untested.
Potash or Bohemia....	Salt dome	Plaquemine	Louisiana	1928	Humble Oil & Refining Co.	Humble Oil & Refining Co.			Untested.
? Barrataria Bay......	Salt dome?	Plaquemine	Louisiana	1928		?			Untested.
Lake Grand Ecaille (Bay Long).........	Salt dome	Plaquemine	Louisiana	1928	Gulf Refining Co.	Geophysical Research Corpn.			Untested.
Lake Salvador or Bayou Couba.......	Salt dome	St. Charles	Louisiana	1928	Louisiana Land & Exploration Co.	Geophysical Research Corpn.			Untested.
Little Chenier........	Salt dome	Cameron	Louisiana	1928	Pure Oil Co.	Geophysical Research Corpn.			Untested.
? Quarrantine.........	Salt dome?	Plaquemine	Louisiana	1928					Untested.
Bayou La Fourche....	Salt dome	La Fourche and Jefferson	Louisiana	1928	Gulf Refining Co.	Geophysical Research Corpn.			Untested.
Delta	Salt dome	Plaquemine	Louisiana	1928	McCollom Exploration Co.				Untested.
? Lake de Cade.......	Salt dome?	Terrebonne	Louisiana	1928					Untested.

* Primary credit for the discovery of these domes belongs to the torsion balance. † Name changed to Shell Petroleum Corpn.

Two-thirds of the domes discovered in the area of the interior Texas domes would have been discovered ultimately on the basis of geologic work supplemented by core or shallow drilling. Irregular dips, salines, brackish springs, topography or drainage faintly suggestive of the presence of a dome led geologists to recommend the shooting of those localities but ultimately would probably have led to drilling them and the discovery of some of the domes without the introduction of any geophysi al method.

The general effect of the introduction of the seismic method into the salt dome area of Texas and Louisiana has been to speed up the discovery of salt domes about 75 years.

Commercial Success of Seismic Prospecting

The discovery of those domes has not been merely an academic success for the seismic method but subsequent exploration by the drill has led to the discovery of a series of commercially important oil fields and at least one sulfur deposit. The Sorrento dome and East Hackberry seem probably to have oil of the first class. Fausse Pointe and Starks at times have seemed to look as if a first-class oil field might be developed, but at present the indications are rather of a second-class oil field, although the presence of a first-class oil field has not been disproved. An oil field at least of the third class is present on the Orchard dome and a small well has been completed in the cap of the Long Point dome, although numerous flank tests and other tests to the cap were dry. The indications are that a commercial deposit of sulfur is present on the Long Point dome, and an interesting show of sulfur was found in the single well into the cap of another dome.

The cash market value, at discovery, of the salt domes discovered by the seismic method in the Gulf Coast is approximately equal to the amount that has been spent in seismic prospecting, without regard to the potential value of any commercial deposits that subsequently may be discovered on them. The cash value of a guaranteed but undrilled salt dome may be estimated conservatively at \$150,000 and more liberally[8] at \$500,000. Prices of that magnitude are reported to have been paid for large blocks of acreage on the discovery of such domes as Moss Bluff and Dewalt, and the cost of discovering a dome by the old methods of drilling faintly suspicious prospects would average far above \$300,000. As some 45+ salt domes have been discovered in the Gulf Coast, their cash value before exploration amounts to \$6,000,000 to \$12,000,000.

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[8] By a recent deal, The Texas Co. has acquired seven untested seismic domes, two drilled and confirmed seismic domes, and much acreage with one producing well on the East Hackberry (seismic) dome from the Louisiana Land and Exploration Co. for the consideration of the assumption of \$1,800,000 of bonds of the L. L. Expl. Co., one-quarter royalty and certain drilling obligations.

The amount that has been spent in the operation of seismic exploration in the Gulf Coast is of the order of $6,000,000 to $9,000,000.

A saving of considerable lease money should also be credited to the seismic method. The larger companies were probably carrying a total of 1,000,000 or 2,000,000 acres of leases scattered here and there over the Gulf Coast at localities with no merit above the average for the Gulf Coast. The lease rental probably averaged more than $1 per year per acre. Much of this acreage is being shot carefully and the leases dropped with the consequent saving to the company of the annual rental. If the geophysical methods had not been introduced, the policy of promiscuous leasing and holding of land would probably have continued indefinitely. The companies are not satisfied as yet with the thoroughness and finality of the condemnation of areas by the geophysical methods and are still holding much area of no known merit but as the Gulf Coast is shot and reshot, the amount of such acreage probably will gradually diminish, and ultimately the seismic method and to a less extent the torsion balance method will have to be credited with a yearly saving in lease rentals of a million or more dollars.

The ability and the reliability of the seismic method to detect and map deeply buried salt domes in the Gulf Coast have been questioned not only by skeptical geologists and laymen but also by some geophysicists. Lost Lake, one of the first of the deep domes discovered by the seismic method, has been carried with a question mark by many. Thick, characteristic anhydrite cap rock, definitely indicative of the presence of a salt dome, has recently been drilled into at a depth of 3955 ft. The estimated depth based on a shrewd guess from the seismic survey was 4000 ft. The seismograph, therefore, has proved its ability to detect a deeply buried salt dome and to predict its depth with fair accuracy.

The discovery of the Lost Lake dome was a very pretty piece of engineering. It had been drilled by the Roxana Petroleum Corpn. on the basis of a torsion balance indication of a dome and apparently condemned as a prospect by a moderately deep dry test. The Pure Oil Co. considered it faintly suspicious, withdrew one of its Geophysical Research Corpn. crews from under the eyes of the watching rival scouts, and brought it in from 200 miles away in Louisiana. The troop began shooting as soon as it came on the ground, shot 4 hr., discovered the dome, and returned to its former assignment in Louisiana before the rival scouts had time to trace it and find out what it was doing.

The seismic method has been used successfully in California, Mexico, Venezuela and elsewhere, but the details of the application of the method in those areas are not known to the writer. Hans Mothes experimented with determination of the thickness of the ice on the Hintereisferner glacier in the Oetz Valley of the Alps and obtained results which checked well with the determinations by other methods.

Costs

The costs of operating a standard seismic troop in the Gulf Coast salt dome area is commonly reported to average between $15,000 and $20,000 per month. The acreage covered in reconnaissance shooting ranges from 150,000 to 300,000 acres per month per troop. In steady progressive reconnaissance of coastal prairie, the rate of 300,000 acres per month has been maintained by a troop for several months. In similar reconnaissance of marsh lands and bays, 200,000 acres per month can be covered. If a troop is moved rapidly from place to place, it may not be able to cover more than 150,000 acres per month. The per acreage cost of seismic reconnaissance in the Gulf Coast ranges from 5 to 12½ c. per acre. These costs include the fee to the consulting seismic company, the salaries and expenses of the crew, cost of the dynamite, the shooting permits for the tracts on which the firing is done, filling the explosion craters, damages, but not the cost of leasing land for exploration.

The per acreage cost of detailing a salt dome is considerably higher and varies with the dome and the degree of detail. The time necessary ranges from a week on a small dome in coastal prairie country to six weeks for a large dome in the Louisiana marshes. The area covered in detailing a dome ranges from 1000 to 10,000 acres. The per acreage cost of detailing a dome ranges therefore from $3 to $5.

The per acre cost of seismic exploration in the area of the interior salt domes is reported to have been about 40 per cent. higher than the costs given in the preceding paragraphs. The higher costs are due to the fact that the presence of high-speed beds relatively close to the surface forced a reduction of the shot length, the necessity of greater care in the work, and caused a very considerable reduction in the acreage that it was possible to cover in a month.

Most of the seismic work in this country has been and is being done by two consulting companies, the Geophysical Research Corpn., and the Seismos Gesellschaft. The common practice of both companies on long contracts is to supply a skeleton troop of three, four or five technical men and the technical apparatus. The oil company hiring the troop supplies all the nontechnical men, nontechnical apparatus and transportation and pays all expenses and salaries, except the salaries of the technical men. The fee to the consulting company ranges from $6000 to $8000 per month on contracts for 6 months to a year. For short jobs a complete crew is furnished at a rate ranging from $1000 per day for a job of a few days to $14,500 per month for longer jobs, plus the cost of the dynamite which will range from $1500 to $3000 per month. For shooting in marsh, water or other areas where the ordinary automobile equipment can not be used, the rate is slightly higher.

Present Activity

The present-day activity in the use of the seismic method known to the writer is given in Table 5:

Table 5.—*Seismic Prospecting Now in Progress*

Company†	Locality	Number of Troops
Geophysical Research Corpn:		
Gulf Oil Corpn	Gulf Coast and West Texas	2
Pure Oil Corpn	Gulf Coast	2
The Texas Co	Gulf Coast	1
Amerada Petroleum Corpn	Oklahoma and West Texas	2
Miscellaneous	Mexico, California, and elsewhere	4
Seismos Gesellschaft:		
Gulf Oil Corpn	Gulf Coast	1
Roxana Petroleum Corpn	Gulf Coast	1
Miscellaneous	Gulf Coast	1
In Mexico		?
Humble Oil & Refining Co	Gulf Coast	3
The Texas Co	Gulf Coast	1
McCollum Exploration Co		
Consulting in Gulf Coast		1
In Mexico		1
In Venezuela		1
Shell Petroleum Corpn*	Gulf Coast	2
Geophysical Exploration Co	Gulf Coast	1
Sun Oil Corpn	East Texas and Gulf Coast	1
Petty Geophysical Engineering Corpn		
Consulting out of San Antonio, Texas		1
Frank Rieber		
Consulting in California		2

* Formerly Roxana Petroleum Corpn.

† Marland Oil Co., Phillips Petroleum Co. and others have started seismic work in the past with company troops but are not known to be active at the present time.

Acknowledgment

Grateful acknowledgment is due to G. M. Bevier, Frank Neumann and others for constructive criticism of this paper.

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Modern Instruments and Methods of Seismic Prospecting

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(Boston Meeting, August, 1928)

For a long time it has been known that it is possible to deduce conclusions about the physical constitution of the interior of the earth from the records of natural earthquakes obtained by stationary seismographs. In view of this fact it is surprising how late, relatively speaking, geologic prospecting by seismic methods has been taken up, although the first suggestion was made 40 years ago[1]—that travel-time curves of artificial explosions might be used for the study of the change of the velocity of seismic waves as depth. Further suggestions of practical applications of artificial explosions for geologic purposes were made by Belar,[2] by von dem Borne,[3] by Benndorf[4] and by Galitzin,[5] but it was not until 1919 that Mintrop obtained a patent on the seismic prospecting method which he has applied commercially on a large scale ever since.

Dr. Mintrop came to this country with his method at about 1923. Since that time, several American oil companies have developed seismographs of the mechanical type for the vertical component similar to that of Mintrop's (Humble Oil Co., Dr. Ricker). Other consulting geophysical companies designed other types of seismic devices, the simplest of them for the recording of the first impulse being the geophone (McCollum Geophone Co.). The galvanometric recording method as first proposed by Galitzin is being used (Geophysical Research Corpn.?) and use is being made even of the piezoelectric properties of crystals for portable seismographs (F. Rieber). There is such an enormous variety of all kinds of portable seismographs, vibrographs, accelerographs, etc., that it is difficult to decide just what kind is preferable. Unfortunately, nothing has been published as yet about American designs of portable seismographs, except the Wood-Anderson torsion-seismometer. After Mintrop's design of portable seismographs, other constructions were made

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1 A. Schmidt: *Jahresh.* d. Vereins f. Naturk. Wuerttemberg (1888).

2 A. Belar: Die Erdbebenwarte, 60. 1901–02.

3 G. von dem Borne: *Gerlands Beiträge zur Geophysik* (1908) 378.

4 H. Benndorf: Über die Bestimmung der Geschwindigkeit transversaler Wellen in der aeussersten Erdkruste. *Phys. Ztsch.* (1912) **13,** 83.

5 B. Galitzin: Vorlesungen über Seismometrie, 117, 153, 438, 1914. Petersburg Acad. Impér. des Sci. (1913) **3,** 350.

in Europe by Reutlinger, Hecker, Mainka, Ambronn (Accelerometer), Whipple, Schweydar, Duckert (the latter uses the mass of a vertical seismograph as one part of a condenser in an audiofrequency circuit).

As emphasized before, it is very difficult to decide what kind of a seismograph should be used in practice; it depends largely on the completeness that is desired in regard to the interpretation of the seismogram. In seismic prospecting in general, fewer data are employed than in pure seismology. Theoretically any explosion produces longitudinal and transversal waves, the former being the faster. In practice, we use now only longitudinal waves and not the transversal ones which are always noticeable on seismograms of distant natural earthquakes, but disappear practically on records of artificial explosions.

For instance, in case of a horizontal discontinuity in the subsoil, there will be three types of longitudinal waves: (1) those traveling directly from the shot point to the seismograph; (2) those being refracted and traveling part of their way through the lower layer and eventually overtaking the wave through the upper layer; (3) the reflected wave. All three types have a horizontal and vertical component in regard to the amplitude of the vibration produced, and they also possess a definite period. All these quantities naturally depend on the geologic condition underneath. If complete information is desired about them, it will be advisable to observe as many physical quantities of the seismic waves as possible. The greatest number of physical data seem to be obtainable by a mechanical seismograph (Mintrop's, for instance); because (1) this type may be easily built for two components, (2) it has a definite individual period and magnification, and (3) the recorded movements and amplitudes especially may easily be referred to the actual motion of the ground. The electromagnetically recording seismographs, however, would not be so suitable for this type of work, because the recorded amplitudes do not depend on the actual amount of displacement of the ground, but on the velocity of the displacement. While it is advantageous to have several electromagnetic seismographs produce their records on one sheet of the oscillograph paper, it is sometimes impracticable in the field to make connections of a number of seismographs by wires to a common recording station, especially if the stations are very far apart. Thus every type of seismograph has more or less its advantages and disadvantages, and it all depends on the purposes for which a special type is to be used.

In the educational and research work of the Department of Geophysics at the Colorado School of Mines, it was our aim to have a seismograph that will allow a clear interpretation of as many physical details as possible; that is, period, sharp impulses, angle of emergence, etc. Therefore we selected a new type of mechanical seismograph built by Schwey-

dar[6] for the observation of the vertical and horizontal component. As this seismograph has never been described in English, a brief description is given here.

Schweydar's Two-component Seismograph

The seismograph consists of two parts, the double pendulum and the recording device. The recording device is placed about 1 m. distant from the pendulum, connected with it by a rigid tunnel (Fig. 1) or a telescope-jointed tube (Fig. 2) to prevent outside light from getting to the sensitized paper. The recorder consists essentially of a gramo-

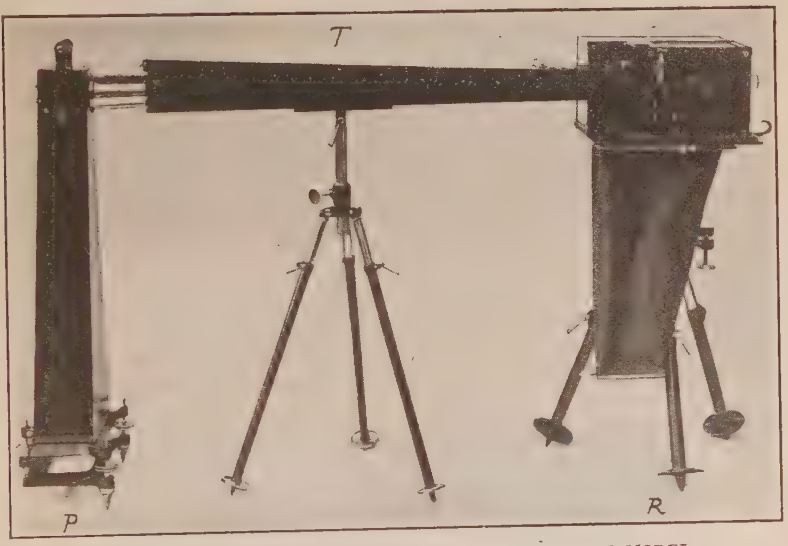

Fig. 1.—Schweydar's field seismograph, first model.
P = double pendulum; T = light tunnel; R = photographic recording apparatus.

phone clockwork which drives photographic paper about 6 cm. wide across a slit in front of a cylindrical lens; there the light which has been reflected from the mirrors of the seismograph hits the recorder. The speed of the paper may be adjusted from 7 to 10 cm. per second. The exposed paper may be cut off with a knife and dropped into a bag in the older type of recording apparatus (Fig. 1). The new type (Fig. 2) has a detachable container for the paper. Beside the mechanism for driving the paper there is a 3.5-volt lamp on the apparatus, which furnishes the light for the seismograph. All of the light, however, does not go to the seismograph; part of it passes at intervals through a small pinhole in a sheet of metal attached to a little pendulum inside the recorder. With this arrangement the time marks are obtained on the seismogram. The time interval of these marks is 0.07 sec. (Fig. 7). When the switch

[6] W. Schweydar and H. Reich: Künstliche elastische Bodenwellen als Hilfsmittel geologischer Forschnung. *Gerlands Beiträge zur Geophysik* (1927) **17**, 121.

that starts the paper is turned, the pendulum begins to oscillate. There is a small mirror provided in the recording apparatus which is connected with the armature of an electromagnet; the mirror makes a line on the seis-

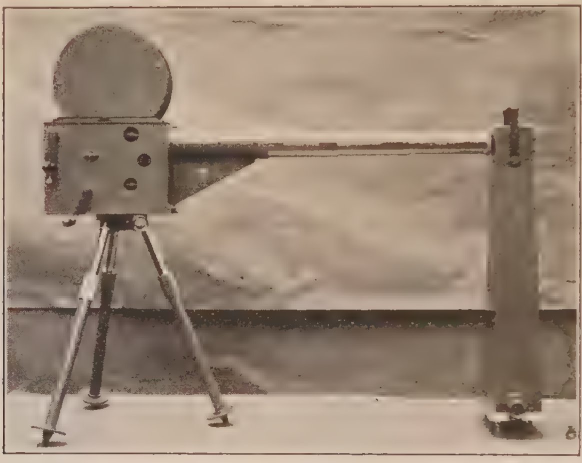

FIG. 2.—TWO-COMPONENT FIELD SEISMOGRAPH ASSEMBLED, NEW TYPE.

mogram when current goes through the magnet; at the moment of the explosion, the current is interrupted and the line ends, thus marking the moment sharply on the record from which time must be counted (Fig. 7).

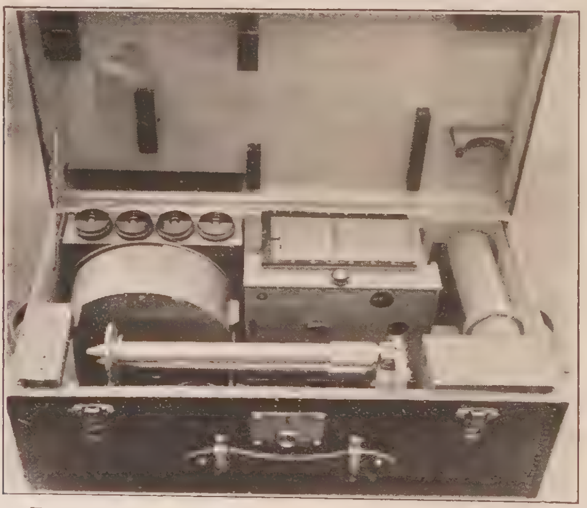

FIG. 3.—RECORDING APPARATUS OF NEW TYPE SEISMOGRAPH IN BOX.

For this purpose the recording apparatus may be connected by a wire with the spotpoint, or with a receiver upon which the moment of the explosion is transmitted by radio. Another possibility is to transmit

a periodic signal and record it on the seismogram by means of a simple small oscillograph which is placed at the same distance from the recorder as the pendulum, and to interrupt the signal at the moment of the explosion. A third possibility is to use the record made on the seismogram by the sound wave and compute with it the moment of the explosion, using the velocity of sound through air at known atmospheric pressure and temperature, wind intensity and direction. This method does away with the necessity of transmitting by wire or radio the moment of the explosion to the recorder, but involves other data which must be observed, and also is not very dependable if the charges are buried in the ground because then not a very sharp impulse of the sound is recorded. Hubert's results proved that by burying the charge the intensity of the explosion is increased 50 to 100 times. Fig. 3 shows how the recording apparatus is packed for transportation. It weighs, without the box, about 30 pounds.

The seismograph consists of two pendulums, one for the vertical and the other for the horizontal component. The construction of the pendulum for the vertical component is very similar in principle to that applied by Mintrop; a cross-section of this apparatus may be found in a previous publication of the author's.[7] This seismograph is essentially a mass attached to a horizontal blade spring, of which the vibrations are magnified by a cone-shaped light aluminum lever. In Mintrop's apparatus, the end of this lever turns by friction a vertical axle to which a mirror is fastened; in Schweydar's seismograph, the end of this lever carries a very thin wire which is wound once around the vertical axle and is held under tension horizontally by a small spring. In Schweydar's apparatus the heavy mass weighs about 1800 gm.; the reduced length of the pendulum is 6.89 cm.; the natural period, about 0.07 seconds.

The vertical movement of the ground is made approximately horizontal by the aluminum lever and mechanically magnified about eight times; this mechanical magnification is increased considerably by the combination of the friction of the wire on the vertical axle with the optical lever. This magnification depends on the ratio of the focal length of the lens in front of the seismograph mirror divided by the radius of the vertical axle (lever principle) to be doubled on account of the reflection. As the diameter of the little axle is about 2 mm. and the distance of recorder and pendulum about 1 m., the combined magnification by friction and optics is about 2000. Hence, together with the mechanical magnification, these portable seismographs of Schweydar's have a magnification of about 16,000 for infinitely small periods. It is obvious

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[7] C. A. Heiland: Instruments and Methods for the Discovery of Useful Mineral Deposits. *Engng. & Min. Jnl.* (1926) **121**, 47. Geophysical Methods in Oil Finding. *Oil & Gas Jnl.* (July, 1926) **25**, 30. Seismograph and Other Methods Used for Locating Structures (August, 1926) **25**, 64-B.

that this magnification could readily be increased by double reflection, smaller radius of the axle and larger mechanical magnification, so that smaller charges of dynamite could be used and less damage be caused. There is, however, a practical limit to it, because the microseismic move-

FIG. 4.—A SHOT.

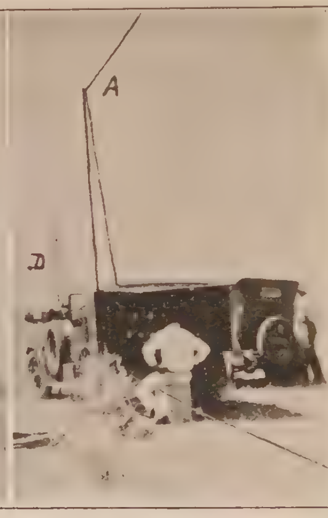

FIG. 5.—RADIO TRANSMITTING STATION CLOSE TO SHOT POINT.
A = antenna; D = derrick.

FIG. 6.—SEISMOGRAPH FIELD STATION.
P = pendulum; O = oscillograph; S = seismic recorder; R = radio receiver.

ments are magnified by the same amount as those caused by the explosions. In other words, the vibrations caused by the friction of the wind on the ground, traffic, movements of derricks, pumping of oil wells, etc., would be so strong that it would be impossible to determine the impulse of the first longitudinal waves in the seismogram. However, it is reported that consulting companies in Europe employ mechanical

seismographs of their own design with magnification as large as one million, which is adjustable.

In Schweydar's seismograph, an arrangement could readily be made whereby the magnification could be adjusted optically between very wide limits and equally for both components. Then it would not be necessary to use larger charges of dynamite for more distant shots than for close ones.

The seismograph contains not only a pendulum for the vertical but also for the horizontal component, with the same arrangement for mechanical and optical magnification, except that the blade spring revolves about another axis. This axis is usually placed at right angles to the seismic profile. The magnification of both the vertical and the horizontal pendulums, as well as their natural periods, etc., are exactly alike. The pendulums may be used either damped or not damped. The new types have an arrangement for oil damping.

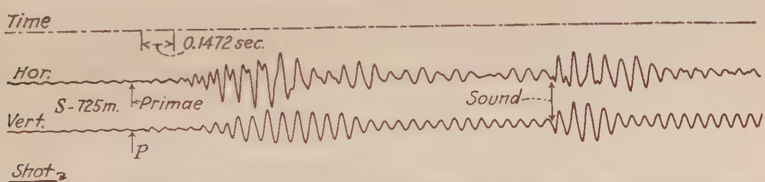

FIG. 7.—SEISMOGRAM OBTAINED WITH THE SCHWEYDAR TWO-COMPONENT PENDULUM.

Figs. 4 to 6 show the method in operation. Fig. 4 shows the explosion of a charge buried in a hole which was drilled in the ground by a readily portable derrick *D* which is seen in Fig. 5. In the foreground of the same figure is the radio transmitting station with the antenna *A*. Fig. 6 shows the tent with the radio receiver *R*, the seismic recorder *S*, the double-pendulum seismograph *P* and the shotmarker (oscillograph) *O*. The proper adjustment of recorder in reference to the pendulum can be made much faster than is ordinarily thought. In fact, to get a seismic station ready takes so little time that the seismograph parties must generally wait until the shooting party is ready. Fig. 7 represents a seismogram obtained with the Schweydar instrument.

INTERPRETATION OF SEISMOGRAMS

Very little has been published about the interpretations of the seismograms; practically nothing in English. Very great secrecy is being exercised in regard to the methods of interpretation, but most of it is very simple indeed. As far as I know, so far it has not even been necessary in practice to consider curved paths of seismic waves, which we have to assume in pure seismology. The increase of pressure, temperature and density as depth is negligible in the depths now accessible to seismic exploration (within one layer; compare footnote 16), or, if not negligible,

its influence is at least smaller than that of other sources of error. Considering only straight paths of the seismic waves, the theory underlying the interpretation may be handled by applying (1) the optical laws of refraction and reflection, (2) Fermat's principle, which states that any light ray travels in such a manner that the time required to pass from one point to another will be a minimum (brachystochronic principle). If an interpretation is being based also on the periods of the seismic waves, which depend on the period by which the various strata perform oscillations, the laws of free and forced oscillations of bodies with definite dimensions would have to be applied, but to my knowledge an attempt to use periods only for quantitative interpretation has not as yet been made. Neither have angles of emergence been used exclusively, nor the intensities of the arriving seismic energy. Striking changes of the latter have been observed occasionally (in crossing fault lines or strata in different stages of decay, etc.) but an attempt has not as yet been made to use intensities exclusively in interpretation.

The datum that is generally employed at present is the time which elapses between the moment of explosion and the arrival of the wave at the station. The explosion is made at one point and a number of seismographs are set up in a straight line. For complete information these lines should be parallel to as well as at right angles to the strike of the formations. Sometimes it is necessary also to shoot at two different distances from the seismographs.[8] Instead of using an explosion and four or five seismographs in line, one seismograph may be used and explosions may be made on various points on the line. This has, however, the practical disadvantage that the cost for dynamite and damages becomes extremely high.

The interpretation of the results begins with the analysis of the seismogram. First, the impulses must be located which correspond to the arrival of different types of waves. In practice, only impulses of longitudinal or first impulses are noticed. The very first impulse always is very noticeable. If seismic measurements are made above two horizontal strata, for instance, the lower of which has the greater speed of propagation, it happens at a certain distance that the wave through the lower stratum arrives first and causes a deflection of the straight line. Then there may be a second impulse due to the wave that traveled in the upper layer on a straight path from shotpoint to station. There may also be a third impulse due to a wave that has been reflected by the lower stratum. The first impulse may be always readily located, the other impulses are often hard to find and therefore are seldom used. After the impulses have been located the time is taken from the seismogram which elapses between the moment of the shot and the impulse. This time is plotted for the various stations against their distance from

[8] B. Gutenberg: *Lehrb. der Geophys.* (1926) **3**, 596.

the shotpoint. Thus a graph is obtained, which is usually called the "travel-time curve." As said before, this curve in most cases will contain only the times of the arrival of the very first impulse. If the curve is straight, there is no geologic structural unconformity in the soil that may be found by seismic prospecting. If the curve has breaks, there are unconformities, the depth and location of which may often be calculated from the travel-time curves. It is advisable to have a number of structures for which the travel-time curves have been computed theoretically and to compare these graphs with the field results so as to draw a first general conclusion. Such theoretical travel-time curves were mentioned first by Mintrop in his application for a patent for his method in 1919. In 1921, Ambronn[9] published a curve for a horizontal discontinuity. In 1924, 1925 and 1926 the author[10] published curves for several types of structures; in 1926, Gutenberg[11] computed such curves also for various structures and published the formulas; in 1927, Meisser[12] independently published similar formulas and applied them in several practical cases. In the same year, Pautsch[13] published formulas similar to those of Gutenberg and Meisser. Schweydar and Reich,[14] also, in 1927, investigated travel-time curves not due to refraction but independent oscillations of the lower layers.

On account of the fact that there is a keen need, especially in our educational work, for English publications on this subject, a brief and elementary résumé of formulas and methods used in the interpretation of seismic prospecting is given here. For results on known geologic conditions, the references must be consulted.

Groups of Travel-time Curves

The interpretation of travel-time curves may be classed in three decidedly different groups according to the origin of the impulses. (The theoretical considerations hold for longitudinal as well as for transversal waves; the velocity of the latter is less than that of the former, and their impulses are seldom used, as emphasized before.) For "alternating" waves (partly longitudinal, partly transversal) the formulas must be somewhat modified. The groups are as follows:

1. Impulses due to directly arriving and refracted waves.
2. Impulses due to reflected waves.
3. Impulses due to transmission of the tremor through oscillation of the lower layers.

[9] R. Ambronn: *Jahrb.* des Halleschen Verbandes (1921) 27.

[10] C. A. Heiland: *Op. cit.*

[11] B. Gutenberg: *Op. cit.*

[12] O. Meisser and H. Martin: *Ztsch. f. Geophys.* (1927) 106.

[13] E. Pautsch: Methods of Applied Geophysics, 32. 1927. Gulf Pub. Co., Houston, Tex.

[14] W. Schweydar and H. Reich: *Op. cit.*

IMPULSES DUE TO DIRECTLY ARRIVING AND REFRACTED WAVES

Group 1 is by far the most important at present and will be discussed first.

The simple relation of travel-time curve and geologic structure is illustrated by the simplest example of seismic prospecting,[15] that of a vertical fault separating two strata (Fig. 8). The velocity of the waves may be v_1 in the left and v_2 in the right stratum; let v_1 be 2000 m. per sec., v_2 4000 m. per sec. Let us assume that the 0 point is the shot point and seismographs are set up at the points 1, 2, 3, 4, 5 at 400-m. intervals. Then the tremor reaches point 1 in 0.2 sec. after the explosion; point 2 in 0.4 sec.; point 3 in 0.6 sec. but it reaches point 4 in 0.7 sec., point 5 in 0.8 sec., and so on. Plotting these times against distance, we obtain a curve of the shape indicated in Fig. 8, with a break above the fault. Since the geologic structure demonstrated in Fig. 9 also produces a travel-time curve of the same shape, a shot may be fired at point B, which may have the distance $x' - y'$ from the first shot point. In case of a vertical fault, the distance of the break from the 0 point remains the same; if there is a horizontal unhomogeneity, the distance of the break from the zero point becomes $(x' - y')$ m. greater. Thus it is possible to locate a vertical fault without knowledge of geology. In other words, we apply the following relations for the interpretation:

$$a' = x', \text{ if shot point is at } A;\ b' = y' \text{ if shot point is at } B \qquad [1]$$

The inclination of the travel-time curve furnishes the velocity, as the tangent of the angle equals the quotient of time divided by distance, or

$$\text{cotang } \gamma = v_1, \text{ cotang } \beta = v_2 \qquad [2]$$

Horizontal Layer

A second possibility of a very common geologic structure is a horizontal stratum underneath the surface at a depth d. This structure will also produce a travel-time curve with a break as shown in Fig. 9. The problem is to find the depth of the stratum d. To obtain the necessary equations for this case, we first consider the possible paths of the longitudinal waves through this structure. The explosion is made at point I. Waves radiate from this point in all directions. Suppose there is a receiver at point VI. One wave will travel directly to this point horizontally in the upper layer. If the distance I − VI is s, the time t_1 at which the impulse arrives is $t_1 = \frac{s}{v_1}$. The first part of the travel-time curve with the inclination γ is produced by this wave, as cotang $\gamma = \frac{ds}{dt_1} = v_1$.

In order to find the path of the wave that travels through the lower layer, we apply the laws of optical refraction. If we have two media of

[15] C. A. Heiland: Seismograph and Other Methods Used for Locating Structures. *Oil & Gas Jnl.* (August, 1926) **25, 64-B.**

different indices of refraction, a light ray passing through the two media is being deflected. If the incident ray makes the angle φ with the vertical upon the boundary and the deflected ray the angle ψ, the relation holds that $\frac{\sin \varphi}{\sin \psi}$ = index of refraction, or $\frac{v_1}{v_2}$, where v_1 and v_2 respectively are the velocities of light in the two media. There will be refraction only, however, if the angle φ does not exceed a certain maximum, which may be denoted by i. For a definite index of refraction, there may be found, from the above equation, an angle φ for which $\sin \psi$ becomes one. That is, ψ is 90° or the refracted beam of light does not penetrate the lower medium but travels along the boundary surface. If the angle of incidence φ becomes greater than this "boundary-angle of total reflection" i, reflection will take place on the boundary. From the above equation, it follows that $\sin i = \frac{v_1}{v_2}$, or, denoting the quotient $\frac{v_1}{v_2}$ by q, $\sin i = q$. This angle i is of great importance for our problem. Suppose we have a seismic wave traveling from I to VII; being refracted into the lower medium, it will not reach the receiver at all. Take another beam which strikes the boundary at a different angle and travels from I to III and IV; this will undergo total reflection. Apparently the only[16] ray that will reach the receiver by refraction will be the one traveling horizontally on the boundary of the lower medium. This ray must strike the boundary at the angle of total reflection i, as we have noted before, and consequently must leave it at the same angle. Making this assumption, we apply Huygen's principle that each point of a wave may be considered as a source of new waves. We have thus defined geometrically the path which is the only one that the seismic ray may travel if it is to reach the receiver by refraction.[16] Defining this path by an equation, we have to differentiate this equation, put the differential equal to zero and thus find the path which the ray may travel through the lower medium in a minimum of time, according to the principle of Fermat. The location of the break in the travel-time curve is then defined by the moment at which the upper and lower waves arrive simultaneously, or $t_1 = t_2$. From this relation, the depth of the boundary may be computed as follows:

The wave travels, according to Fig. 9, the path I–II–V–VI. I–VI being s, II–V = a, it covers

$$\sqrt{d^2 + \left(\frac{s-a}{2}\right)^2},\ a, \text{ and } \sqrt{d^2 + \left(\frac{s-a}{2}\right)^2}. \quad \text{Hence,}$$

$$t_2 = \frac{a}{v_2} + \frac{1}{v_1}\sqrt{(s-a)^2 + 4d^2} \qquad [3]$$

[16] In reality, some waves will reach the receiver which penetrate the lower medium, because the seismic paths are curved (concave to the surface) on account of the increase of pressure at depth. The surface wave in the upper stratum is also somewhat concave.

As the time must be a minimum for the distance a,

$$\frac{dt_2}{da} = 0 \text{ or}$$

$$\frac{dt_2}{da} = \frac{1}{v_2} + \frac{1}{v_1} \cdot - \frac{s - a}{\sqrt{(s-a)^2 + 4d^2}} = 0. \qquad [4]$$

Leaving out the intermediate steps for briefness, we have from [4],

$$a = s - \frac{2dq}{\sqrt{1 - q^2}} = s - 2dq \cdot \sec i \qquad [5]$$

Substituting this in [3], we obtain

$$t_2 = \frac{s}{v_2} + \frac{2d}{v_1}\sqrt{1 - q^2} = \frac{s}{v_2} + \frac{2d}{v_1}\cos i \qquad [6$$

Hence, $\dfrac{ds}{dt_2} = \text{cotang}\,\beta = v_2$. In other words, equation [2] also holds for a horizontal layer; the velocities, and therefore q, may be determined from the travel-time curve. For $t_1 = t_2$ is $s = x$ (Fig. 9). We thus obtain from [6] after substitution of these quantities for the depth

$$d = \frac{x}{2} \cdot \sqrt{\frac{1 - q}{1 + q}} = \frac{x}{2} \cdot \sqrt{\frac{v_2 - v_1}{v_2 + v_1}} \qquad [7]$$

Thus the depth may be determined from the travel-time curve only without further geologic assumptions. The question whether there is a horizontal layer and not a vertical fault must first be ascertained by shooting at a second point B. There are two other data of the travel-time curve from which the depth may be computed. Extending the second part of the curve with the inclination β toward the 0 point, we obtain an intersection with the ordinate and thus the time t_x; we further obtain an intersection with the abscissa, the distance of which from the 0 point may be D. Then the depth is

$$d = \frac{t_x}{2} \cdot \frac{v_1}{\sqrt{1 - q^2}} = \frac{t_x}{2} \cdot v_1 \cdot \sec i \qquad [8]$$

$$\text{and } d = \frac{Dq}{2\sqrt{1 - q^2}} = \frac{D}{2}\,\text{tang}\, i \qquad [9]$$

From [7] it follows that the lower wave always arrives earlier than the upper one for distances $x > 2d\sqrt{\dfrac{1-q}{1+q}}$. From the same formulas it is seen that the depth always is smaller than $\dfrac{x}{2}$. These equations may readily be applied for more than two horizontal strata one below the other. They may also be combined for laterally limited blocks, with the first equations given for vertical faults. Irregular profiles of structures often may be combined of horizontal parts. For dipping strata the formulas may also be applied (to determine the depths of flanks of anticlines, for instance) if the profiles are made parallel to the strike and if the axis of the anticline is fairly horizontal. It is also possible to use

graphical methods for the geometrical derivation of the depth from the travel-time curve; such methods have been published by Schweydar[17] for horizontal layers and by Meisser[18] for inclined beds. These diagrams may be constructed easily when the geometrical meaning of the above equations is taken into consideration. To give a detailed account of these is beyond the scope of this paper, which is to describe only the fundamental principles upon which the interpretation is based.

For the same reason, details of the derivation of the equations for an inclined stratum are not given. They may be derived by applying the laws of refraction and the principle of Fermat, as described. The derivation of the equation shows that it is not possible to determine with one shot the dip and depth of the stratum. One possibility[19] is to shoot at two different distances from the seismographs (points A and B). The other possibility[20] is to exchange shot point and receiver. The resultant equations for both methods will be given.

Inclined Layer

Gutenberg's Method, Shooting at A and B.—AB is y; the distance of seismograph and receiver $= AR = s$. The surface of the inclined stratum with the dip α (positive in the direction shown in Fig. 10) should be extended to the intersection with the surface; AI is D_1, $BI = D_2$. The depth below the shot points are d_1 below A and d_2 below B. Shooting at A, we have as before $t_1 = \frac{s}{v_1}$ and $\frac{ds}{dt_1} = \text{cotang}\ \gamma = v_1$;

$$t_2 = \frac{1}{v_1}(2D_1 + s)\sqrt{1 - q^2}\sin\alpha + \frac{1}{v_2}\cdot s \cdot \cos\alpha \qquad [10]$$

If the stratum dips in the opposite direction, in reference to the relative position of shot point and receiver, α and D must be taken negative. The angle β of the second part of the curve is smaller as compared to that above a horizontal stratum, if the stratum goes up in the direction from A to R. It is larger if the stratum dips in that direction. (These relations are not shown in the generalized figures.)

If a shot is fired at point B, the intersection of t_1 and t_2 will lie higher for the second travel-time curve than for that of the first if the stratum dips in the direction from A to R; it will be lower if the stratum goes up (Figs. 10 and 11). If shots are fired at A and B, the second parts of the two curves may be extended so as to intersect the ordinate, thus giving t_1' and t_2' (see Fig. 10). It can then be shown that

$$D_1 = y \cdot \frac{t_1'}{t_2' - t_1'} \text{ and } D_2 = y \cdot \frac{t_2'}{t_2' - t_1'} \qquad [11]$$

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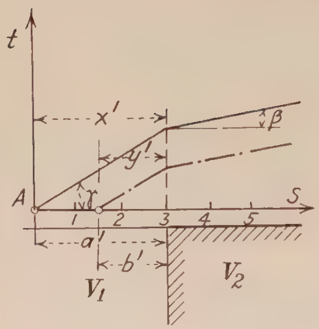
[17] W. Schweydar and H. Reich: *Op. cit.*
[18] O. Meisser and H. Martin: *Op. cit.*
[19] B. Gutenberg: *Op. cit.*
[20] O. Meissner and H. Martin, W. Schweydar and H. Reich: *Op. cit.*

8


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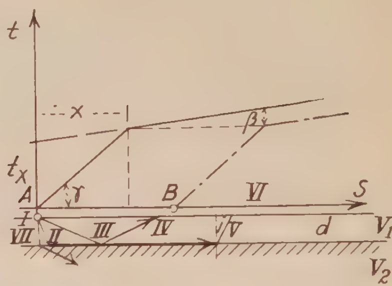

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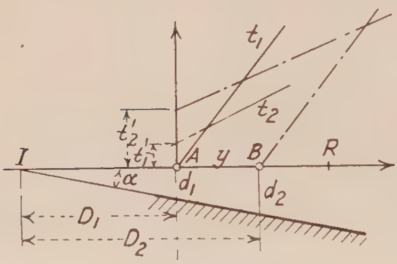

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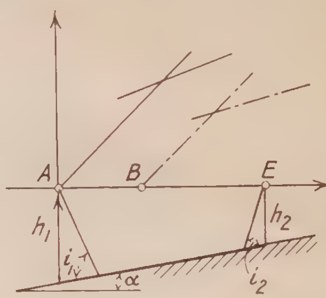

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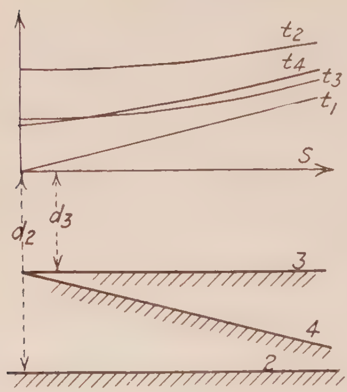

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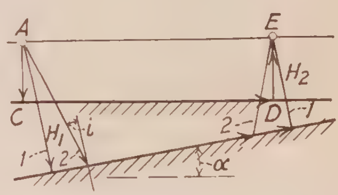

FIG. 8.—TRAVEL-TIME CURVE OF A VERTICAL FAULT SEPARATING TWO STRATA.
FIG. 9.—TRAVEL-TIME CURVE OF A HORIZONTAL DISCONTINUITY.
FIG. 10.—METHOD OF GUTENBERG FOR INCLINED STRATUM.
FIG. 11.—METHOD OF MEISSER FOR INCLINED STRATUM.
FIG. 12.—TRAVEL-TIME CURVES FOR REFLECTED WAVES.
FIG. 13.—ILLUSTRATION OF IMPULSES PRODUCED BY OSCILLATION OF LAYERS.

Furthermore, if the distance $s = 2D_1$ or $2D_2$ respectively, where the times t_1'' and t_2'' may be observed,

$$t_1'' = \frac{-2D_1}{v_2} \cos \alpha, \text{ and } t_2'' = \frac{-2D_2}{v_2} \cos \alpha \qquad [12]$$

The depth of the stratum below the shot points will be

$$d_1 = y \cdot \frac{t_1'}{t_2' - t_1'} \text{ tang } \alpha \text{ and } d_2 = y \cdot \frac{t_2'}{t_2' - t_1'} \text{ tang } \alpha \qquad [13]$$

Method of Meisser and Schweydar, Interchanging Shot Point and Receiver.—According to Fig. 11, E is now the point of explosion, A the receiver. $AE = s$. $i_1 = i_2 = \text{arc cos} \cdot q$. In this method, the depth below shot point h_2 and receiver h_1 are used. (Different symbols have been chosen for the sake of differentiation from Gutenberg's depths d below shot points.)

Shooting, first, from E to A (positive direction),

$$t_{2+} = \frac{2h_1}{v_1} \sqrt{1 - q^2} \cos \alpha + \frac{s}{v_2}\left(\cos \alpha - \frac{1}{q}\sqrt{1 - q^2} \sin \alpha\right) \qquad [14]$$

$$\frac{dt_{2+}}{ds} = \text{tang } \beta_+ = \frac{1}{v_{2+}}\left(\cos \alpha - \frac{1}{q}\sqrt{1 - q^2} \sin \alpha\right) = \frac{1}{v_{2+}} \qquad [15]$$

Reversing the shooting from A to E (negative direction),

$$t_{2-} = \frac{2h_2}{v_1} \sqrt{1 - q^2} \cos \alpha + \frac{s}{v_2}\left(\cos \alpha + \frac{1}{q}\sqrt{1 - q^2} \sin \alpha\right) \qquad [16]$$

$$\frac{dt_{2-}}{ds} = \text{tang } \beta_- = \frac{1}{v_2}\left(\cos \alpha + \frac{1}{q}\sqrt{1 - q^2} \sin \alpha\right) = \frac{1}{v_{2-}} \qquad [17]$$

The dip of the stratum may then be computed by combining [15] and [17] or

$$\text{cotang } \alpha = \frac{1 + p}{pq}\sqrt{1 - q^2}, \text{ where } 2p = \frac{v_{2+} - v_{2-}}{v_{2-}} \qquad [18]$$

In formulas 14 to 18, $\frac{1}{q}\sqrt{1 - q^2}$ may be replaced by cotang i.

In other words, the inclination of the second parts of the travel-time curves no longer furnishes the velocity in the lower medium; it is

$$v_2 = v_{2-} \cdot \frac{1 + 2p}{1 + p} \cos \alpha \qquad [19]$$

The depth below the receiver is computed from the distance x of the break in the curve and the ordinate:

$$h = \frac{x}{2} \frac{1 - q \cos \alpha + \sqrt{1 - q^2} \sin \alpha}{\sqrt{1 - q^2} \cos \alpha} \qquad [20]$$

The foregoing formulas may be simplified[21] by using the depths H_1 and H_2 perpendicular to the lower stratum (Fig. 13, path 2) E being shot point.

$$t_{2+} = \frac{H_1 + H_2}{v_1} \cos i + \frac{s}{v_2} \cos \alpha \text{ (compare with [14])} \qquad [21]$$

[21] W. Schweydar and H. Reich: *Op. cit.*

and similarly, t_{2-} if A is the shot point. The inclinations of the second part of the travel-time curves are:

$$\frac{dt_{2+}}{ds} = \text{tang } \beta_{+} = \frac{1}{v_1} \sin (i - \alpha) \text{ and } \frac{dt_{2-}}{ds} = \text{tang } \beta_{-} = \frac{1}{v_1} \sin (i + \alpha) \quad [22]$$

By combining the last equations, i and α are found; as v_1 is known, v_2 may be determined from i. H_2 may be expressed by H_1, α and s; thus the depth may be computed from [21].

Summary.—The three types of geologic structure which have been discussed so far are, in almost any practical case, sufficient to interpret more complicated types of structures. In general, it will be quite sufficient to assume straight boundaries of the structure. For curved boundaries the equations become complicated; it is then easier to derive the travel-time curves from a geometric construction of the wave path, using the laws of refraction.

Methods Using Impulses Due to Reflected Waves

These methods are very seldom used, because of the fact that the impulses of reflected waves are very hard to recognize on a seismogram; the impulses frequently get lost in the general movement of the ground and the pendulum which has been produced by the wave that passes horizontally from the shot point to the receiver. Publications about observations of reflected waves in applied seismology are not frequent. Hubert, using a stationary seismograph with more than two million magnification, claims that he was able to observe impulses of waves which were reflected on discontinuities in several kilometers' depth; Barton[22] states that an American geophysical company has perfected a device to observe waves that are reflected at a high angle. The theory of impulses produced by reflected waves is simple. The intensity of the reflected waves is not great. If the angle at which a wave strikes the unconformity underneath is smaller than the angle of total reflection, most of the energy is refracted into the lower layer and never reaches the receiver; only a small portion is reflected. If this angle of incidence is greater, the total energy is reflected, but this occurs in greater distances only, where in turn energy is absorbed by the longer path. Lack of intensity is also a reason why the observation of reflected waves is difficult. To derive the travel-time curves for reflected waves we have to mention that the first impulse in the seismogram will again be produced by the directly passing wave, for which $t_1 = \frac{s}{v_1}$ (see Fig. 12). For the reflected wave the reflection points lie in distance units of 1, 2, 3, 4, etc. from the

[22] D. C. Barton: Applied Geophysical Methods in America. *Econ. Geol.* (1927) **22**. 649.

shot point if the seismographs are placed at intervals of 2, 4, 6, 8, etc. at the surface. Hence,

$$t_2 = \frac{1}{v_1}\sqrt{s^2 + 4d^2} \qquad [23]$$

so that the depth of the stratum

$$d = \frac{v_1}{2}\sqrt{t_2^2 - \frac{s^2}{v_1^2}} \qquad [24]$$

If s is zero or not very large (very slow rise of the curves in Fig. 12)

$$t_2 = \frac{2d}{v_1} \text{ and } d = \frac{t_2 \cdot v_1}{2} \qquad [25]$$

The travel-time line is not straight, but curved, as

$$\frac{dt_2}{ds} = \frac{s}{v} \cdot \frac{1}{\sqrt{s^2 + 4d^2}} \qquad [26]$$

In Fig. 12 these curves are illustrated, two of which represent horizontal strata at greater (d_2) and smaller (d_3) depths. It is readily seen that the amplitude of t increases with depth. The velocity v_1 may be obtained from the first impulse, that is from the t_1 curve. In Fig. 12 there is also represented a curve for waves reflected on a dipping stratum; there is not much difference in amplitude of t_4 from t_3, which infers that this method is not very suitable to determine dips at right angles to the strike. It will be more accurate to shoot several profiles parallel to the strike with this method and calculate the dip from the depth values thus obtained. The curves of t_4 and t_3 intersect, as for an inclined stratum at the shot point $t_4 = \frac{2d}{v_1}\cos\alpha$, while $t_3 = \frac{2d}{v_1}$. The formulas given above for this method may be readily applied to locate also the distance of structures cut off by vertical faults: for this purpose it is advisable to use a horizontal component seismograph.

Methods Using Impulses Produced by Oscillation of Lower Layers

The use of the two-component seismograph as described permits of deriving the apparent angle of emergence e' of the seismic waves from the amplitude of the horizontal component A_H and that of the vertical A_Z:

$$\text{tang } e' = \frac{A_Z}{A_H} \qquad (27)$$

If a longitudinal seismic wave strikes an unconformity, four new waves are originated: two reflected waves (transversal and longitudinal), provided that the angle of incidence is not greater than the angle of total reflection. Considering that by any elastic impulse longitudinal as well as transversal are originated which both strike the unconformity, it follows that eight new waves are produced. While it is certain that difficulties in the interpretation of seismograms are frequently due to

such complications, we have previously dealt only with the longitudinal waves, because we have considered only the first impulse of the fastest wave; it is well known that the velocity of the longitudinal waves is greater than that of the transversal waves.

If a longitudinal wave strikes the earth's surface we have to deal only with reflected waves, one longitudinal and one transversal. Theoretically it can be shown that the energy of the incident longitudinal wave for certain angles of incidence goes almost entirely into the reflected longitudinal, while for others it goes more into the transversal wave.

As a consequence of this distribution of intensity (causing the amplitude A) in the reflection, the "apparent angle of emergence" e' is not the same as the "actual angle of emergence" e; for longitudinal waves,

$$\cos e = \frac{v_l}{v_t}\sqrt{\frac{1 - \sin e'}{2}} = 1.79\sqrt{\frac{1 - \sin e'}{2}} \qquad [28]$$

where v_l is the velocity of the longitudinal and v_t that of the transversal waves.

It follows, therefore, that if a longitudinal wave emerges steeply (that is, with a great angle of emergence), most of the reflected energy will be again longitudinal, so that apparent and true angles of emergence are almost alike. If, however, the longitudinal wave arrives at a flat angle, so that the angle of total reflection is exceeded, total reflection will occur, transferring the longitudinal into polarized transversal energy, so that there will be great differences of apparent and true angles of emergence. We know, from pure seismology, that in addition to this there are a number of other complications, primarily the composition of the ground upon which the seismograph rests; loose ground is readily induced to oscillate, so that for certain periods there may be even resonance, depending on the thickness of the top layer.

This experience gained in seismic station work, that oscillations of layers are possible, has been applied in seismic prospecting. Schweydar[22] observed above glacial top layers at all distances angles of emergence which were greater than those to be expected by applying the laws of refraction. He consequently concluded that the tremor is propagated the shortest distance from the shot point down to the lower layer and induces it to oscillate. This oscillation transmits the wave vertically through the top layer back to the receiver.

In this case, it may be conceived that (Fig. 13) the seismic waves go vertically down from A to C, travel along the boundary of the lower layer to D and go up vertically to E. A similar path may be constructed for the inclined layer (Fig. 13, path 1). For the first case, Schweydar publishes the formulas

$$t_2 = \frac{2d}{v_1} + \frac{s}{v_2}, \text{ so that again } \frac{ds}{dt_2} = \text{cotang}\,\beta = v_2 \qquad [29]$$

In other words, also with this assumption, there must be a break in the travel-time curve. From its distance from the ordinate, x, the depth of the layer may again be computed:

$$d = \frac{x}{2}(1 - q) \qquad [30]$$

Assuming a path for an inclined layer in the same way as for a horizontal stratum, the formulas may be derived accordingly. The angle of dip and depths in this case can also be derived only from the travel-time curves if the profile is shot twice in a positive and negative direction.

It is of very great practical importance to know for which geologic problem the formulas using the laws of refraction or the formulas based on the assumption of independent layer-oscillations should be applied. A distinction should be possible by the observation of angles of emergence, according to Schweydar's investigations.

There is, however, one objection to it. While there are numerous practical cases where it could not be determined whether the first or the second assumption is right on account of negligible differences, there have been other cases where the actual depths or the depths drilled later on checked the first assumption better than the last.

It seems, therefore, that the problem is to determine which method is right. We have had very little experience in seismics with angles of emergence and it does not seem unlikely that such great angles have been produced for some other unknown reason but the propagation by oscillation of the lower layer. I believe it might be possible that the transmission of the waves actually follows the law of refraction, but that the *top* layer is induced to oscillate by the seismic energy, not the lower layer. This assumption seems to be in accordance with Schweydar's observation that the observed periods of the waves varied with the thickness of the glacial top layer. In one locality, the thickness of this layer was 8 to 15 m. and the period of the first waves 0.006; in the second locality, 200 m. and the period 0.02 seconds.

From the foregoing, it may be seen that it is of great importance to devote more study to the angles of emergence and to determine which of the assumptions is correct.

DISCUSSION

F. Rieber, San Francisco, Calif. (written discussion).—Dr. Heiland appears to have drawn the material for his excellent paper largely from experience with earthquake seismology, including, of course, such direct adaptations of earthquake instruments as the mechanical seismographs used in geophysical work.

He says that he has been handicapped in presenting a more complete paper by the fact that practically nothing has been published, to date, with regard to the electrical methods of receiving and recording vibrations, but seems to feel that such methods would prove less satisfactory than mechanical registration because they could not make records from which amplitude, angle of emergence, or frequency could be obtained.

Knowing the importance of these factors in earthquake work, and assuming that they would be equally important in geophysical work, he turns naturally to the mechanical seismograph as the instrument of choice.

While I cannot pretend to speak for others who are using electrical recording—for, in fact, the lack of published information on these methods places me at an equal disadvantage with Dr. Heiland—I can at least outline the developments with which I have been associated in California, and which have led in some instances to conclusions differing from those put forward in the paper under discussion.

The first problem undertaken in our work, which is now entering its sixth year, was the development of a two-component electrically operated seismograph, which would permit the following factors to be deduced from the records:

1. Elapsed time between an explosion and the first resulting vibrations to reach the instrument.
2. Angle of emergence of the wave, as deduced from its vertical and horizontal components.
3. Relative amplitudes of waves.
4. Frequency of vibrations.

In planning this instrument, we were guided by suggestions from J. B. Macelwane, S. J., and his successor, Dr. Perry Byerly, both of the department of seismology, University of California, who emphasized the importance of two-component registration of vibrations, as in earthquake work.

Elapsed Time

Our instrument was finally developed to a point of satisfactory operation—only to show us that the factor of elapsed time was by all odds the most important thing to be learned from our records, and that the other three (angle of emergence, amplitude, and frequency) not only had very little significance geologically but were extremely difficult and tedious to compute.

We also found that if we were to make a record of the full wave train, in two components, we could do so only at a considerable sacrifice either of time accuracy or of powder consumption. These rather radical conclusions, which eventually led to the development of our present form of apparatus, may possibly require amplification.

Angle of Emergence

The first wave in a train of earthquake waves arrives at the surface as a motion having a certain definite direction—angle of emergence. By comparing the magnitudes of the vertical and horizontal components of this first motion, as recorded on two seismographs, this angle may be calculated.

After the arrival of the first impulse, the motion usually becomes so complicated that its direction is difficult to derive accurately from the records.

When dealing with artificial elastic waves from explosions, however, we soon found that for explosions at any distance from the apparatus, the horizontal component of the first motion was negligible, and the computed angle of emergence rapidly approached 90°. This fact has been noticed and commented on by others—notably Schweydar,[23] who found, with the type of seismograph used by Dr. Heiland, the following values for the angle of emergence, at various distances from an explosion:

Distance Meters	Angle
15.2	76°
30.0	66°
60.0	79°
103.0	90°

[23] W. Schweydar and H. Reich: *Op. cit.*

Various reasons have been advanced for this observation. My own feeling is that it is due to refraction of the waves by relatively thin and soft surface layers, in which they travel with very low velocities. Such layers would scarcely affect earthquake waves, with their wave lengths measured in thousands of feet, but would have great effect on the much shorter waves such as result from an explosion.

Having found that the first wave, at any useful distance, emerges in a practically vertical direction, the question naturally arose: Why record the horizontal component at all? Would it not be better to make both our instruments of the vertical-component type, and secure two readings at different places. (As a matter of fact, if we do this, we can obtain the angle of emergence very accurately by computations from the time-distance graphs plotted for the two vertical-component instruments.)

Amplitude

Considering next the matter of amplitude, we must also take into account the width of our record strip. If we are to keep this strip of convenient size—for the sake of argument, say we wish to allow a width of 50 mm. as the maximum excursion of our largest waves—and if we wish to record all waves that reach our instrument with amplitudes proportionate to the true earth motions, the smallest waves may become very small indeed.

It is not uncommon for the largest vibrations arriving from an explosion to have an amplitude several hundred times that of the smallest waves. In the case of our 50-mm. record, this might mean that the smallest waves would appear on the record with amplitudes less than 0.1 mm.—far too small to be observed correctly.

This state of affairs would be permissible, except for the fact that the really valuable information in such a record resides in the first arriving waves, which are almost uniformly the smallest in amplitude. It seems scarcely justifiable, therefore, to suppress the amplitude of these important first waves, in order to be able to read, on a proportionate scale, the magnitude of a later wave of doubtful significance.

On this matter of amplitude, one further comment seems to be in order. Dr. Heiland mentions, and quite correctly, that the ultimate sensitivity to which a seismograph can be adjusted is limited by the amount of vibration already present in the earth. Pumps, highway travel, railroads, winds, all contribute their quota to this vibration. Even in areas entirely remote from culture, the effects of winds, temperature changes, and minute earth motions combine to keep the earth in a slight but not entirely negligible condition of vibrations.

If we continue to make our receiver more and more sensitive, we will eventually be able to see such vibrations, which may be classed under the term "earth unrest," on our own records. When this occurs, we have reached the maximum sensitivity that it is practical to use.

If we wish to record a wave from an explosion, and to recognize it definitely as such, it is quite obvious that this wave must produce at our receiver a vibration noticeably larger than those of the "earth unrest." To make such a wave will require, for any given distance from the receiver, and for any given type of soil, a certain charge of dynamite, which we may term the minimum permissible shot. It seems quite obvious that this minimum permissible charge is a direct function of the unrest at the receiver; if this unrest doubles, we must double our charge, and so on.

Dr. Heiland mentions the desirability of adjusting the magnification of the Schweydar instrument—which is undoubtedly a good idea—but I cannot agree with him when he says this would make it unnecessary to use larger shots at greater distances. At any given distance, it is desirable first to adjust the instrument for maximum sensitivity consistent with unrest, and then to judge the charge required according to this sensitivity. If our instrument is not at the maximum sensitivity, we use more

dynamite than is really needed, that is all; and this minimum amount of dynamite—for a constant sensitivity of the instrument—increases more rapidly than the square of the distance. All that we could hope to do, therefore, in the way of making all charges equal at different distances would be to overcharge the nearer shots and use less sensitivity—a procedure which has no advantages.

Frequency

The meaning of frequency in records is still in dispute. Certainly it has little geological significance in the light of our present knowledge. We know that any explosion radiates a group of waves of assorted frequencies. Some of these waves may encounter, at some point in their path, a body resonant to their own frequency, and thus be intensified, as a sound of the right pitch is amplified in a closed bare-walled room. Or—and this is most often the case—our receiving device may prefer certain frequencies, which will be accented in the record.

I cannot, therefore, see any definite advantage to be gained from the study of frequencies at the present time; and, as a matter of fact, Dr. Heiland has not pointed out any either. As I understand him, he simply feels that there might be such advantages and therefore we should record true frequencies.

On the other hand, there is a real advantage to be gained from *not recording* the full range of frequencies—as can be shown. "Earth unrest" consists of vibrations covering a certain range of frequencies. Dynamite causes waves covering a certain other range of frequencies. These two frequency ranges do not coincide, the unrest being spread over a far wider range than the useful part of the explosive waves. If we restrict our receiver to this useful part of the explosive range, we thereby exclude the unrest to a greater degree than we do the waves from the dynamite, allowing greater amplification at the apparatus and the use of less dynamite in our "minimum permissible shot."

Summary

Summing up: the first arrival time has great value, and the precision with which we can read it is greatly increased by modifying the construction of our instrument and by using electrical rather than mechanical registration.

Such a modified instrument, while it does not permit complete evaluation of angle of emergence, amplitude, and frequency, from the records in the manner used in earthquake work, really loses nothing, since these factors are of far less value and meaning in geophysics than in earthquake seismology.

Dr. Heiland has presented the case for mechanical registration, and presented it well. Many of the points he makes are valuable, but since the case for electrical registration has never been presented with equal clarity, his paper is unavoidably deficient at that point.

From our experience with electrically operated elastic-wave recorders, the following opinions may be added:

1. If we desire to construct an instrument for registering earth movements in two components, and with known magnification, this can be done electrically in a completely satisfactory manner, with the special advantage that vibrations from a number of stations may be recorded on a single record strip.

2. Two-component registration, on a true scale, of the actual earth movements seems to us to possess fewer advantages than can be obtained by certain modified arrangements—and these modifications can best be obtained by electrical means

Some of the points mentioned in this discussion might profitably be illustrated from reproductions from actual records.

Dr. Heiland illustrates his article with a two-component seismogram, taken at 725 m. with a Schweydar two-component seismograph (Fig. 7). Since the time of the first arrival is the *only factor* referred to in his formulas, which can be obtained directly from the seismograms, with the exception of a vague reference to the use of the angle of emergence, we may first consider time as obtainable from these records. *P* marks the instant of first arrival as read by him on both records. Considering first the vertical component, we can see a brief series of sharp waves at this point, although the natural vibration of the instrument has produced a continuous record which is several times the amplitude of the actual waves.

Turning to the horizontal component, I am sorry to say that *P* and the arrow mark a spot where we should certainly expect a horizontal component, if there were one, but where I can see absolutely nothing but the "unrest" vibration of the instrument. Enlarged views of both wave trains in their earlier parts are included to enable confirmation of this fact (Fig. 14). Reading time from records like this—

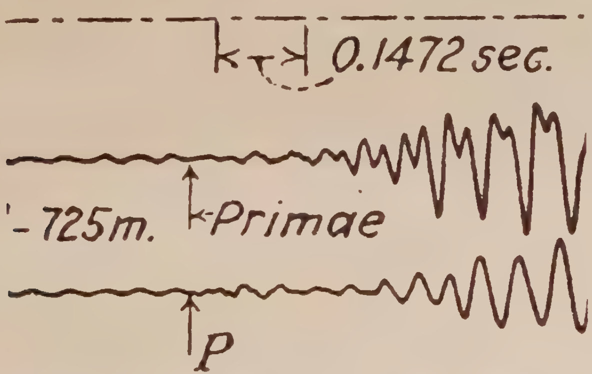

Fig. 14.—Enlarged views of wave trains.

and especially attempting to plot angles of emergence from the two components—frankly appears to me to come rather too close to guessing to be truly practical.

By contrast there are reproduced five typical records (Fig. 15) taken with an electrically registering vertical-component instrument, all at a distance of 2500 ft.—approximately the 725 m. of the Schweydar record just shown. These records *do not* reproduce actual earth motion in its true magnitude but they *do* enable the determination of time to a degree of certainty not possible with other forms, as may be readily seen from the time scale attached.

Attention is also called to the comparative size of the "unrest" and "explosion" waves, quite the reverse of those for the mechanical instrument first shown.

Further, these records are fairly typical of a series of over 12,000 films now taken by this method, and it is believed that they give a true idea of the possibilities of modified electrical registration.

The magnification available with the apparatus is on the order of 5×10^{13}. For practical field operation, a magnification in excess of 10^{13} is seldom used, and even this factor must frequently be reduced when excessive "unrest" is encountered. This large magnification—compared to the magnification 16,000 of the Schweydar instrument, and the several million power of the best stationary mechanical instruments, is only possible of use by reason of the modified demands of the recording apparatus, and the elimination of frequency and total amplitude from the factors derived from the records.

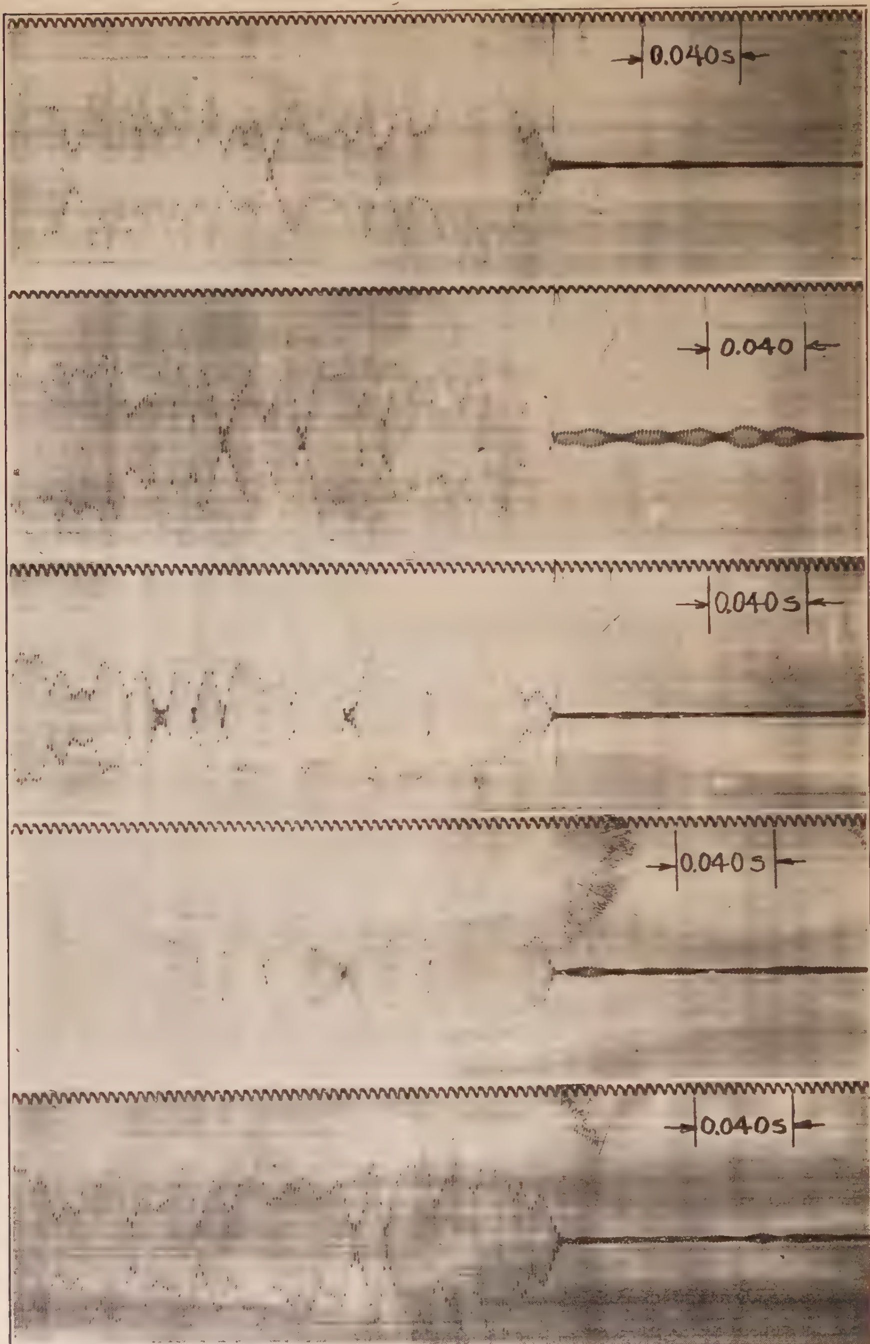

FIG. 15.—TYPICAL RECORDS TAKEN WITH ELECTRICALLY REGISTERING VERTICAL-COMPONENT INSTRUMENT AT 2500 FT. DISTANCE.

C. A. Heiland (written discussion).—Mr. Rieber deals with an electromagnetic seismograph designed by himself and aims to prove that this type of seismograph, which is generally known as the Galitzin type of seismograph, is more advantageous in applied seismic work than the mechanical type.

From the nature of his discussion the reader will probably be led to believe that I dealt at some length with the electromagnetic seismographs and came to the conclusion that the mechanical type was very much superior to the former. I fail to see how Mr. Rieber could have so completely misunderstood my opinion about these seismographs, which I summed up, on the second page of my paper, in the following sentences:

"As emphasized before, it is difficult to decide what kind of seismograph should be used in practice; it depends largely on the completeness that is desired in regard to the interpretation of the seismogram . . . . Thus every type of seismograph has more or less its advantages and disadvantages, and it all depends on the purposes for which a special type is to be used."

It is generally known that a Galitzin seismograph frequently records first impulses well on account of the fact that the recorded amplitude depends on the current induced in the coils by the movement of the heavy mass, the magnitude of which in turn depends on the velocity of this movement; frequently the first arrival of the waves is characterized by a quick displacement of the ground (called "impetus," while a slow rise is "emersio"). However, the true amplitude and the true frequency of the ground movement are more difficult to derive from the seismogram than the moment of the arrival of the first wave. While, thus, an electromagnetic seismograph is very advantageous if one wants to confine the interpretation to time differences, a mechanical seismograph is better if one wants not only these time differences but true amplitude and true frequency. Besides, the terrain under investigations makes it often very troublesome to connect up four or five seismographs by wires with a common receiving apparatus.

Mr. Rieber says that I have not dealt in my paper with interpretation of the phenomena of amplitude, frequency, etc., although I gave preference to a seismograph which furnishes these data. The reason for this is, as expressly stated in the introduction, that the formulas given were to be only a résumé of what had been published. I also said that hardly anything had been published about the interpretation of these data but that they are being used more and more. In order to be able to contribute to this development, I have given preference to a seismograph which permits of a clearer interpretation of these data than other types, which may, however, be better for other purposes. I hope to be able in the near future to publish experiences obtained in analyzing the behavior of these quantities on known geologic conditions.

Mr. Rieber also says that my paper is deficient in presenting the facts about electromagnetic seismographs. He is quite right on this point, because the object of the paper was only the description of the mechanical two-component seismograph and the methods of time analysis thus far published. The brief mention of the electromagnetic type was made incidentally only to explain why we had purchased the mechanical type. While a detailed description of the mechanical type, together with an exhibition of its physical constants, is given in my paper, attention must be directed to the fact that Mr. Rieber, although criticizing the deficiency of my paper in reference to electromagnetic seismographs, has never given us such a detailed description of his seismograph, a section of it or a statement about its physical constants nor travel-time curves on proved geologic structures. The only datum given in his discussion is the magnification, but it is difficult to make practical use of this statement, as the magnification depends on the natural period of seismograph and galvanometer (or oscillograph respectively) and no reference is made to these periods. Generally speaking, geophysicists doing research in scientific institutions are greatly handicapped by the fact that the results of commercial researches in this line are not published.

The administration of the institution with which I am connected has made several efforts to obtain the cooperation of consulting geophysical companies, but has not been successful. If it thus happens that geophysicists not connected with commercial consulting companies publish information not as far advanced as the data which the latter have acquired, the companies that have declined to cooperate should be the last to criticize their efforts.

Value of Two Components

There are several points in Mr. Rieber's discussion which obviously are not quite in accord with the experience of others and with published data. Mr. Rieber first refers to the observation of both components and suggests that the observation of the horizontal component should be abandoned because it has been observed on several occasions that even in smaller epicentral distances this component is very small; that is, the apparent angles of emergence are almost 90°. Personally, I think that we do not yet possess enough corroborative evidence of this phenomenon (it has been published only twice), and chiefly, we do not know at all what causes it. Neither do we know that there are not geologic conditions which may change these steep angles. One explanation is that the wave fronts emerge from any subsurface discontinuity parallel to its surface and not according to the laws of refraction described in my paper. That is, it may be assumed that the waves are propagated by oscillations of the lower layer (see page 641). The truth of that could be determined by seismic observations above formations dipping at a steep angle, but nobody seems ever to have tried it. However, the data so far obtained seem to indicate that this view is *not* correct, because depth determinations made on the basis of the laws of refraction have so far checked the actual geologic conditions better than the determinations based on the assumptions given.

Furthermore, nobody has ever tried to determine the true angle of emergence from a graphic differentiation of the travel-time curve of artificial explosions, nor compared it with the apparent angle as obtained from the records of both components and thus computed Poisson's ratio for the formations close to the surface. I do not believe in discontinuing the observation of a physical phenomenon just because an explanation or a relation to known phenomena does not present itself immediately. Mr. Rieber claims that the observation of two components is more troublesome than the observation of one component, but evidently he thinks only of the electromagnetic seismographs. For two recording elements (oscillographs) are required for a two-component electromagnetic pendulum, while this is not necessary for a mechanical seismograph, where both components may be readily recorded on one piece of paper. It is just as easy and economical to operate a two-component mechanical seismograph as it is to operate a one-component recorder. Thus the continued observations of the horizontal component may eventually reveal the true reason for the steep angles of emergence and may then prove very helpful in the analysis of special cases. However, we know now definitely that the observation of the horizontal component is useful in observing waves that travel through or are reflected from vertical faults and similar features, and also in the analysis of polarization effects. Evidently, Mr. Rieber has not thought of these two applications.

Reliability of Records

In order to corroborate his statement that a Galitzin seismograph furnishes sharper impulses than a mechanical seismograph—a statement of which the truth has never been doubted—Mr. Rieber has enlarged the seismogram of Fig. 7, to show that "reading time from records like this seems to come too close to guessing to be truly practical."

I am not surprised at this remark. I remember distinctly that when I began to interpret records of mechanical seismographs my teacher pointed out several "well marked" impulses in the seismogram, and I thought that he was telling stories. However, the location of impulses turned out to be very simple—by an analysis of the frequency. I am sure that when Mr. Rieber applies this analysis to the record of the horizontal component on the original seismogram of Professor Schweydar, he will agree that the arrow belongs where it has been placed. I do not claim that the impulses in a mechanical seismograph are as sharp and as easily recognized as in an electromagnetic device; however, I do say that with the proper analysis the determination of time intervals is reliable in both types of instruments.

The final proof in such discussions about the reliability of one method or apparatus as compared to another is always the success. Taking the statistics recently published on 47 salt domes discovered by geophysics in the past four years in Texas and Louisiana, we find that 29 domes were located by companies with mechanical seismographs and 18 domes by companies using electromagnetic devices. Of the domesl ocated by mechanical seismographs, 41 per cent. have been proved by drilling and of the domes located by geophones or electromagnetic types, 28 per cent. have been proved. How is this possible if, as Mr. Rieber claims, the interpretation of records obtained with mechanical seismographs is too close to guessing?

Mr. Rieber claims[24] that only by appropriate design of apparatus, especially greater sensitivity, and extension of the mathematical treatment of the results, is it possible to attempt work on such unconsolidated formations as occur in California. I have been informed that the Schweydar seismographs are applied successfully in the same territory (San Joaquin Valley) and that an extension of the mathematical analysis over and above the known methods published in my last paper has not been necessary. I believe that an analysis of the California seismograms is often possible even with methods older than those published in my article; namely, with the Wiechert-Herglotz, or similar methods based upon some law of variation of elastic constants with depth.

I am sorry that Mr. Rieber took the trouble to enlarge the seismogram of Fig. 7. The history of this illustration is as follows: Professor Schweydar reproduced his original seismogram on tracing cloth; this diagram was photographed and printed in a scientific magazine; from that magazine, I took this diagram and had it reprinted in my paper. Therefore little information may be derived from an enlargement of such a diagram. Besides, the record happened to have been taken for special purposes without the damping attachment. Had I anticipated that the time of the impulses would be doubted, a record with a much sharper impulse (also in H) could have been demonstrated.

Importance of Frequency and Amplitude

Mr. Rieber feels that only my assumption that amplitude, frequency, etc., are equally as important in applied seismic work as they are in earthquake seismology has made me give preference to the mechanical seismograph, and aims to prove that these factors have little significance geologically and are "extremely difficult and tedious to compute." While I agree with Mr. Rieber that the time analysis is the most important factor, I do not believe that the other factors are of so little significance as to be neglected in favor of the former. On the contrary, the time analysis has more or less exhausted its possibilities and progressive consulting companies nowadays also devote considerable attention to the possibilities of amplitude, frequency, etc.

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[24] F. Rieber: Adaptation of Elastic-wave Exploration to Unconsolidated Structures. See page 654.

This may be readily illustrated by referring to one of these factors, the frequency, of which Mr. Rieber says that: "The meaning of frequency in records is still in dispute. Certainly it has little geological significance." This is a mistake. First of all, the frequency is influenced by the thickness of the individual formations traveled through, and often dispersion is caused; *i. e.*, the velocity changes with the frequency. However, we have few data on this at present. The frequency is of practical and thus geologic significance in the following respect: Experience has shown that under most geologic conditions the reflected waves have a frequency which differs from the frequency of the directly transmitted and the refracted wave. As the magnification of any seismograph depends on its natural period, and as it is possible to design seismographs with adjustable natural periods, we may change the period of the seismograph until we have practically "tuned out" the directly arriving or refracted wave and increase the amplitude of the reflected wave for the geologic conditions under investigation. The intensity of the reflected energy (or the attenuation factor on the point of reflection) depends on the angle of incidence; therefore, also, the amplitude must be investigated to determine the optimum distance of shotpoint and receiver. It is also advantageous to investigate the state of polarization of the reflected wave from the amplitudes recorded in the various directions if opportunity offers. This reflection method at high angle is often advantageous for certain geologic structures, as for instance basement rocks buried at great depth, etc. It has been applied recently by several companies in oil fields of Kansas and Oklahoma. Of course, it is possible to make most of these investigations with an electromagnetic seismograph, as long as they are confined to an analysis of the *relative* amplitude and frequency. Speaking of relative amplitude, this factor has also been found to be of geologic significance. Faults which do not manifest themselves by a difference in elasticity of the two adjoining formations but represent zones of disintegration may be detected by the attenuation of the energy of waves passing through them.

Permissible Sensitivity

The importance of frequency and amplitude demonstrated by the recent development in seismic prospecting brings me to my last point, a discussion of the following statement made by Mr. Rieber:

"The minimum permissible charge is a direct function of the unrest of the receiver . . . I do not agree with Dr. Heiland that adjustable magnification makes it unnecessary to use larger shots at greater distances . . . at any given distance it is desirable to adjust the instrument for maximum sensitivity consistent with unrest, etc."

This does not seem quite in accordance with published data and introduces complications which may be eliminated by using a seismograph with adjustable natural period. Then the ground unrest which otherwise would determine the permissible sensitivity may be "tuned out" and thus in certain distance intervals the decrease in energy of the waves may be compensated by an increase in magnification. Of course, I do not hesitate to admit that this cannot be indefinitely applied to the entire distance covered in a profile, as most likely the energy of one kind of shot decreases more rapidly in too great intervals than can be compensated for by an increase in magnification. The elimination of the ground unrest, as described, can, of course, also not be carried to the extremes, depending on the difference in the period of unrest and the period of the effective wave energy.

Advantages of Mechanical Seismograph

In the foregoing discussion I have naturally attempted to point out the good features of the mechanical seismograph, while Mr. Rieber has emphasized in his discussion the advantages of his instrument over the mechanical type. I have found

especially in recent technical publications that authors believe that the mechanical type is not capable of furnishing clear records on account of resonance, etc. I believe, however, that in proper operation—*i. e.*, with a high rate of damping and if possible with adjustable period and magnification—the mechanical type is in no way inferior to the electromagnetic type. As said before, I have selected it because it furnishes with careful handling and analysis not only all quantities that may be obtained with an electromagnetic seismograph, but some others besides, of which the importance is just beginning to be recognized. Finally, it must be remembered that into such discussions about the advantage of one type of instrument over another there enters always a good deal of personal viewpoint, and, as an old proverb says: *De gustibus non est disputandum.*

Adaptation of Elastic-wave Exploration to Unconsolidated Structures

By Frank Rieber,* San Francisco, Calif.

(Boston Meeting, August, 1928)

The study of earthquakes long ago developed the fact that by studying the travel times of the various groups of waves from the same earthquake, as received on seismographs at varying distances, major discontinuities in the earth's crust could be reasonably well defined. Using this original discovery, and adapting the mathematical methods originally devised in seismology for determining the depth below the surface at which such discontinuities occur, other investigators were able to develop a new branch of seismology which promises to be extremely useful in extending the boundaries of our information on local conditions below the earth's surface.

As an outstanding example of the applications of this work, we have the so-called "seismograph" exploration of the Gulf Coastal region, which has resulted in the discovery of a large number of salt domes, from many of which oil fields may be developed.

The original seismological investigations, from which different major discontinuities in the earth's crust were deduced, as well as these later successful experiments in locating salt-dome structures, involve relatively simple problems. In each case we are dealing with surface materials in which elastic waves will be propagated with a fairly definite velocity. Under this material we have something quite different, in which the elastic waves will be propagated with a much higher velocity.

The transition from one medium to the other is likewise relatively abrupt. For example, in the Gulf Coastal region the sedimentary formations have a velocity of 6000 or 7000 ft. per sec., while the velocity of elastic waves in the cap rock of a salt dome may exceed 15,000 ft. per sec. Under such conditions, delineation of the formation is a relatively simple matter, and, furthermore, there is very little opportunity for ambiguous results.

Development of Elastic-wave Method

The success of this Gulf Coast work immediately suggested the advisability of attempting to extend elastic-wave methods to a point

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where they could deal with structures of a more general type, in which the differences in the velocity of the various parts of the formation were less pronounced, and the transitions from one type of material to another were less abrupt. The writer has been concerned for the last three or four years in the development of such elastic-wave methods for investigating structural conditions in California, with especial reference to the location of possible oil fields. Although the prospects for a successful outcome of this work were at first extremely discouraging, later developments have proved the work well worth while.

At the present writing, we can be fairly certain that elastic-wave methods may be used in a wide variety of locations, and in relatively unconsolidated sedimentary materials, to give a very accurate picture of structural conditions. This work, of which a brief account is given in this paper, has involved the development of new types of apparatus and also a large number of extensions of the earlier mathematical methods of treating the results.

Chief Requirements in Apparatus

The chief items to be developed, from the standpoint of apparatus, include the most sensitive possible means for recording vibrations and the most accurate methods possible for estimating the elapsed time between an explosion and the recording of the resulting vibration as it arrives at a distant receiving instrument. Such development involves a series of compromises, and is limited by certain natural factors that are extremely difficult to get around.

For example, take the question of the sensitivity of the recording device. The earth's surface, even at points well removed from surface disturbances, is in a state of continual vibration. This agitation probably proceeds from a number of causes, such as the action of wind on the surface, small changing stresses due to the gravitational action of the sun and the moon, temperature changes, barometric changes and the like. Some portion of the vibration probably proceeds from minute motions along the planes of active faults in the vicinity. Therefore, if we place a receiver of vibrations in a spot that is relatively quiet, and if we gradually increase the sensitivity of this receiver, we will eventually reach a point at which it will begin to record these various spontaneous vibrations.

If we wish to record, with this same receiver, the arrival of a train of elastic waves resulting from an explosion, it is quite obvious that this wave train must reach our recording device with an amplitude considerably greater than that of the average accidental vibrations. This requirement automatically determines the minimum amount of explosive that can be used at various distances from the receiver to generate a wave train of this required magnitude.

It is quite apparent that if we are trying to use our apparatus in a locality in which an unusually large amount of vibration is already present in the earth—such as would be expected in the vicinity of an oil field where a large number of pumps are in use, and occasional trucks are passing the location of our apparatus—the wave train arising from our explosion must be likewise much larger, in order to compete successfully with these accidental vibrations.

Therefore the "powder schedule" depends, among other things, on the amount of accidental vibration already present in the area in which we have to work.

It is evident that there is no object in increasing the sensitivity of a receiving apparatus indefinitely. If the receiver is sufficiently sensitive so that it responds noticeably to the accidental vibrations present in the earth, increasing this sensitivity can have no useful results, as such increase will simply magnify both the accidental vibrations and the wave trains from our explosions, without giving any advantage to the latter.

The matter of accuracy in the determination of time is also subject to similar natural limits. The explosion of a quantity of dynamite sends into the earth a complex vibration, having components at a large number of differing frequencies.

If we desire to do so, we can make a receiving device which is far more sensitive for some one of these vibration frequencies, and extremely insensitive for all others. It is obvious that if we select a very high frequency for such a device, and if a wave train at this frequency comes through the earth and reaches our receiver, it should be possible to tell very precisely just when such an abrupt wave starts to agitate the apparatus.

On the other hand, if we adapt our receiving device particularly to the reception of some one of the lower frequency vibrations, the arrival of this vibration as recorded on our records will be very gradual, and it will be difficult to tell the precise instant when the vibration starts. Therefore, if we desire to produce an accurately timed record, it would seem highly desirable to adapt our receiver to operate at a relatively high vibratory frequency. Unfortunately, however, an explosion of dynamite in any reasonable quantity, if analyzed into its component parts, will be found to radiate a majority of its energy at very moderate frequencies, and to put out very little energy at the higher frequencies. Further, the transmitting characteristics of the earth must be taken into account. Here again, we find that the transmission of vibrations at the lower frequencies is far better than the higher ones. Therefore, if we are to produce a workable apparatus, we must operate it at a compromise frequency, sufficiently high to be readily timed, but not so high as to be beyond the range where vibrations can be efficiently generated by explosives or efficiently transmitted by the earth.

After a rather extensive study of all of the factors involved, apparatus has been developed which seems to possess the best attainable working

FIG. 1.—INSTRUMENT TRUCK IN ACTION.

characteristics. The sensitivity is so great that it has never been possible to operate the apparatus at more than 10 per cent. of its maximum capacity, without reaching the limit at which accidental vibrations are present

FIG. 2.—DYNAMITE TRUCK FIRING A CHARGE.

The operator at the left is holding the safety switch closed. The actual firing of the charge is done, over the telephone line, from the instrument truck several miles away.

in the earth. As a measure of this sensitivity, vibrations considerable smaller in amplitude than the diameter of the hydrogen molecule are readily recorded with the new apparatus. The time of arrival of wavy

trains can also be determined with this equipment to the nearest one or two thousandths of a second.

FIG. 3.—RECEPTOR BEING LOWERED INTO PLACE.

FIG. 4.—TYPICAL FIELD PARTY.

FIELD OPERATION

The receiving and recording apparatus is carried in a specially built truck (Fig. 1), while the equipment for laying and firing the explosive

charges is carried in a second truck (Fig. 2). The recording truck commonly takes up a station in the area to be investigated, at which the vibration sensitive device, termed a "receptor" (Fig. 3), is buried in the earth. This receptor is usually placed several hundred feet from the instrument truck itself, to avoid disturbing sounds, and is buried about 6 ft. underground. A shielded electric cable runs from the receptor to the truck.

Within the instrument truck (Fig. 1) are the various amplifying devices, together with a photographic instrument for recording the earth vibrations. The truck also carries a dark room for developing

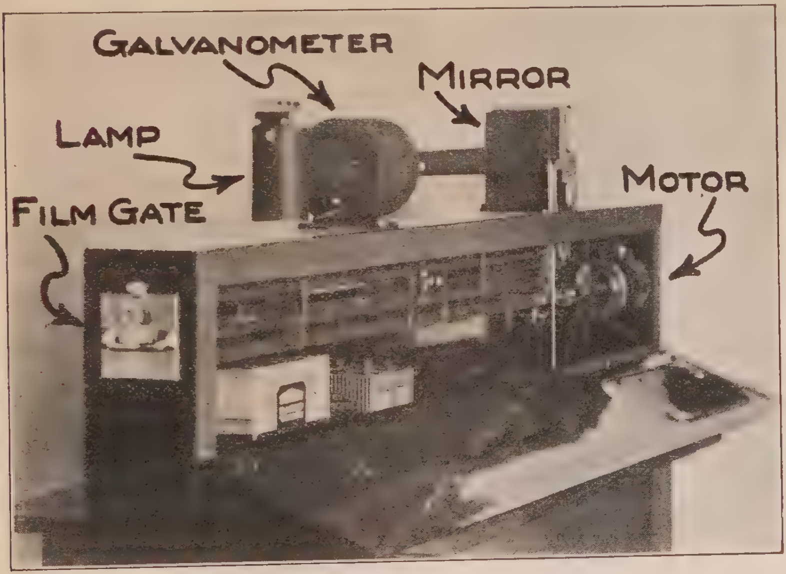

FIG. 5.—THE PHOTOGRAPHIC RECORDER.

these records, and a telephone equipment for communicating with the dynamite truck.

This second truck takes up its position at varying distances from the instrument truck, depending on the type of investigation in progress, and lays behind it as it proceeds an insulated wire, by means of which telephone conversations may be maintained in both directions. This wire also serves to carry the electrical signal which fires the explosive, and to carry back to the instrument truck a signal indicating when the explosion has taken place.

Radio control may be used in territory in which the laying of such a wire is not practical. The relative simplicity of a wire, however, together with the extreme ease of maintenance of the equipment, makes the employment of radio control a very doubtful advantage in most locations.

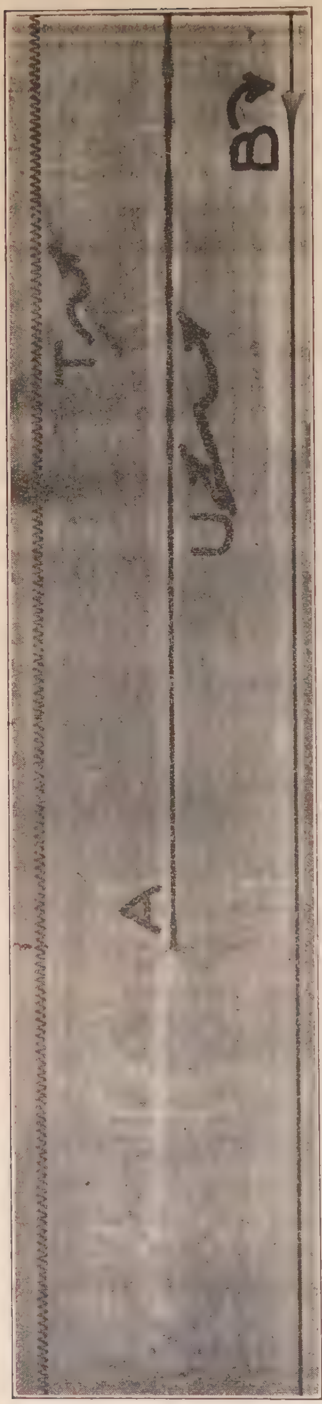

Fig. 6.—Record of earth vibrations. Accidental vibrations shown at U; waves due to explosion at A; record of timing fork at T; instant of explosion at B.

The field crew and equipment are shown in Fig. 4; *i. e.*, the party chief's car, the dynamite truck, the recorder (or "instrument") truck, and the operating personnel.

Fig. 5 shows the photographic recorder. Film in cut strips is introduced from the dark room into the film gate, and returned as records. Fig. 6 shows a record of earth vibrations.

Interpretation of Results

Before proceeding to a description of the adaptation of elastic-wave methods to such unconsolidated structures as we have in California, it may be well to review briefly some of the basic principles, discovered earlier in seismology, upon which this general type of elastic-wave work rests. An approximate description will suffice.

Wave travel may be considered as proceeding along straight lines, and refraction of these waves may be considered as occurring sharply at the boundary between the two media. While these conditions are not strictly true in practice, they simplify the problem of illustration.

Let us assume that we are conducting an examination in an area where the surface material having a velocity of V_1 lies in a layer of uniform thickness on top of suspected deeper material having an unknown but higher velocity V_2. Let us further assume that we desire to learn the thickness of the layer of surface material, by means of a simple elastic-wave exploration.

If we have set up a receiver such as is shown at R, Fig. 7*a*, and successfully fire explosions at the points *a*, *b*, *c*, *d*, *e*, *l*, *g*, we will obtain, for each explosion,

a corresponding elapsed time, or travel time, for the *first arriving waves.* We can now plot the distances from the receiver to the respective explosions against the corresponding travel times for the first waves from each, obtaining such a graph as that shown in Fig. 7*b*.

Referring to Fig. 7*b*, it will be seen that the explosive waves from the near-by points *a*, *b*, *c* arrived in proportionate time. That is, the *b* wave took twice as long as the *a* wave to come twice the distance, and the *c* wave three times as long to come three times the distance. When we reach the *e*, *f*, and *g* waves, however, we find a departure from this ratio, and the establishment of a faster travel rate. Thus we can be sure that, however the waves reached the receiver, they certainly did not reach it by

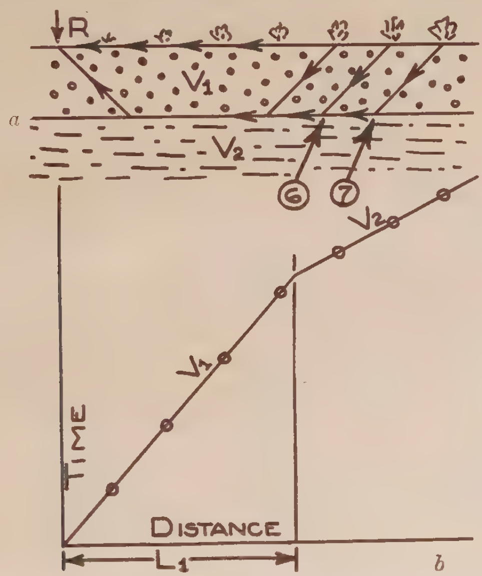

FIG. 7.—TRAVEL TIME FOR FIRST ARRIVING WAVES.

traveling in a medium with the same velocity as that which carried the *a*, *b* and *c* waves. We may readily assume, instead, that the waves traveled, for part of their path, in a medium whose velocity was higher than that which carried the *a*, *b* and *c* waves.

Hence the arrival times, for near-by explosions, in our experimental case, are proportionate to the distances from these explosions to the receiver, but the more distant explosions arrive in less than this proportionate time.

If we assume that the surface material, with its uniform velocity V_1, is underlaid by a horizontal stratum of another material, having a higher velocity V_2, we may arrive at a possible explanation of the graph Fig. 7.

Referring to the explosions *f* and *g*, these shocks radiate waves in all directions. Some part of each explosion would therefore pass along the

surface directly to R. Another portion, however, might travel down from the explosion and strike on the lower medium at such an angle that it would refract into the upper surface of that medium. Here it would travel at a higher velocity than in the surface material, and eventually some portion of the energy could be refracted back towards the surface, and reach the receiver.

Waves traveling by this indirect path, partly in one medium and partly in another, would be able under some conditions to reach the receiver ahead of the direct surface waves, in spite of the shorter path of the latter. We cannot assume that the refracted waves will remain in their full strength in the lower medium until they have reached the appropriate point near the receiver, and that then they will be refracted out again in their entirety and reach the receiver. We should rather expect, once the energy has been refracted into the lower medium, that a small but continuous amount of it would be refracted back into the first medium as the wave progressed, until the wave had reached a point where the refracted portion passing into the upper medium would strike the receiver.

Also, assuming equal angles of incidence, the waves from f will strike the lower medium at 6 and those from g at 7. The distance f-g will then equal 6-7. Thus we see that the only change we have made in the total wave path by moving the explosion from f to g is to add an amount of the lower material to the path. This increment of path will result in an increment of time on the graph, and the ratio of the two, expressed in δ_1/δ_t will be the velocity V_2 in the lower medium.

Referring to Fig. 8, Snell's law for refraction on passage from one medium to another may be stated as

$$\sin A/\sin B = V_1/V_2$$

Where

A = angle of incidence, measured from normal
B = angle of refraction, measured from normal
V_1 = velocity of waves in first medium
V_2 = velocity of waves in second medium

For the condition shown in Fig. 8b where $B = 90°$ and the refracted ray skims the surface between the media, $\sin B = 1$, and the above expression simplified to

$$V_1/V_2 = \cos \alpha$$

Thus, if we have two media, for which we can ascertain the wave velocities V_1 and V_2, we can then determine the angle α at which waves must strike the interface between the media in order to be refracted at a grazing angle along the interface.

V_2 must always be greater than V_1, for otherwise V_1/V_2 would become greater than unity. As a practical matter, if the velocity in the lower

medium is less than that in the upper, the beam will be bent toward the normal, and not toward the grazing angle necessary if the phenomena here described are to take place. In this case, the time travel graph will show only the first slope, and no break in the line will occur.

From the foregoing, we have concluded that, if a stratum of homogeneous material, of uniform thickness, and of velocity V_1, is underlaid by a second stratum of uniform velocity V_2, we may cause explosions at the surface of the first medium, and the resultant vibrations will be able to reach a distant receiver by one of the two following paths:

1. Directly along the upper surface by the shortest path, traveling at a velocity V_1.

2. Downward through the upper medium, at a velocity V_1, then bent by refraction, and skimming along in the upper surface of the lower medium at velocity V_2, part of the energy being continually refracted

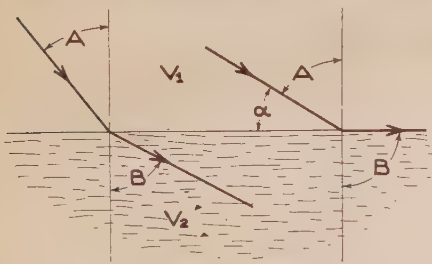

Fig. 8.—Illustrating Snell's law of refraction.

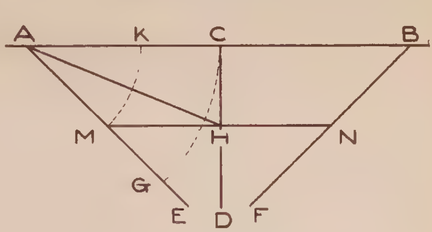

Fig. 9.—Graph of expression for depth.

back into the upper medium until the wave front has progressed to a point where the part refracted upward can arrive at the receiver by a final path through the upper medium at velocity V_1. We have further concluded that the angle α at which the energy is refracted into or out of the lower medium (measured from the interface in both cases) is determined as follows:

$$V_1/V_2 = \cos \alpha$$

We have arrived also at the following additional conclusions with respect to the travel-time graph, plotted from the distances from our various explosions to the receiver, and the corresponding travel times of the first arriving waves:

3. The first arriving waves from near-by explosions will take the direct path along the surface, as requiring the shortest possible time. These arrivals will plot a straight line, where $\delta_1/\delta_t = V_1$.

4. For distant explosions, however, the longer path described in conclusion 2 will be the shortest, in point of time, since the longer distance will be more than compensated by the higher velocity in the second medium. Thus, the *first* waves arriving from distant explosions will come

partly through the lower medium, while the other waves, traveling along the surface path, will arrive later. In the case of such distant explosions, the slope of the second branch of the graph plotted from them will depend entirely on V_2, as follows:

$$\delta_1/\delta_t = V_2$$

Therefore we may conclude that there must be some distance from the receiver at which waves from an explosion would arrive simultaneously by both paths 1 and 2. Graphically, we see that this condition is fulfilled by an explosion at distance L_1, where L_1 is the distance at which the travel-time graph changes in slope. Assume that we have an upper and a lower medium, as described, and that by the procedure of recording the first arrival times of waves from a succession of explosions, we have developed a travel-time graph from which V_1, V_2, may be obtained, we may then develop graphically an expression for depth, as shown in Fig. 9.

Construction of Graph

Lay off $AB = L_1$. Determine value of α ($V_1/V_2 = \cos\alpha$). Construct angles BAE and ABF each $= \alpha$. Draw CD normal center AB. With center A, radius AC, draw arc cutting AE at G. Erect GH normal AG. Through H draw MN parallel AB. With center A and radius AM, draw arc cutting AB at K. Join AH. The line MN now represents the lower stratum, at depth CH.

Proof

(1) Travel time AB = time $AMNB$ (AB chosen $= L_1$)
(2) Therefore, dividing by 2, time AC = time AMH
(3) Time AK = time AM (equal distance in same medium)
(4) Time KC = time MH (from 2 and 3)
(5) Time KC = (dist. KC) $/V_1$, and time MH = (dist. MH)$/V_2$
(6) Therefore $KC/MH = V_1/V_2$ (4 and 5)
(7) But $V_1/V_2 = \cos\alpha$. Therefore $KC/MH = \cos\alpha$
(8) $KC = MG$, by construction. Therefore $MG/MH = \cos\alpha$

Referring to Fig. 9, it will be seen that angle $GMH = \alpha$, and that $MG/MH = \cos\alpha$.

Further, empirically, it is impossible to draw any other line parallel to MN, draw two arcs similar to KM and CG, and thereby construct a triangle similar to GMH, with the angle $MGH = 90°$. Therefore the depth CH is the only depth that will satisfy the conditions $AB = L_1$ and $\alpha = \cos^{-1}(V_1/V_2)$.

Development of Formula

While the foregoing gives a construction and proof for the fact that the depth CH is the depth of the surface stratum under the assumed

conditions, it does not furnish a readily applicable formula for solution. Such a formula will now be developed.

From construction, $AC = AG$ and $CH = GH$
Therefore angle CAH = angle HAG (similar triangles)
But $CAH + HAG = \alpha$
Therefore $CAH = \frac{1}{2}\alpha$
$CH = AC \cdot \tan CAH = AC \cdot \tan \frac{1}{2}\alpha$
$AC = \frac{1}{2}L_1$
Therefore CH = depth $= L_1 \cdot \frac{1}{2} \tan \frac{1}{2}\alpha$

This formula will serve to determine the depth to any stratum where the second velocity is higher than the first, provided that the interface between the strata is parallel to the surface. It will also operate where the interface is not parallel to the plane of the surface, provided the line of shots is placed parallel to the line of intersection of the buried stratum with the surface (that is, parallel to the strike of the buried stratum). In this latter case, the depth obtained from the solution will not be the vertical distance to the plane of the interface, but rather the distance from any point on the line of shots to the interface, measured normal to the plane of the latter.

This formula does *not* apply in cases where the submerged stratum either dips or rises in the line of the shots, nor does it apply in cases where the velocity of propagation of waves in either medium is other than constant. Given a sufficient number of measurements taken on any certain territory, however, it is possible to determine whether the lower stratum is dipping or rising in the line of the shots, or whether the velocity in either medium is varying, and to derive expressions permitting corrections to be applied for these conditions, so that final solutions may be obtained to a fair degree of accuracy in most cases.

Application to Unconsolidated Sedimentary Formations

The elementary case just considered, where a medium overlies one of higher velocity, may by appropriate means be extended to include the delineation of a succession of strata, where the velocity of any layer is always greater than that of any layer above it. Such a condition is shown in Fig. 10*a*, where a succession of layers with an increasing range of velocities, V_1, V_2, V_3, V_4, V_5, when examined by the use of a series of elastic-wave paths, may be computed as to thickness from the factors obtained from the travel-time graph, Fig. 10*b*.

It will readily be seen that, if the layers become thin enough, and the changes in velocities become small enough, we will approach a condition where the true travel-time graph ceases to be a series of straight lines and becomes a gradual curve, denoting a velocity increasing as a continuous rather than a discontinuous function of depth.

Fig. 11 shows a condition that is frequently met with in formations such as are encountered in the San Joaquin Valley in California, where considerable experience has been obtained in the use of the apparatus here described. A surface layer of recently deposited unconsolidated valley fill overlies a planed-off fold of older sedimentary material. The

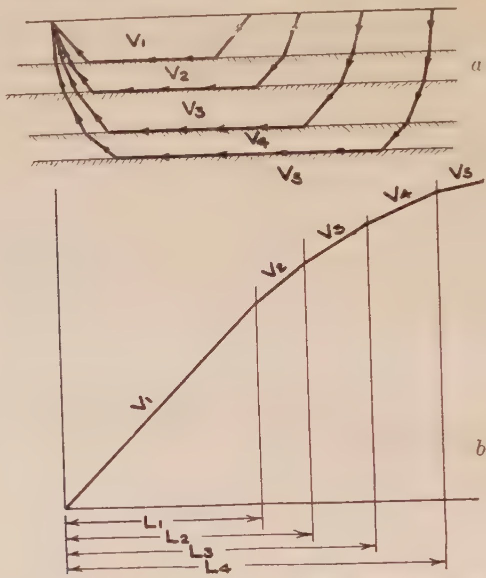

FIG. 10.—*a*. SUBSURFACE CROSS-SECTION SHOWING WAVE PATHS THROUGH SUCCESSIVE LAYERS OF MATERIAL OF INCREASING VELOCITY. *b*. TIME TRAVEL GRAPH FOR CONDITION SHOWN IN *a*.

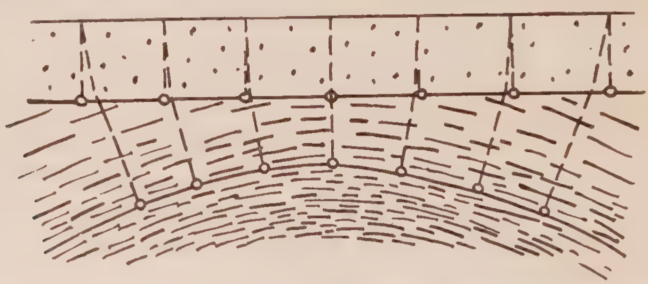

FIG. 11.—SURFACE LAYER OF RECENTLY DEPOSITED UNCONSOLIDATED VALLEY FILL OVERLYING PLANED-OFF FOLD OF OLDER SEDIMENTARY MATERIAL.

velocity of this lower material increases as a continuous function of its depth, as originally deposited.

By means of a series of elastic-wave determinations made along lines normal to the section, and at the indicated points, it is readily possible to determine: (1) the depth of the unconsolidated valley fill, at each observation station, and (2) the depth in the lower material, at which the velocity has risen to any assigned value. This velocity would seem to

depend principally on the degree of consolidation of the formation which, again, is probably a function of the pressure exerted by the overburden during the period of consolidation.

Hedberg[1] has worked out probable relations between compaction and depth which are quite consistent with such information as the writer has obtained on the relation of velocity to depth. From the experience so far gained in this work, it would seem feasible, with proper apparatus and methods, to attempt the delineation of such structures as those with which Hedberg was principally concerned; namely, structures formed by settling of sedimentary materials over granite knobs and the like.

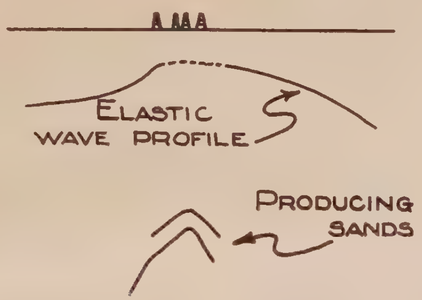

FIG. 12.—PROFILE TAKEN ACROSS LOST HILLS OIL FIELD, CALIFORNIA.

As an example of the accuracy with which structural conditions may be determined, Fig. 12 is offered. This represents an actual profile taken across the Lost Hills oil field, in the San Joaquin Valley region, near Bakersfield. The producing sands in this field, as determined from the logs of a large number of wells, will be seen to correspond very strikingly with the elastic-wave profile. When it is considered that the elastic-wave determinations apply to formations of too low a degree of consolidation to be recognized or correlated by core drilling, and that no other means of recognizing structural conditions at these depths could be successfully employed, the utility of elastic-wave exploration for oil structures in similar areas will be readily appreciated.

CONCLUSIONS

The extension of elastic-wave methods of exploration to include the delineation of relatively unconsolidated sedimentary formations, in which abrupt changes in characteristics are absent, has hitherto been regarded as unpractical. By appropriate design of the apparatus, principally to obtain greater sensitivity, and also through extensions of the mathematical treatment of results, it now seems possible to attempt work on such formations, under suitable conditions with reasonable hopes for valuable results.

[1] H. Hedberg: Effects of Gravitation Compaction on Structure of Sedimentary Rocks. *Bull.* Am. Assn. Petr. Geol. (1926) **10**, 1035.

INDEX

(Note: In this index the names of authors of papers and discussions and of men referred to are printed in SMALL CAPITALS, and the titles of papers in *italics*.)

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